4. \( x' = y, \ y' = x \): \( a(e^t + e^{-t})' = a(e^t - e^{-t}) \) and \( (a(e^t - e^{-t}))' = a(e^t + e^{-t}) \) (where we are assuming that \( a \) is a constant); so this pair of functions of \( t \) satisfies the differential equations in \( x \) and \( y \).

5. \( x' = y, \ y' = -x \): \( (a \sin t)' = a \cos t \) and \( (a \cos t)' = -a \sin t \), so this pair of functions is a solution to the system of differential equations.

7. \( x' = x, \ y' = -y \): \( (ae^t)' = ae^t \) and \( (be^{-t})' = -be^{-t} \), so this pair of functions is a solution to the systems of differential equations.

9. (a) From \((1,0)\) the flow line goes straight up, but as soon as it leaves that point, there is a movement to the right: (III). The arrows are tricky: they point up on the through \((1,0)\), right on the one through \((0,1)\), down on the one through \((-1,0)\), left on the one through \((0,-1)\), away from the origin on the two halves of the line \(y = x\), and toward the origin on the two halves of the line \(y = -x\). (b) From \((1,0)\) the flow line goes straight up, but as soon as it leaves that point, there is a movement to the left: (I), with arrows counterclockwise. (c) From any point the movement is directly away from the origin: (II) (d) The movement is roughly circular counterclockwise, like (b), but whenever we are not exactly on the \(x\)-axis (e.g., at \((0,5)\)), there is a small additional movement away from the \(x\)-axis: (V) (spiraling away from the origin). (e) The movement is roughly circular counterclockwise, like (b), but whenever we are not exactly on the \(x\)-axis (e.g., at \((0,5)\)), there is a small additional movement toward the \(x\)-axis: (VI) (spiraling toward the origin). (f) By elimination: (IV). Or better, all the vectors in the field have slope 1, so all the flow lines have slope 1. When \(x>y\), they go right and up; when \(x<y\), they go down and left; so the line \(x=y\) divides those that go in one direction from those that go in the other (and along that line the vector field is \(\vec{0}\)).