

Section 1.6: The sum of positive divisors

Definition 1.20 For n a natural number, let

$$\sigma(n) = \sum_{1 \leq d \leq n \text{ and } d|n} d.$$

NB. $\sigma(n)$ sums the positive divisors of n .

Theorem 1.21 Let $n > 1$ and write $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$. Then

$$\sigma(n) = \prod_{i=1,2,\dots,r} \left(\frac{p_i^{k_i+1} - 1}{p_i - 1} \right).$$

Corollary Let m and n be natural numbers with $(m, n) = 1$. Then $\sigma(mn) = \sigma(m)\sigma(n)$.