Section 1.7: Functions

Definition 1.22 Let A and B be sets. A function from A to B is any assignment of objects in A to objects in B so that for each a in A, the function assigns a unique object in B to a.

Write $f : A \to B$ to denote a function from A to B and $f(a)$ to denote the unique object in B that f assigns to a.

If $A = \mathbf{N}$, then we say that f is a number theoretic function.

Definition 1.23 Let $f : A \to B$ be a function.

(i) We say f is one-to-one if

$$a_1 \neq a_2 \text{ in } A \text{ implies } f(a_1) \neq f(a_2) \text{ in } B.$$

(ii) We say f is onto if for each b in B, there is an a in A with $f(a) = b$.

Definition 1.24 Let $f : \mathbf{N} \to B$ be a number theoretic function.

(i) We say f is multiplicative if $f(mn) = f(m)f(n)$ whenever $(m, n) = 1$.

(ii) We say f is completely multiplicative if $f(mn) = f(m)f(n)$ for any pair of natural numbers, m and n.