Section 1.8: The number of positive divisors

Definition 1.25 Let $\nu : \mathbf{N} \to \mathbf{N}$ be the number theoretic function with $\nu(n)$ equal to the number of positive divisors of n. That is,

$$\nu(n) = \sum_{1 \leq d \leq n \text{ and } d|n} 1.$$

Theorem 1.26 Let $n > 1$ and write $n = p_1{}^{k_1} p_2{}^{k_2} \ldots p_r{}^{k_r}$. Then

$$\nu(n) = \prod_{i=1,2,\ldots r} (k_i + 1).$$

Corollary The function ν is multiplicative.

Theorem 1.27 For $n > 1$,
$$n^{\frac{\nu(n)}{2}} = \prod_{1 \leq d \leq n \text{ and } d|n} d.$$