Group Assignment 2 Brian Dinneen, Heather Clark, Shirley Han, Kellen Myers

7.11) Let $f : S \to T$ and $g : T \to U$.
(a) If $g \circ f$ is one-to-one, must both f and g be one-to-one?
(b) If $g \circ f$ is onto, must both f and g be onto?

i) If $g \circ f$ is one-to-one, f must be one-to-one.

Consider $f(s_1) = f(s_2)$ for $s_1, s_2 \in S$
$g(f(s_1)) = g(f(s_2))$ since g is a function, and $f(s_1),\ f(s_2) \in T$
$s_1 = s_1$ since $g \circ f$ is one-to-one
Thus, f is one-to-one

ii) If $g \circ f$ is onto, g must be onto.

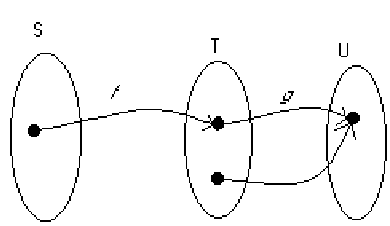
Since $g \circ f$ is onto, $\forall\ u \in U, \exists\ s \in S \ni g(f(s)) = u$
Take $t = f(s)$, where, definitively, $t \in T$
So, $\forall\ u \in U, \exists\ t \in T \ni g(t) = u$
And g is onto.

iii) If $g \circ f$ is one-to-one, g is not necessarily one-to-one.
iv) If $g \circ f$ is onto, f is not necessarily onto.

Consider such functions and sets defined as follows:
$S = \{s_1\}$
$T = \{t_1, t_2\}$
$U = \{u_1\}$

$f(s_1) = t_1$
$g(t_1) = u_1$
$g(t_2) = u_1$
$g \circ f(s_1) = u_1$

or, in set notation (see pages 59-60 of the textbook):
$f = \{(s_1, t_1)\}$
$g = \{(t_1, u_1), (t_2, u_2)\}$
$g \circ f = \{s_1, u_1\}$



There is no $s \in S$ such that $f(s) = t_2$.
$g(t_1) = u_1 = g(t_2)$ but $t_1 \neq t_2$.
In this examle, $g \circ f$ is both one-to-one, and onto. However, f is not onto (iv), and g is not one-to-one (iii).