Unit 6: Probability

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10/06/2011

Chapter 13: What is a probability

Definition

The probability that an event happens is the percentage of the time it happens when the same process is repeated many times in identical conditions.

Examples

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- Coin toss

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- Coin toss
- Die roll

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- Coin toss
- Die roll
- Drawing a card from a pile

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- Coin toss
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- Picking a ball from a box (lottery)

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- Coin toss
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- Flipping a coin 1000 times, how many tails do you expect?

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- Coin toss
- Die roll
- Drawing a card from a pile
- Picking a ball from a box (lottery)
- Flipping a coin 1000 times, how many tails do you expect?
- Rolling a die 6000 times, how many 2's do you expect?

Examples

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- Coin toss
- Die roll
- Drawing a card from a pile
- Picking a ball from a box (lottery)
- Flipping a coin 1000 times, how many tails do you expect?
- Rolling a die 6000 times, how many 2's do you expect?
- Rolling a die 6000 times, how many 1,3,4,5, and 6s do you expect?

Rules

Fact (Rules of probability)

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2. *Prob (event A happens)=1 - Prob (event A does not happen)*
3. $$Prob(X) = \frac{\#\ of\ ways\ X\ can\ occur}{total\ number\ of\ outcomes\ (equally\ likely)}.$$

Conditional probabilities

Definition

The probability of X given that Y happened is

$$P(X \mid Y) = \frac{\#\text{ of ways } X \text{ and } Y \text{ happen}}{\#\text{ of ways } Y \text{ occurs}}.$$

Example

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Suppose that we have an urn containing 3 red balls and 4 green balls.

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- What is the probability that you extract one red ball?

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- What is the probability that you extract one red ball?
- What is the probability that you extract one red ball if you extract first a green ball?

Independent Events

Definition

We say that X and Y are independent if

$$P(Y \mid X) = P(Y).$$

Example

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Suppose that you are flipping a coin 5 times.

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Suppose that you are flipping a coin 5 times.

- What is the probability that the 4^{th} trial is head?

Example

Example

Suppose that you are flipping a coin 5 times.

- What is the probability that the 4^{th} trial is head?
- What if you know that the first four trials were all tails?

Chapter 14: AND Rule

Fact (AND Rule)

$$\begin{aligned} P(X \text{ and } Y) &= P(X) \cdot P(Y \mid X) \\ &= P(Y) \cdot P(X \mid Y) \end{aligned}$$

Examples

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Find the probability of each of the following events:

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Find the probability of each of the following events:

- Drawing the ace of clubs and then the two of diamonds.

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Find the probability of each of the following events:

- Drawing the ace of clubs and then the two of diamonds.
- Drawing a club and then a heart.

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- Drawing five clubs

Examples

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Find the probability of each of the following events:

- Drawing the ace of clubs and then the two of diamonds.
- Drawing a club and then a heart.
- Drawing five clubs
- Rolling 2 sixes

The paradox of the Chevalier de Méré

Example

The Chevalier de Méré made the following computations:

- Rolling a die
  - In one roll of a die, I have $1/6$ of a chance to get an ace (a one).
  - So in 4 rolls, I have $4 \times 1/6 = 2/3$ of a chance to get at least one ace (a least a one)

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- Rolling a pair of dice
  - In one roll of a pair of dice, I have $1/36$ of a chance to get a double-ace (two ones).
  - So in 24 rolls, I must have $24 \times 1/36 = 2/3$ of a chance to get at least one double-ace.

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  - So in 24 rolls, I must have $24 \times 1/36 = 2/3$ of a chance to get at least one double-ace.

Do you agree with his statements?

OR Rule

Fact (OR Rule)

$$P(X \text{ or } Y) = P(X) + P(Y) - P(X \text{ and } Y).$$

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Find the probability of the following events

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Find the probability of the following events

- You draw a Heart or a Queen.

Examples

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Find the probability of the following events

- You draw a Heart or a Queen.
- You pick a two or a five.

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Find the probability of the following events

- You draw a Heart or a Queen.
- You pick a two or a five.
- You pick a red card or a jack.

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Find the probability of the following events

- You draw a Heart or a Queen.
- You pick a two or a five.
- You pick a red card or a jack.
- You pick an Ace and then a King.

Examples

Examples

Find the probability of the following events

- You draw a Heart or a Queen.
- You pick a two or a five.
- You pick a red card or a jack.
- You pick an Ace and then a King.
- You roll 0 sixes in 5 rolls

Mutually exclusive events

Definition

We say that X and Y are mutually exclusive if

$$P(X \text{ and } Y) = 0.$$

Example

Example

Suppose that you role a die. What is the probability that you get a 2 and a 3?

What if it is hard to find the probability of an event?

Fact

Keep in mind the following: If the probability that an event A happens seems hard to compute, then try to find the probability that A does not happen, that is, $1 - P(A)$.

Chapter 15: Binomial Formula

Example

Chapter 15: Binomial Formula

Example

- How many ways can we arrange 3 books on a table?

Chapter 15: Binomial Formula

Example

- How many ways can we arrange 3 books on a table?
- How many ways can we arrange 5 balls with 3 red and 2 green?

Chapter 15: Binomial Formula

Example

- How many ways can we arrange 3 books on a table?
- How many ways can we arrange 5 balls with 3 red and 2 green?
- How many ways can a class of 21 students be divided into a group of 10 and another group of 11?

Factorials and the binomial coeffiecient

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- n-factorial is
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- The binomial coefficient
$$\frac{n!}{k!(n-k)!}$$

More examples

Example

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Example

- How many ways can 21 dice come up with 10 sixes?

More examples

Example

- How many ways can 21 dice come up with 10 sixes?
- How many way can 4 dice come up with 4 sixes?

Binomial Formula

Fact

If p is the probability of success on each trial and we do n independent trials, then

$$P(k \text{ successes out of } n \text{ trials}) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}.$$

Example

Example

Example

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- When drawing 6 balls with replacement from a box with 8 green and 5 red balls, what is the probability of getting 3 red and 3 green?

Example

Example

- When drawing 6 balls with replacement from a box with 8 green and 5 red balls, what is the probability of getting 3 red and 3 green?
- If 10 draws are made with replacement, what is the probability that two or less balls are red?

Example

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Example

Example

- What is the probability of rolling 3 sixes on 6 rolls?

Example

Example

- What is the probability of rolling 3 sixes on 6 rolls?
- What is the probability of rolling at least 4 sixes on 6 rolls?