# Unit 7: The Law of Averages

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# Chapter 16: Law of Averages

**Fact (Commonly said)**

*In the long run, the expected result will happen; or the luck evens out.*

# Law of Averages

Fact (Actually)

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*The formula says that*

$$\textit{Actual value} = \textit{Expected} + \textit{chance error}.$$

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*So you are likely to be farther from expected, but down in % terms:*

- *If you toss a coin 1000 times and there are 550 heads, then the error is 50 and the relative error is 5%.*
- *If you toss a coin 1,000,000 times and there are 501,000 heads, then the error is 1000 and the relative error is 0.1%.*

# Law of Averages

**Example**

Question: if you play a basketball game against Kobe Bryant for \$1 million, do you want to play 1 basket or 20 baskets?

# The Sum of Draws

**Example**

Suppose that we have a box containing the numbers 1, 2, 3, 4, 5, and 6. Extract 100 numbers *with replacement* from the box. What do you expect their sum to be?

# Casino play

**Example**

Recall that in a casino game there are 18 reds, 18 blacks, and a 0 and 00. Suppose that you are playing roulette and you bet one dollar for red. How much do you expect to win if you bet 100 times? Is this a fair game?

# Casino play

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Recall that in a casino game there are 18 reds, 18 blacks, and a 0 and 00. Suppose that you are playing roulette and you bet one dollar for red. How much do you expect to win if you bet 100 times? Is this a fair game?

## Fact

*To answer these questions (and the previous example) we can use the "box model".*

# Box model

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**Fact**

- *Box models form a useful tool to study events that repeat a lot.*
- *Turn any repeated independent random process into drawing from a box.*
- *They summarize the key ingredients of the problem.*
- *They analyze chance variability.*

# Examples

**Example**

Make a box model for

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- Roll a die and sum results

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- Roll a die and count fours

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- Draw a card and count the hearts

# Examples

## Example

Make a box model for

- 50 flips of a coin counting heads.
- Roll a die and sum results
- Roll a die and count fours
- Draw a card and count the hearts
- Multiple choice exam

# Chapter 18: Expected Value

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# Chapter 18: Expected Value

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- What is the expected number of heads if you toss a coin when $n = 10, 100, 1000$?
- What is the expected number of fours if you roll a die $n = 60, 600$?
- What is the expected number of even values if you roll a die $n = 60, 600$?

# Expected Value

**Definition (General Formula)**

The expected value of a repeated independent process is

$$\mathrm{EV}_{\mathrm{sum}} = n \cdot \mathrm{AVG}_{\mathrm{box}}.$$

# Example

## Example

Find the expected value if you roll a die and sum the results for $n = 1, 10, 100$.

# Spread from EV (SE)

**Fact**

*Law of Averages says that the spread from EV (SE) is proportional to* $\sqrt{n}$*:*

$$SE = \sqrt{n} \cdot SD_{box}.$$

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## Fact (Useful formula)

*If the box contains just two numbers then there is a shortcut for the SD of the box:*

$$SD_{box} = (high - low)\sqrt{(fraction\ of\ high) \cdot (fraction\ of\ low)}$$

# A box model for the roulette

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- What is the average of the box?

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- There are 18 winning tickets: we put in the box 18 tickets with +1.
- There are 20 losing tickets: we put in the box 20 tickets with -1.
- What is the average of the box?
- What is the spread?

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- Suppose that you roll a die and you win 2 dollars if 1 and 6 show up and you lose 1 dollar if 2, 3, 4, or 5 show up. Would you play this game?

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- Suppose that you roll a die and you win 2 dollars if 1 and 6 show up and you lose 1 dollar if 2, 3, 4, or 5 show up. Would you play this game?
- In roulette, betting on four numbers pays 8 to 1. What do you expect to win after 100 plays if you bet 1 dollar? What is the expected number of wins? Find the spread for both questions.