Unit 7: The Law of Averages

Marius Ionescu

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Chapter 16: Law of Averages

Fact (Commonly said)

In the long run, the expected result will happen; or the luck evens out.

Law of Averages

Fact (Actually)

There are two parts to the law of averages:

1. *The absolute error goes up as the number n of trials increases*
2. *The relative (%) error goes down as n increases*

The formula says that

$$\textit{Actual value} = \textit{Expected} + \textit{chance error}.$$

Law of Averages

Fact

So you are likely to be farther from expected, but down in % terms:

- *If you toss a coin 1000 times and there are 550 heads, then the error is 50 and the relative error is 5%.*
- *If you toss a coin 1,000,000 times and there are 501,000 heads, then the error is 1000 and the relative error is 0.1%.*

Law of Averages

Example

Question: if you play a basketball game against Kobe Bryant for \$1 million, do you want to play 1 basket or 20 baskets?

The Sum of Draws

Example

Suppose that we have a box containing the numbers 1, 2, 3, 4, 5, and 6. Extract 100 numbers *with replacement* from the box. What do you expect their sum to be?

Casino play

Example

Recall that in a casino game there are 18 reds, 18 blacks, and a 0 and 00. Suppose that you are playing roulette and you bet one dollar for red. How much do you expect to win if you bet 100 times? Is this a fair game?

Fact

To answer these questions (and the previous example) we can use the "box model".

Box model

Fact

- *Box models form a useful tool to study events that repeat a lot.*
- *Turn any repeated independent random process into drawing from a box.*
- *They summarize the key ingredients of the problem.*
- *They analyze chance variability.*

Examples

Example

Make a box model for

- 50 flips of a coin counting heads.
- Roll a die and sum results
- Roll a die and count fours
- Draw a card and count the hearts
- Multiple choice exam

Chapter 18: Expected Value

Example

- What is the expected number of heads if you toss a coin when $n = 10, 100, 1000$?
- What is the expected number of fours if you roll a die $n = 60, 600$?
- What is the expected number of even values if you roll a die $n = 60, 600$?

Expected Value

Definition (General Formula)

The expected value of a repeated independent process is

$$\mathrm{EV}_{\mathrm{sum}} = n \cdot \mathrm{AVG}_{\mathrm{box}}.$$

Example

Example

Find the expected value if you roll a die and sum the results for $n = 1, 10, 100$.

Spread from EV (SE)

Fact

Law of Averages says that the spread from EV (SE) is proportional to $\sqrt{n}$:

$$SE = \sqrt{n} \cdot SD_{box}.$$

Fact (Useful formula)

If the box contains just two numbers then there is a shortcut for the SD of the box:

$$SD_{box} = (high - low)\sqrt{(fraction\ of\ high) \cdot (fraction\ of\ low)}$$

A box model for the roulette

Example

- Recall that we bet 1 dollar for red and we lose one dollars for black. In addition, we lose if 0 or 00 come out.
- There are 18 winning tickets: we put in the box 18 tickets with +1.
- There are 20 losing tickets: we put in the box 20 tickets with -1.
- What is the average of the box?
- What is the spread?

Other games

Example

- Suppose that you roll a die and you win 2 dollars if 1 and 6 show up and you lose 1 dollar if 2, 3, 4, or 5 show up. Would you play this game?
- In roulette, betting on four numbers pays 8 to 1. What do you expect to win after 100 plays if you bet 1 dollar? What is the expected number of wins? Find the spread for both questions.