Unit 8: Normal Distribution

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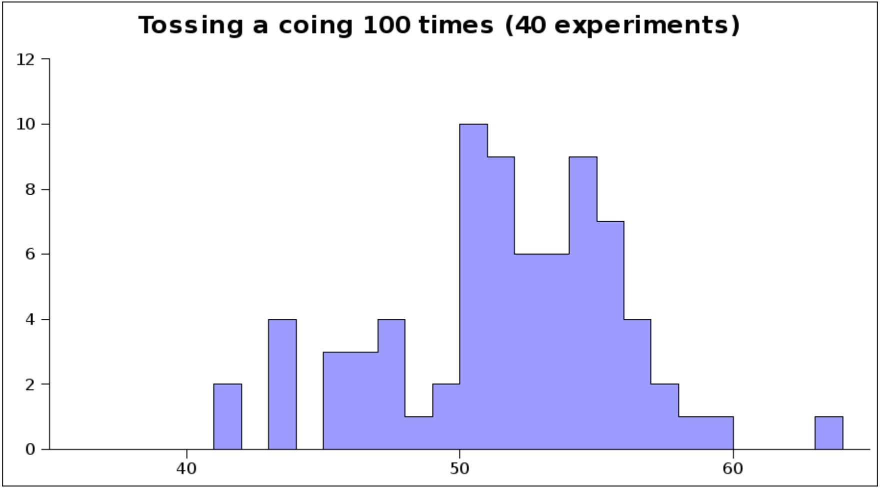
- Suppose that we repeat the experiment for 40 times, each time writing down the number of heads.
- We can create a histogram in which each bin contains the number of times a specific number showed up.

Tossing a coin



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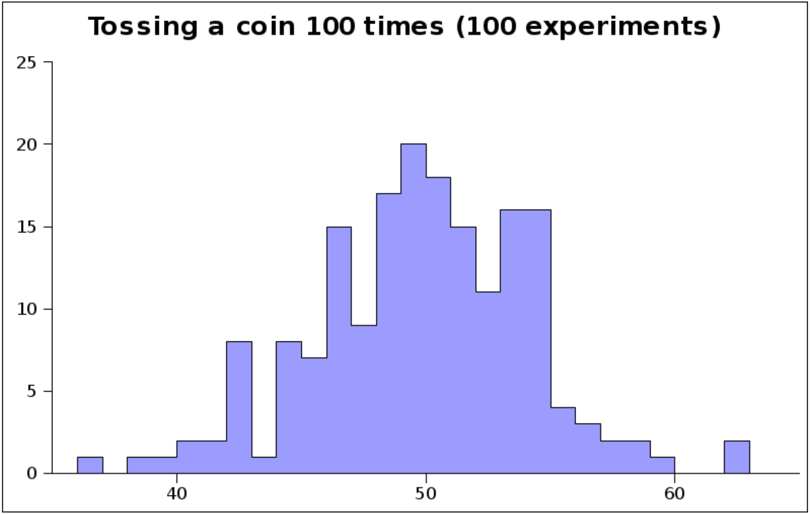

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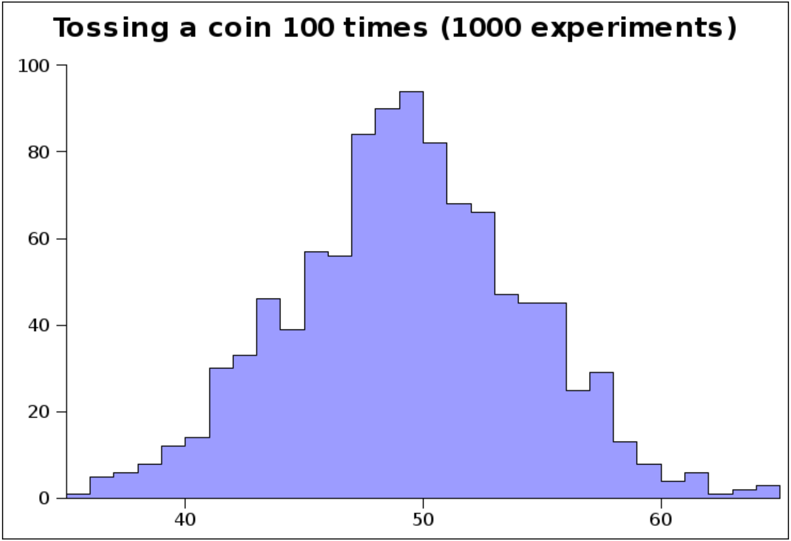

The Central Limit Theorem

Fact (The Central Limit Theorem)

When drawing at random with replacement from a box, the probability histogram from the sum will follow the normal curve, even if the contents of the box do not. The number of draws must be reasonable large and the histogram must be put into standard units.

Using the Central Limit Theorem

Fact (For **COUNTING/SUMMING**)

Suppose that X represents the outcome of counting/summing the outcome of repeating for a large number of times an experiment.

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$$P(N - 0.5 \leq X \leq N + 0.5).$$

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- *If we want to compute $P(X = N)$, where N is some integer, then we can approximate it with*

$$P(N - 0.5 \leq X \leq N + 0.5).$$

- *If we want to compute the probability that X is greater or equal than N and smaller or equal than M then we can approximate it with*

$$P(N - 0.5 \leq X \leq M + 0.5).$$

Using the Central Limit Theorem (cont'd)

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Using the Central Limit Theorem (cont'd)

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- *If we want to compute the probability that X is strictly greater than N and strictly smaller than M then we can approximate it with*

$$P(N + 0.5 \leq X \leq M - 0.5).$$

Using the Central Limit Theorem (cont'd)

Fact

- *If we want to compute the probability that X is strictly greater than N and strictly smaller than M then we can approximate it with*

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- *If we do not know if the endpoints N and M are or not included, then we can approximate it with*

$$P(N \leq X \leq M).$$

Important!

Fact

Don't forget the change to standard units!

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- Find the probability that there are exactly 50 heads in 100 tosses (without using the normal approximation)?

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- Find the probability that there are exactly 50 heads in 100 tosses (without using the normal approximation)?
- Use the normal approximation to find the probability that there are exactly 50 heads?

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Suppose that we make 100 draws with replacement from a box containing three zeros, a 1, and a 2. What is the chance that exactly 20 times we draw a 1? Give both the exact formula and the normal approximation.

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Suppose that you take a 100 multiple choice question exam. You get 4 points for each correct answer and 1 point off for a wrong answer (5 choices per question). If you guess at random, what is the chance of getting a score of 10 or more?

Example: Roulette revisited

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- What is the expected number of wins?

Example: Roulette revisited

Example

In roulette, betting on four numbers pays 8 to 1.

- What do you expect to win after 100 plays if you bet $1 each time?
- What is the expected number of wins?
- Assuming a normal distribution, what is the chance of breaking even or winning money?

Average of the draws

Example

Suppose that a city has an average income of $24,000 with SD of $10,000. A survey collects data at random from 400 households. What do we expected for the average of the survey?

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Using the Central Limit Theorem

Fact (For **AVERAGE**)

Suppose that X represents the outcome of taking the average of the outcome of repeating for a large number of times an experiment.

- *If we want to compute the probability that X is greater or equal than N and smaller or equal than M then we can approximate it with*

$$P(N \leq X \leq M).$$