Unit 11: More tests for averages (Chapter 27)

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   - Null hypothesis: the samples come from boxes with the same average.
2. Experiments:
   - Null hypothesis: the treatment does not affect the response.

Differences

Fact

To test the difference between two (independent) samples:

$$z = \frac{\textit{observed difference - expected difference}}{\textit{SE for difference}}.$$

What is SE for difference?

Fact

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If $a = SE$ of the first quantity and $b =$SE of the second quantity and the quantities are independent then

$$SE \text{ for difference} = \sqrt{a^2 + b^2}.$$

Example

Example (Two boxes)

Suppose that we have two boxes: the first one contains 1 red ball and 2 green balls; the second one contains 3 heads and 4 tails. 100 draws are made from box 1 and 200 from box 2. We count the number of red balls from box 1 and the number of heads from box 2. Find the EV and SE for the difference between the number of red balls and number of heads.

Example

Example (Differences between groups)

We ask a survey if they went out drinking this week. The sample has 100 men and 200 women in it. 46 of the men went out, 114 of women did. Does this mean that drinking behaviour is different between women and men this week?

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- What is the Null hypothesis?

Example

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Freshmen nationwide at public universities work an average of 12.2 hours per week for pay. Data is obtained from a sample of 4000 students and the SD is 21. At private schools another survey of 1000 students has an average of 9.2 hours/week with SD=9.9. Is this difference significant?

Experiments

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Fortunately the two cancel each other and we can still use the methods that we learned to study experiments.

Experiments

Example (Wheaties for breakfast)

General Mills sent a team to Colgate to study the effect of eating Wheaties for breakfast on the grades. Suppose that 499 students agree to participate. After the midterm 250 are randomly chosen for the treatment (Wheaties), and 249 for the control group (sugar pops).

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- The average of the final scores were 66 for the treatment group (with an SD of 21) and 59 for control (SD=20). What do you conclude?

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- The average of the final scores were 66 for the treatment group (with an SD of 21) and 59 for control (SD=20). What do you conclude?
- Suppose that the midterm scores were 61 for the treatment group (with SD of 20) and 60 for the control group (SD=19). Were the groups comparable at the beginning?

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General Mills sent a team to Colgate to study the effect of eating Wheaties for breakfast on the grades. Suppose that 499 students agree to participate. After the midterm 250 are randomly chosen for the treatment (Wheaties), and 249 for the control group (sugar pops).

- The average of the final scores were 66 for the treatment group (with an SD of 21) and 59 for control (SD=20). What do you conclude?
- Suppose that the midterm scores were 61 for the treatment group (with SD of 20) and 60 for the control group (SD=19). Were the groups comparable at the beginning?
- What if the midterm scores were 68 for treatment (with SD=21) and 59 for control (SD=20)?

χ^2-test

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Often a test of significance will involve many comparisons instead of one. For example:

- *Is a die loaded?*
- *Is the distribution of voting behavior random?*

χ^2-statistics

Fact

We test these situations using the χ^2-statistic

$$\chi^2 = \text{sum of } \frac{(\text{observed frequency} - \text{expected frequency})^2}{\text{expected frequency}}.$$

Example

Example

One evening you roll a die with your best friend. After a while you assume that he/she is using a loaded die. You record his/her last 60 rolls. They are as follow:

Die roll	Frequency
1	5
2	7
3	17
4	16
5	8
6	7

Is the die indeed loaded?

How to make a χ^2-test

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2. *form the χ^2-statistic*
3. *The degrees of freedom $= n - 1$.*
4. *Find the P-value*

Example

Example

Suppose that you record 600 rolls and you get the following data:

Die roll	Frequency
1	50
2	70
3	170
4	160
5	80
6	70

What do you think now?

Example

Example

Let's see if the age distribution of jurors is representative of the entire county:

Age	%of county	# of jurors	Expected
21-40	42%	5	
41-50	25%	9	
51-60	16%	19	
61 and over	17%	33	
	100%	66	

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What is the Null hypothesis?

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- *Usually data is given as a table.*
- *Null hypothesis is that the two variables are independent; that is, all groups have same proportions.*

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- *Degree of freedom=(# of rows -1)(# of columns -1).*

Test of Independence

Fact

To test for independence

- *Find the overall percentage in each row.*
- *Form an expected table of counts assuming Null hypothesis*
- *Find χ^2.*
- *Degree of freedom=(# of rows -1)(# of columns -1).*
- *Finally, use the χ^2-table*

Example

Example

Is voting independent of sex or is it correlated? Assume that we have the following data:

	Men	Women	Total
Voted	2792	3591	6383
Didn't vote	1486	2131	3617
Total	4278	5722	10000

Example

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	Men	Women	Total
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Null hypothesis The voting is independent, that is, the % of voting would be the same for each sex.

Example

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Students at Colgate are polled if they want to take statistics before graduating. The results are as follows

	1st year	Soph	Jr	Sr	Totals	%
want to	99	98	98	97	392	
don't want to	121	112	92	83	308	
total	220	210	190	180	800	

Test the hypothesis that the results are the same regardless of class.