Chapter 4: The Average and the Standard Deviation

Marius Ionescu

09/06/2011 and 09/08/2011

Center and spread of a histogram

Fact

- *A histogram can be used to summarize large amounts of data*
- *Often the histogram is summarized by two numbers: the center and the spread*
- *The center represents the "level" or "position" of the distribution*
- *The spread represents the variation within population*
- *However, things do not always work so well*

Measures of the centers

Definition

- The **average** is the sum of all values divided by the number of values
- The **median**: the value with 50% of the values higher and 50% lower

Average

Example

Find the average

- 1, 1, 1, 1, 2, 2, 2, 2, 15
- 1, 1, 1, 1, 2, 2, 2, 2 ,3 ,3, 4
- Add 5 to the last example and find the average
- Multiply each number by 5 and find the average

Example

If the average of the day temperature during the last month is 27F what is the average in terms of Celsius?

Rules

Fact

- $average(x{+}5){=}average(x){+}5$
- $average(x \cdot 5){=}average(x) \cdot 5$

Median

Example

Find the median for each of the following sequence of numbers:

- 1, 1, 1, 1, 2, 2, 2, 2, 15
- 1, 1, 1, 1, 2, 2, 2, 2 ,3 ,3, 4
- 8, 10, 15, 20

Average and median in Excel

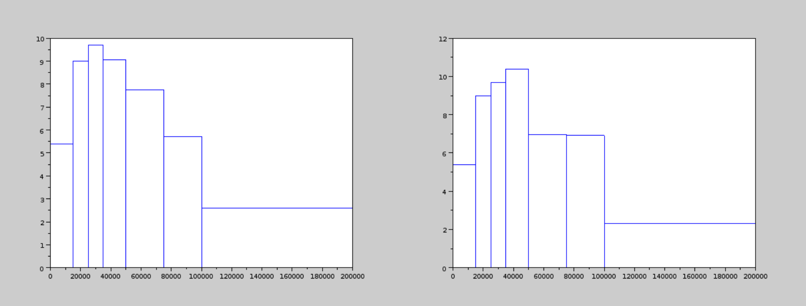
Fact

- *To find the average of the cells A1:A10 you need to write* ***=average(A1:A10)***
- *To find the median of the cells A1:A10 you need to write* ***=median(A1:A10)***

Average, median, and the histogram

Example

Which histogram has higher average? Which histogram has higher median?



Average, median, and the histogram

Fact

- *Average is the point at which the distribution balances.*
- *Median is the point for which 1/2 of the area is on the left and 1/2 is on the right.*
- *Median describes a "middle" individual, a typical subject.*

Income in US

Example

For income in US, which would you expect to be larger? The median or the income?

Answer: In 2008

- the median income was $61,521
- the average income was $79,634

Standard Deviation

Fact

Standard deviation *(SD) is a common way of measuring the spread around the average.*

The Root-mean-square

Definition

Root-mean-square= the square root of averages of square

Example

The root-mean-square of $5, -5, 0, 6$ is

$$\sqrt{\frac{5^2 + (-5)^2 + 0^2 + 6^2}{4}} \quad = \quad 4.6368.$$

Standard Deviation

Definition

SD= root-mean-square of distance to the average.

Example

Find the standard deviation of 20, 10,15, 15.

$$\text{Avg} = \frac{20+10+15+15}{4} \quad = \quad 15.$$

$$\text{SD} = \sqrt{\frac{5^2+(-5)^2+0^2+0^2}{4}} \quad = \quad 3.5355$$

Standard Deviation

Fact

The SD says how far away the numbers on a list are from their average. Most entries on the list will be somewhere around one SD away from the average.

Is your calculator computing SD or SD^{+}?

Fact

Enter 1 and −1 in your calculator and compute SD. What number do you get?

- *If the answer is 1 then your calculator is computing SD.*
- *If the answer is 1.41..., then your calculator is computing something called SD^{+}.*
- *To find SD from SD^{+} you need to use the following formula*

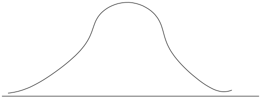
$$SD = \sqrt{\frac{\#\ \text{of entries} - 1}{\#\ \text{of entries}}} SD^{+}.$$

- *To compute the standard deviation in Excel of a sequence of numbers in cells A1:A10 you would write* ***=stdevp(A1:A10)***

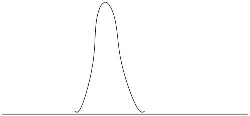
Examples

Example

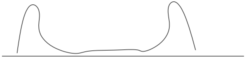
Which of the following histograms has the highest SD and which one has the smallest SD?



a)



b)



c)

Percentiles

Definition

$n\%$: sort the numbers and then find which is bigger than $n\%$ of the others.

Example

- Find the 75th%ile of 1,2,3,4.
- Find the 10th, 50th, 90th percentile of 1,2,3,4,5,6,7,8,9,10.

Percentiles in Excel

Fact

- *To find percentiles in Excel you should use the* ***percentile*** *function.*
- *For example to find the 75th %ile of a sequence of numbers in A1:A10 you would enter =****percentile(A1:A10,0.75)***
- *Notice that you need to enter 0.75 and not 75!*

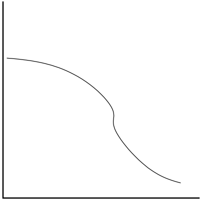
Review Example

Example

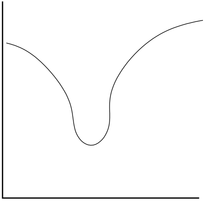
Try to match the following histograms to the following data from a survey of adults in the San Francisco Bay Area:

1. people's height
2. people's weight
3. the distance from a persons home to San Francisco
4. the distance from a persons home to the nearest airport.

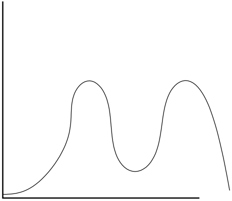
You can use the same diagram more than once or not at all.



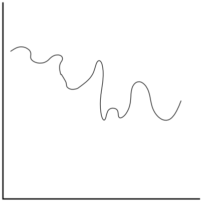
a)



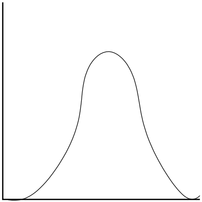
b)



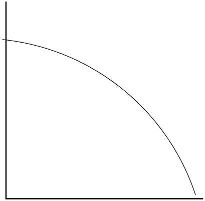
c)



d)



e)

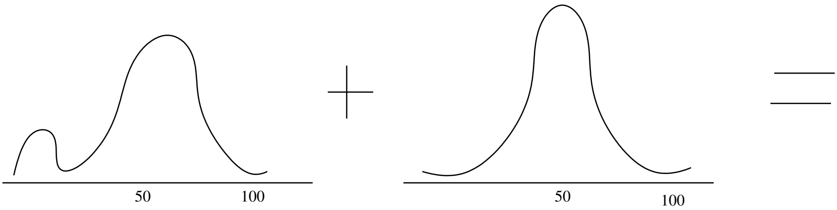


f)

Combining histograms

Example

The following two histograms are for decibel reading at a basketball game and a hockey game. What does the histogram for the combined data look like (for both sporting events)?


50
100
50
100