Unit 3: The Normal Approximation for Data

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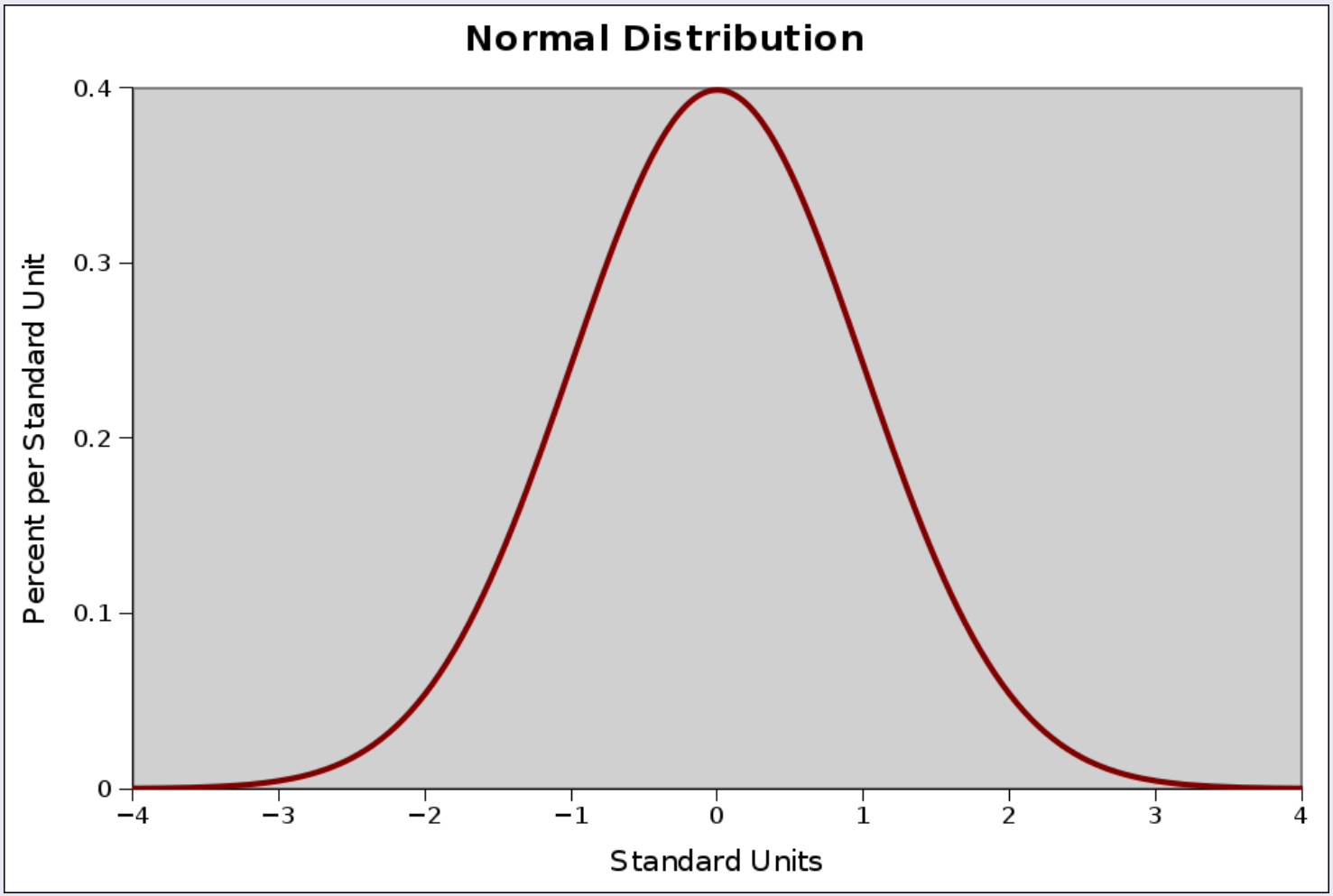
09/13/2011

Fact

The normal distribution is a very common approximate distribution



Properties of the Normal Distribution

Fact

1. *It has a single bump*
2. *It is symmetric about the average*
3. *Its shape depends only on average and SD*
4. *68% of the area lies within 1 SD of the average*
5. *95% lies within 2 SD*
6. *The height is given by*

$$\frac{1}{\sqrt{2\pi SD}} e^{-\frac{1}{2}\left(\frac{x-Avg}{SD}\right)^2}.$$

Standard Units

Definition

If X is approximated by a normal distribution with average Avg and SD, the the standard units are obtain by setting average to zero and scaling SD to one:

$$Z = \frac{X - Avg}{SD}.$$

Standard Units: Example

Example

Suppose that the average of one midterm is 65 and SD is 10. How many SDs away from the average is your score if it is

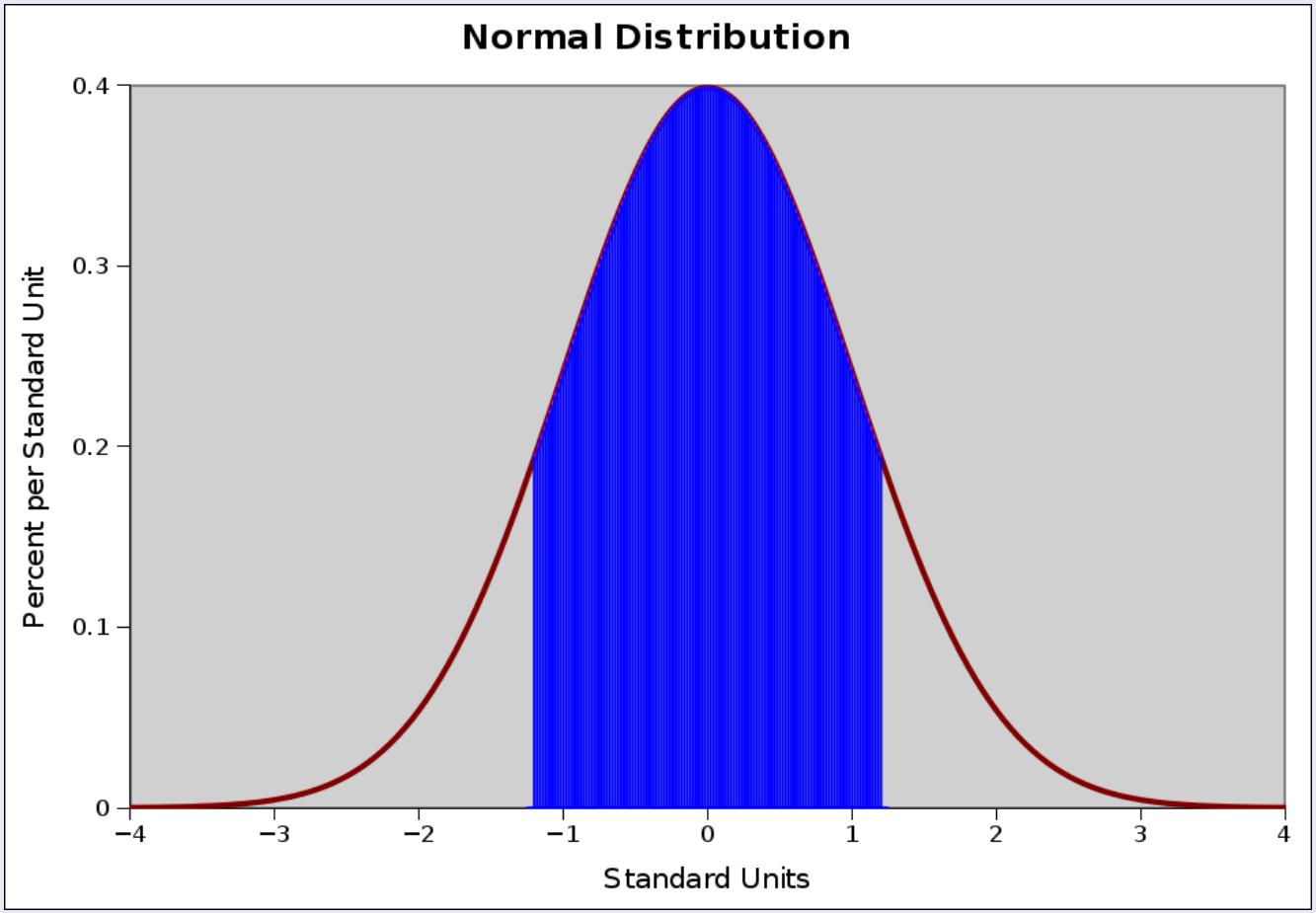
- 68
- 78
- 50
- 60

What was your score if you were 1.75 SD above the average?

Finding areas under the curve

Fact

We use a table to find the area under the normal curve: $A(z)$=Area between $-z$ and $+z$



Examples

Example

- $A(1) = 68.27\%$
- $A(2) = 95.45\%$

Example

Find the area:

- between $-z$ and 0
- between $-z$ and w
- below 0
- above w
- below $-w$

Example

Example

Suppose that the Math SAT has a normal distribution with an average score of 500 and SD=100.

- What percent of scores are above 650?
- What percent of scores are between 600 and 650 or between 350 and 400?
- What percent of scores are between 600 and 650?
- What percentile would 650 be?
- What percentile would 400 be?
- If you score in the 70th%ile, what is your score?

Example

Example

For a certain group of people the 25^{th}%ile is 62.2 inches, the 75^{th}%ile is 65.8 inches. The histogram follows a normal distribution. Find the 90^{th}%ile.

Measurement Error

Fact

Any measurement has error associated with it.

- *Measurement=Exact Value + Variation*
- *Error can be of two types:*
  1. *Bias*
  2. *Chance Error*
- *Measurement = Exact Value + Bias + Chance Error*

Chance Error

Fact

- *Chance Error is due to randomness.*
- *It has zero average*
- *If bias is zero then we can find the exact value by measuring lots of times and taking the average.*

How can we spot bias?

Fact

There is no way to tell if bias exists without comparison with another kind of experiment.

Example

To determine the age of a tree one can do the following experiments:

- carbon dating
- tree ring counting
- gene variation analysis

Outliers

Definition

- If one or two data points are wild results, statisticians often throw them out.
- They assume that something fundametally different occured for that point.
- For example, a bias for that measurement that is not in other measurements.