

Unit 5: Regression

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09/22/2011

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- *Goes through (Avg_x, Avg_y) with slope $\pm SD_y / SD_x$ depending on $r > 0$ or $r < 0$.*
- *Hence, the line is*

$$y = \pm \frac{SD_y}{SD_x} (x - Avg_x) + Avg_y.$$

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- *Clearly, this is not what we want.*

Fact

Regression=Attempt to quantify how one variable depends on another given a certain degree of correlation.

Example

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Suppose that we measure the relationship between height and weight. Let X = height, Y = weight. We have $Avg_x = 70$ in., $Avg_y = 162$ lbs., $SD_x = 3$ in., $SD_y = 30$ lbs., with $r = .47$.

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- A: **Average** weight of all 73 inch tall people in the study.

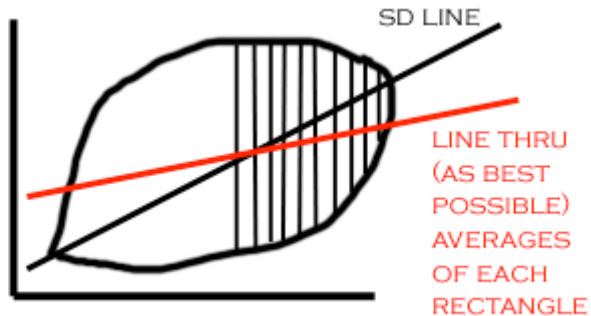
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- Q: So if we know someone is 73 inches tall, what we predict for their weight?
- A: **Average** weight of all 73 inch tall people in the study.
- How do we get this?

The regression line (best fit line)



Fact

The line in red is called the regression line for predicting y given x .

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- *If the data is nice (normally distributed and linear correlated), then the average of y on each rectangle will be on a line.*
- *Graphically the regression line goes through the Avg point **and** the sides of the football.*

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- By construction it goes through the point (Avg_x, Avg_y) .

The regression line in standard units

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- *We can rewrite the regression line as*

$$\frac{y - Avg_y}{SD_y} = r \frac{x - Avg_x}{SD_x}.$$

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- *Thus, in standard units,*

$$Y = rX.$$

Example (Weight vs Height, cont'd)

What is the average weight of those 73" tall?

Example

Suppose that Colgate students who take the LSAT have $Avg_{LSAT} = 600$, $SD_{LSAT} = 50$, $Avg_{GPA} = 3.3$, $SD_{GPA} = 0.3$ and the correlation coefficient $r = 0.6$. What is the regression prediction for the LSAT score of the students with GPA of

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- 3.3?
- 3.6?

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- *A predicted value of y will be close to the average of y (in terms of SDs) than the x value is to the average of x (in terms of SDs).*
- *In virtually all test-retest situations, the bottom group's score will, **on average**, improve and the top group's scores will, **on average**, decline.*

Example

Example (Are sophomores dumber?)

Suppose that the average GPA for the first and second year students at Colgate is 2.8, SD is 0.7, and the correlation coefficient is $r = 0.8$. What is the most likely outcome for the best student (4.0 GPA as a first year student) in the second year?

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- *Regression is an idealized description of a population.*
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- *The previous example does not say that sophomores do worse than in the first year.*

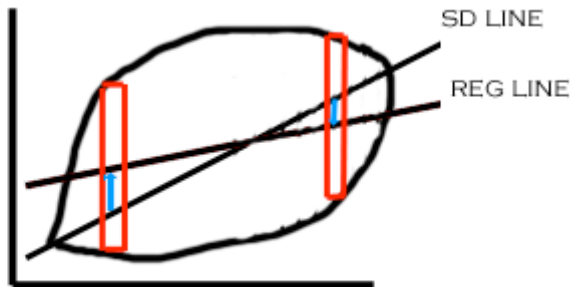
The regression fallacy

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Remember:

- *Regression is an idealized description of a population.*
- *It doesn't describe individual cases.*
- *The previous example does not say that sophomores do worse than in the first year.*
- *It says that the best 1st years will on average do worse in their 2nd year, even though the overall average and SD stay the same.*

The regression fallacy



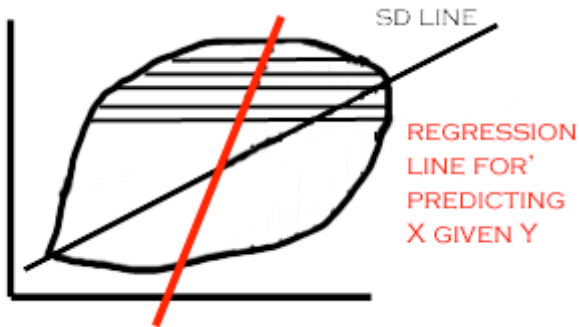
Fact

Blue arrows show increase in scores for those below average and decrease in scores for those above average.

There are TWO regression lines

Fact

One for predicting y given x ; one for predicting x given y .



Example

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- What is the predicted average IQ of women whose husbands have IQ of 140?
- What do you predict for the average IQ of their husbands?

Example (cont'd)

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- *Again, different families of data under consideration:*

Example (cont'd)

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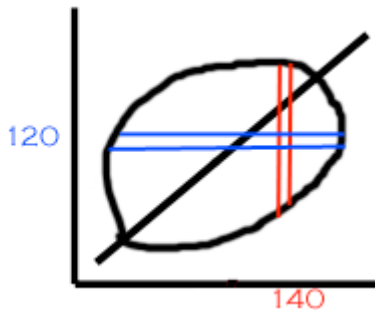
- *So smart husbands have dumber wives, who in turn have dumber husbands.*
- *What gives?*
- *Again, different families of data under consideration:*
 - ① *All wives whose husbands have IQs of 140;*

Example (cont'd)

Fact

- *So smart husbands have dumber wives, who in turn have dumber husbands.*
- *What gives?*
- *Again, different families of data under consideration:*
 - 1 *All wives whose husbands have IQs of 140;*
 - 2 *All husbands whose wives have IQs of 120;*

Example (cont'd)



Chapter 11: Estimates of the error in regression analysis

Fact

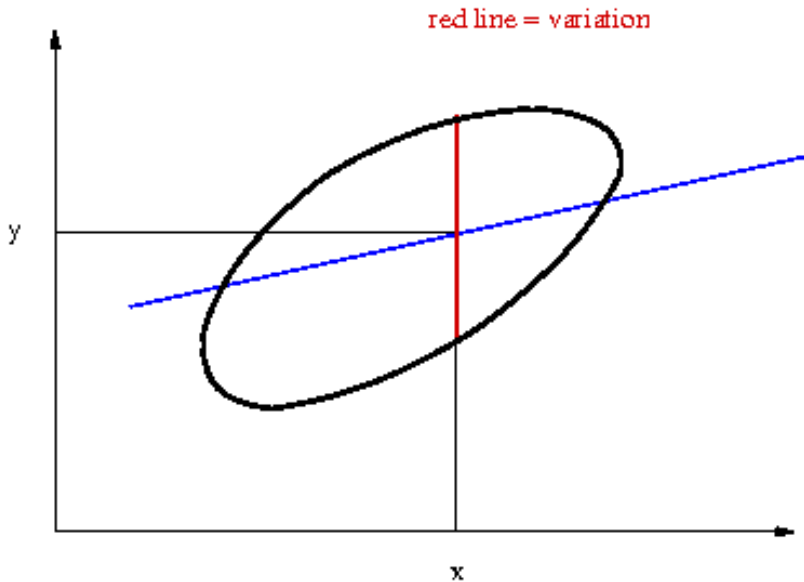
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- Recall that if x and y are linearly correlated and $x = x_1$ then the value y predicted using the regression line is the **average** value

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- What is the variation expected around this?

Variation



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Fact

- *The regression line is the line which minimizes the RMS error.*
- *For this reason the regression line is called also the **least square error** or **least squares line**.*

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- *For ideal situation, 68% of data will fall within 1 RMS error of the regression line.*
- *95% of data will be within 2 RMS errors of the regression line.*
- *The RMS error behaves like the SD from the regression line.*

Formula for computing the RMS

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- *We can compute the RMS using the formula*

$$RMS\ error = \sqrt{1 - r^2} \cdot SD_y.$$

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- *This formula is valid for normally distributed linearly correlated data.*

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Suppose that for Colgate freshmen the average of the SAT verbal is 550, SD is 50, the average of their GPA is 2.8, SD is 0.7, and $r = 0.5$.

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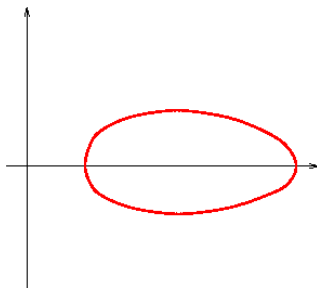
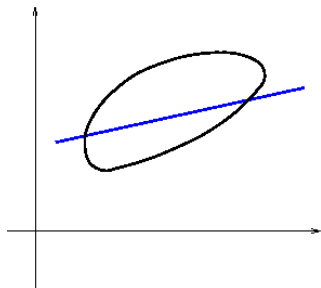
- What is the RMS error when predicting the SAT verbal?

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Suppose that for Colgate freshmen the average of the SAT verbal is 550, SD is 50, the average of their GPA is 2.8, SD is 0.7, and $r = 0.5$.

- What is the RMS error when predicting the SAT verbal?
- For students scoring 650 on the SAT verbal, what is the predicted GPA with error estimate? .

Residuals



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- *The residuals are the errors centered around 0.*

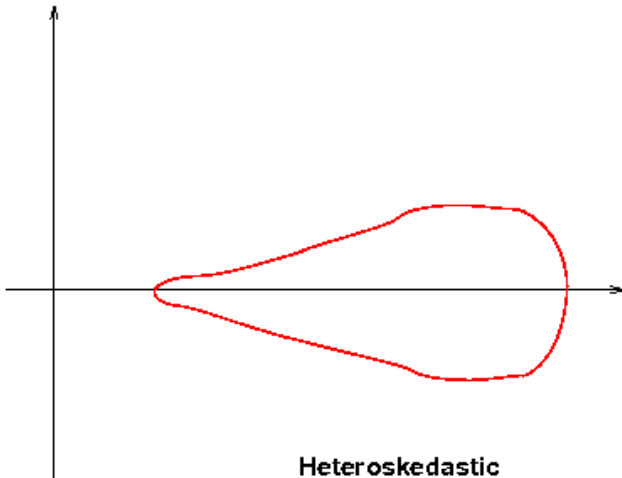
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- *The residuals are the errors centered around 0.*
- *They should have **no** “linear trend” (that is, the slope should be 0).*

Heteroskedastic data

Definition

We say that data is heteroskedastic if the residuals have different distributions

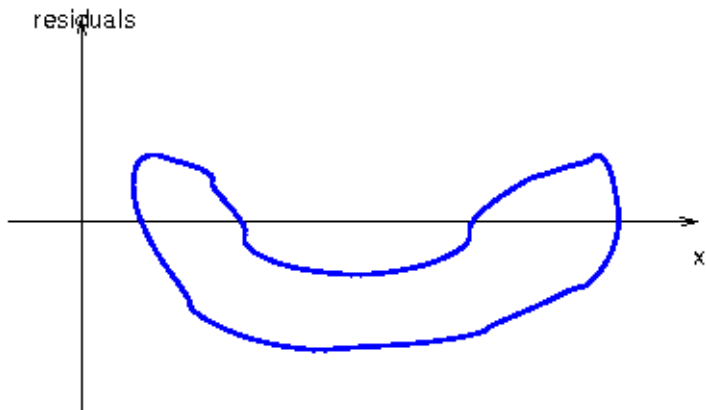


Heteroskedastic

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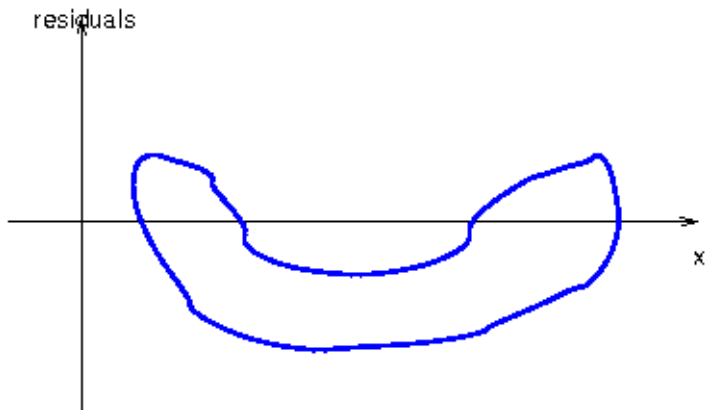
RMS error formula is not a good error estimate for heteroskedastic data.

Nonlinear relationship



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Nonlinear relationship

- Don't use the regression line!