

Regression Examples

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Example

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We study IQ versus Math SAT score. Our group has an average IQ of 100 with a SD of 15, and obtained an average SAT of 550 with SD of 80. We calculated the correlation coefficient to be $r = .6$ and found that the scatterplot was football-shaped.

- If a student scores a 150 on the IQ test, what do you estimate for their SAT score? $X=(150-100)/15=3.33$; $Y=.6*3.33=2$; $y=550+2*80=710$
- If a student scores 710 on SAT, what do we estimate for their IQ? $Y=2$; $X=.6*2=1.2$; $x=100+1.2*15=118$

Fact

Suppose that x and y are normally distributed and linearly correlated (they form a football shaped date). Then

- For each x , the strip above x is normally distributed.
- The average is the predicted value of y
- SD equals RMS error.

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Suppose that the average height of men is 68 inches with $SD = 2.7$, and the average weight of women is 63 inches with $SD = 2.5$. Assume that the correlation between the height of husbands and wives is 0.25 and assume that the data is normally distributed.

- What percentage of women are over 68 inches? $X = 2$; $\% \geq 2 = 2.5$
- What percentage of women married to a man of height 72" are over 68"? $Avg h = 63 + 0.25(2.5/2.7)(72-68) = 63.9$;
 $RMSErr = \sqrt{1 - (1/4)^2} \cdot 2.5 = 2.42$; $Z = (68-63.9)/2.42 = 1.69$;
 $\% \geq 1.69 = 4.5$
- What percentile of women married to a man of height 72" is 68"?
 $100 - 4.5 = 95.5$

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Suppose that in this class the average of the first midterm will be 72, $SD=20$, the average of the final exam will be 75, $SD=10$, and the scores have a correlation coefficient of $r = 0.6$.

- Of all those who score in the 68th%ile on the midterm, what is the expected outcome on the final (with error estimates)? $A(z)=36$; $X=0.48$; $Y=0.6*0.48=0.288$; $Y=75+0.288*10=73$
- Of those who score with 90th%ile on the midterm, what percentile is expected for the final. $X=1.3$; $Y=0.6*1.3=.78$; $A(.78)=56\%$; 79%ile
- Of those who score 75 on midterm what is the percentile we expect for the final? $X=(75-72)/20=0.15$; $Y=0.6*0.15=0.09$; $A(0.09)=8\%$; %ile=54
- If you score 58 on the midterm what do you expect for the score on the final (with error estimates) $X=-14/20=-0.7$; $Y=0.6*(-0.7)=-0.42$; $Y=75+(-0.42)*10=70.8$; $RMSE_r=8$
- If you score 70.8 on the final what is your expected score on the

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Suppose that the average of violent crimes during night in a city is 235, $SD = 34$, the average night temperature is 60, $SD = 7$, and the correlation between crimes and temperature is 0.5.

- If the temperature is 74, how many violent crimes are predicted tonight? 270
- If there were 200 crimes last night, how hot was it? 56.5
- What is the RMS error for these predictions? 6.1
- If tonight is hotter than 68% of all nights, what is the predicted percentile for crimes? $X = .48$; $Y = .24$; 59.5%

Nonlinear relationships

Fact

What happens if the relationship between the variables is not linear?

- For exponential relationships

$$y = Ae^{Bx}$$

we can use linear regression with $\ln y$ and x :

$$\ln y = \ln A + \ln e^{Bx} = \ln A + Bx.$$

- For parabolic data we can try a general quadratic:

$$y = Ax^2 + Bx + C,$$

where we have two variables: x^2 and x .