

Regression Examples

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Example

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We study IQ versus Math SAT score. Our group has an average IQ of 100 with a SD of 15, and obtained an average SAT of 550 with SD of 80. We calculated the correlation coefficient to be $r = .6$ and found that the scatterplot was football-shaped.

- If a student scores a 150 on the IQ test, what do you estimate for their SAT score? $X = (150 - 100) / 15 = 3.33$; $Y = .6 * 3.33 = 2$;
 $y = 550 + 2 * 80 = 710$
- If a student scores 710 on SAT, what do we estimate for their IQ? $Y = 2$; $X = .6 * 2 = 1.2$; $x = 100 + 1.2 * 15 = 118$

Fact

Suppose that x and y are normally distributed and linearly correlated (they form a football shaped data). Then

- *For each x , the strip above x is normally distributed.*
- *The average is the predicted value of y*
- *SD equals RMS error.*

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Suppose that the average height of men is 68 inches with $SD = 2.7$, and the average weight of women is 63 inches with $SD = 2.5$. Assume that the correlation between the height of husbands and wives is 0.25 and assume that the data is normally distribute.

- What percentage of women are over 68 inches? $X=2$; $\%i2=2.5$
- What percentage of women married to a men of height 72" are over 68"? $Avgh=63+0.25(2.5/2.7)(72-68)=63.9$;
 $RMSErr=\sqrt{1 - (1/4)^2} \cdot 2.5 = 2.42$; $Z=(68-63.9)/2.42=1.69$;
 $\%i1.69=4.5$
- What percentile of women married to a men of height 72" is 68"?
 $100-4.5=95.5$

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Suppose that in this class the average of the first midterm will be 72, $SD = 20$, the average of the final exam will be 75, $SD = 10$, and the scores have a correlation coefficient of $r = 0.6$.

- Of all those who score in the 68th %ile on the midterm, what is the expected outcome on the final (with error estimates)? $A(z) = 36$; $X = 0.48$; $Y = 0.6 * 0.48 = 0.288$; $Y = 75 + 0.288 * 10 = 73$
- Of those who score with 90th %ile on the midterm, what percentile is expected for the final. $X = 1.3$; $Y = 0.6 * 1.3 = .78$; $A(.78) = 56\%$; 79th %ile
- Of those who score 75 on midterm what is the percentile we expect for the final? $X = (75 - 72) / 20 = 0.15$; $Y = 0.6 * 0.15 = 0.09$; $A(0.09) = 8\%$; %ile = 54
- If you score 58 on the midterm what do you expect for the score on the final (with error estimates) $X = -14 / 20 = -0.7$; $Y = 0.6 * (-0.7) = -0.42$; $y = 75 + (-0.42) * 10 = 70.8$; $RMSEr = 8$
- If you score 70.8 on the final what is your expected score on the

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Suppose that the average of violent crimes during night in a city is 235, $SD = 34$, the average night temperature is 60, $SD = 7$, and the correlation between crimes and temperature is 0.5.

- If the temperature is 74, how many violent crimes are predicted tonight? 270
- If there were 200 crimes last night, how hot was it? 56.5
- What is the RMS error for these predictions? 6.1
- If tonight is hotter than 68% of all nights, what is the predicted percentile for crimes? $X = .48$; $Y = .24$; 59.5%

Nonlinear relationships

Fact

What happens if the relationship between the variables is not linear?

- *For exponential relationships*

$$y = Ae^{Bx}$$

we can use linear regression with $\ln y$ and x :

$$\ln y = \ln A + \ln e^{Bx} = \ln A + Bx.$$

- *For parabolic data we can try a general quadratic:*

$$y = Ax^2 + Bx + C,$$

where we have two variables: x^2 and x .