February 25, 2003

Math 102 / Core 143 AX and BX — Exam I

Show all work clearly for partial credit. An unsimplified answer like $12\sqrt{3.51}+6/7$ is usually worth more than 23.3, because it is easier to understand where it came from.

1. *(25 points)* Find, for the list of numbers

$$-1, 1, 2, 5, 8 \ ,$$

   (a) the average (b) the standard deviation

   (c) the median (d) the 80th percentile

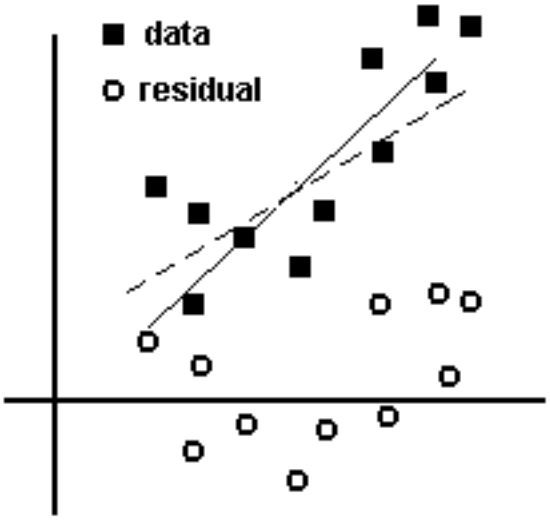
   (e) the skew (i.e., to the left or to the right, as determined by your earlier answers)

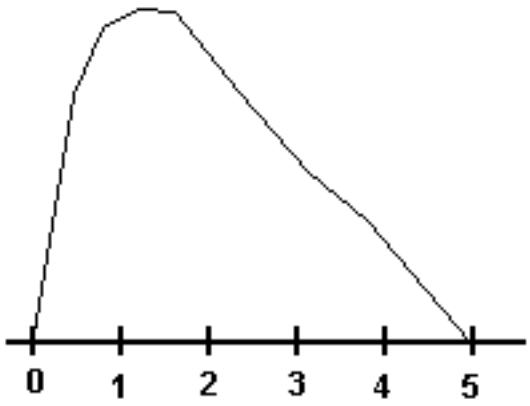
2. *(15 points)* In a test of a fever reduction medication, subjects with fevers are given a dose (different doses for different subjects) of the medicine around noon for 10 days in a row, and the total number of degrees that their temperature falls is recorded (so a data point is the dose per day, in cc's, and the total reduction in temperature for that subject). The scatter diagram is below, including the SD-line, the regression line for projecting fever reduction (on the vertical axis) from dosage (on the horizontal axis) and the residuals.

   (a) Which line (dashed or solid) is the SD-line?

   (b) Is the correlation coefficient closest to -1, $-.5$, 0, 0.2, 0.8, or 1.3?

   (c) Can any information be gained from the residuals? If so, what do they suggest?





3. *(10 points)* For the histogram above:

   (a) Is the average above or below 2.5?

   (b) Is the standard deviation closest to 0, 2, 4 or 6?

   (c) About how high is the highest point, on the density scale (i.e., in percent per horizontal unit)?

4. *(30 points)* A educational psychologist believes that using calculators in the teaching of arithmetic will help children to learn the subject better. She collects data relating the average amount of time per day a child uses a calculator with that child's score on a test of arithmetic skills. Here are the results:

	calculator minutes	test score	
AV	15	80	$r = -.2$
SD	30	10	

   (a) Is the sign of the correlation coefficient consistent with the psychologist's opinion? Explain.

   (b) Using linear regression, guess the test score of a child who uses a calculator 21 minutes a day.

   (c) How far should you expect your estimate in (b) to be off?

   (d) The correlation here is very low, but suppose it had been higher (i.e., close to ± 1). Should the psychologist conclude that students should be encouraged to (or discouraged from) using a calculator, because its use strongly influences their acquisition of arithmetic skills?

   (e) Suppose the children's scores on a reading test have a correlation of 0.7 with their scores on the arithmetic test above. If a child was at the 80th percentile on the reading test, what should we guess was that child's score (not percentile) on the arithmetic test? (Assume the score distributions follow the normal curve.)

5. *(10 points)* The article "'Sham Surgery' aids Parkinson's study" by Jim Erickson says "Sham surgery was included in the study to control for the placebo effect." What is the placebo effect, and how did sham surgery correct for it?

6. *(10 points)* Related to "Tuning in to Violence" by Ron McClamrock: State (at least some of) the author's guesses at what an underlying cause linking TV and aggression might be. Can you think of others?

Some possibly useful formulas:

$$\sqrt{\text{average of } (x - AV_x)^2} \qquad SD_y \cdot \sqrt{1 - r^2}$$

$$r = \text{average of } ((x \text{ in std units}) \cdot (y \text{ in std units}))$$

$$z = \frac{x - AV_x}{SD_x} \qquad x = z \cdot SD_x + AV_x$$

$$y - AV_y = (\text{sign of } r)\frac{SD_y}{SD_x}(x - AV_x) \qquad y - AV_y = r\,\frac{SD_y}{SD_x}(x - AV_x)$$

Math 102 / Core 143 — Solutions to Exam I

1. (a) $(-1+1+2+5+8)/5 = 3$
   (b) $\sqrt{[(-1-3)^2+(1-3)^2+(2-3)^2+(5-3)^2+(8-3)^2]/5} = \sqrt{[16+4+1+4+25]/5} = \sqrt{10} \approx 3.16$
   (c) 2 (d) 5 (probably) (e) Right: average > median.

2. (a) The solid line — it's steeper.
   (b) 0.8 — there is a rather strong positive correlation.
   (c) They indicate a nonlinear association; they show that after the linear tendency, reflected in the regression line, is removed, there is a curvilinear tendency left, because the residuals in the middle x-values have negative y-values, while the ones with high or low x-values have positive y-values.

3. (a) Below — it would fall to the left if supported at 2.5. (b) 2.
   (c) Because the histogram is roughly triangular with a base of 5 units, and the total area has to be 100%, the height must be about 40 percent per horizontal unit, because $\frac{1}{2}(5)(40) = 100$.

4. (a) No: The negative correlation says that as calculator use increases, arithmetic skill decreases.
   (b) 21 minutes corresponds to a z-value of $(21-15)/30 = .2$, so we guess a test score of $(.2)(-.2) = -.04$ in standard units, or in real test scores, $-.04(10)+80 = 79.6$.
   (c) By the RMS error for regression: $10\sqrt{1-(-.2)^2} \approx 9.8$.
   (d) No: The experiment can only show association, not causation. Perhaps the children who have lower arithmetic skills are using the calculator more because they have no confidence in their own skills.
   (e) Start with the reading test: The 80th percentile corresponds to an area of 60% between the z-value and its negative, and hence, by the normal table, to a z-value of 0.84. So on the arithmetic test we should project a z-value of $(0.84)(0.7) \approx .59$, or a test score of $.59(10)+80 = 85.9$.

5. The placebo effect is psychological: A subject reacting as if he/she had received a treatment, though he/she had not received any. (It need not be limited to the deluding of a patient with a sugar pill; relief generated by the reassurance that a condition has been examined and diagnosed by an expert probably fits this description as well.) In this case, sham surgery corrected for it in that every subject, whether in the treatment or control group, had the same evidence of receiving treatment, so that the placebo effect could be assumed to be equally present in both groups.

6. Here are the relevant paragraphs in the article; in grading this question, I would look for some sort of echoes of this (plus, I hope, some ideas of your own): "In what kind of home do children watch less than an hour of TV a day? First, it's an uncommon one: about 12% of the study's total. Second, it could be a situation in which other factors might account for lower levels of aggression. Perhaps there are parents who encourage involvement in productive activities or who at least keep a tighter rein on what the kids are up to. And, likewise, for the kids watching more than three hours of TV a day, it's not at all implausible that they are less closely monitored by parents and their proxies."