10 April 2012

Math 102D — Exam II

An unsimplified answer like $12\sqrt{3.51} + 6/7$ is usually worth more than 23.3, because it is easier to understand where it came from. The total possible points is **85**.

1. *(26 points)* A box of 10 slips is numbered 1 through 10. A slip is drawn at random from the box.
   (a) What is the conditional probability that, if the slip drawn is at most 8, it is divisible by 3?
   (b) If two slips are drawn twice in succession (with replacement), what is the probability that the first slip is at most 8 and the second is divisible by 3?
   (c) If two slips are drawn twice in succession, what is the probability that the first slip is at most 8 or the second is divisible by 3 (or both are true)?
   (d) If 5 slips are drawn in succession, what is the probability that at least one is divisible by 3?
   (e) If 5 slips are drawn in succession, what is the probability that exactly 3 of the slips are divisible by 3?

2. *(12 points)* Which term describes each of the following probability distributions, related to draws from the 10-slip box in Question 1? The choices are: "(approximately) normal", "(approximately) uniform" and "other"; they may be used more than once or not at all.
   (a) One draw from the box.
   (b) For two draws from the box: A 1 if the two draws are equal and a 0 if they are not.
   (c) The sum of many draws from the box.

3. *(12 points)* With the same 10-slip box as above, the average value is 5.5. We could (I) make 15 draws (with replacement), or (II) make 300 draws. In which of these two scenarios, I or II or neither, would the following be more likely?
   (a) The average of the slips drawn is at least 6.
   (b) The average of the slips drawn is at least 4.5.
   (c) The sum of the slips drawn is at least 6 times the number of cards drawn.
   (d) The sum of the slips drawn is at least 6 more than 5.5 times the number of cards drawn.

4. *(10 points)* The test for the disgusting disease lantzeria has a sensitivity of 90% (i.e., if you have it, the test will say you have it 90% of the time) and specificity of 85% (i.e., if you don't have it, the test will say you don't 85% of the time). Suppose that 4% of the population have this disease, and that you test positive for it. What is the probability that you actually have it? (Steps in the solution: 1. Percent of the population who are sick and test positive. 2. Percent who are well and still test positive. 3. Percent who test positive. 4. Percent who are sick and test positive divided by total percent who test positive.)

5. *(15 points)* A game consists of rolling two dice, but if either die comes up a 6, the roll doesn't count and you roll again — in other words, there are only 25 possible rolls. The winning rolls are the ones where both dice come up 3 or less, so there are 9 winning rolls. You pay a dollar to play a round, and if you win you get$1.50 (plus your original dollar). Suppose you play 400 times. Each of the following questions needs two numbers to answer it.

   (a) How much should you expect to win in total, give or take how much?

   (b) What should you expect your average win (over all your plays) to be, give or take how much?

   (c) How many times should you expect to win, give or take?

6. *(10 points)* Relative to the article "DIY statistical analysis: experience of touching real data," by Ben Goldacre: The author says the Poisson distribution is "a bit like the bell-shaped curve you'll be familiar with", but it differs in one important way. How does it differ, and why does that make it a better distribution for bowel cancer mortality?

Some Possibly Useful Formulas:

$$\frac{n!}{k!(n-k)!}\,p^k(1-p)^{n-k}$$

$$\text{EV of sum (count)} = n \cdot \text{(AV of box)} \qquad \text{SE of sum (count)} = \sqrt{n} \cdot \text{(SD of box)}$$

$$\text{EV of avg (\%)} = \text{AV of box} \qquad \text{SE of avg (\%)} = \frac{\text{SD of box}}{\sqrt{n}}$$

$$\text{SD} = \text{(larger} - \text{smaller)}\sqrt{\text{(fraction with larger)(fraction with smaller)}}$$

Normal table (Area between $-z$ and z)									
z	Area(%)	z	Area(%)	z	Area(%)	z	Area(%)	z	Area(%)
0.0	0.0	0.9	63.19	1.8	92.81	2.7	99.31	3.6	99.968
0.05	3.99	0.95	65.79	1.85	93.57	2.75	99.4	3.65	99.974
0.1	7.97	1	68.27	1.9	94.26	2.8	99.49	3.7	99.978
0.15	11.92	1.05	70.63	1.95	94.88	2.85	99.56	3.75	99.982
0.2	15.85	1.1	72.87	2	95.45	2.9	99.63	3.8	99.986
0.25	19.74	1.15	74.99	2.05	95.96	2.95	99.68	3.85	99.988
0.3	23.58	1.2	76.99	2.1	96.43	3	99.73	3.9	99.99
0.35	27.37	1.25	78.87	2.15	96.84	3.05	99.771	3.95	99.992
0.4	31.08	1.3	80.64	2.2	97.22	3.1	99.806	4	99.9937
0.45	34.73	1.35	82.3	2.25	97.56	3.15	99.837	4.05	99.9949
0.5	38.29	1.4	83.85	2.3	97.86	3.2	99.863	4.1	99.9959
0.55	41.77	1.45	85.29	2.35	98.12	3.25	99.885	4.15	99.9967
0.6	45.15	1.5	86.64	2.4	98.36	3.3	99.903	4.2	99.9973
0.65	48.43	1.55	87.89	2.45	98.57	3.35	99.919	4.25	99.9979
0.7	51.61	1.6	89.04	2.5	98.76	3.4	99.933	4.3	99.9983
0.75	54.67	1.65	90.11	2.55	98.92	3.45	99.944	4.35	99.9986
0.8	57.63	1.7	91.09	2.6	99.07	3.5	99.953	4.4	99.9989
0.85	60.47	1.75	91.99	2.65	99.2	3.55	99.961	4.45	99.9991

Answers: **Name:** ______________________________

1. (a)

   (b)

   (c)

   (d)

   (e)

2. (a)

   (b)

   (c)

3. (a)

   (b)

   (c)

   (d)

4.

5. (a)

   (b)

   (c)

Answer Question 6 on the back of this page.

6.

Solutions to Exam II

1. (a) Of 1 through 8, 2 are divisible by 3: $2/8 = 1/4$.
   (b) $(8/10)(3/10) = 24/100$
   (c) $(8/10) + (3/10) - (24/100) = 86/100$
   (d) $1 - (7/10)^5 \approx .83$
   (e) $C(5,3)(3/10)^3(7/10)^2 = .1323$

2. (a) The numbers 1 through 10 are all equally likely: "uniform".
   (b) The outcome 0 is 9 times as likely as the outcome 1: "other".
   (c) By the Central Limit Theorem: "(approximately) normal"

3. (a) I (b) II (c) I (d) II

4. Following the steps in the problem: The fraction of the population who are sick and test positive is $(.04)(.9) = .036$. The fraction of the population who are well and still test positive is $(.96)(.15) = .144$. The fraction who test positive is thus $.036 + .144 = .18$. So the fraction of those who test positive who are really sick is $.036/.180 = .2$, or 20%.

5. (a) The EV of the sum is $[(9/25)(1.5) + (16/25)(-1)](400) = 40$, and the SE is $(1.5 - (-1))\sqrt{(9/25)(16/25)} \cdot \sqrt{400} = 24$
   (b) The EV of the average is $40/400 = .1$ (or 10 cents), and the SE is $24/400 = .06$ (or 6 cents).
   (c) The box is now 9/25 1's and 16/25 0's, so the EV of the count (of wins) is $(9/25)(400) = 144$, and the SE is $(1 - 0)\sqrt{(9/25)(16/25)} \cdot \sqrt{400} = 9.6$.

6. A Poisson distribution, which we've seen a few times in class, has a tail to the right, but not to the left, so it makes better sense as a distribution of mortality: You can't die of bowel cancer before you get it.