November 14, 2006

Exam II — Math 102 / Core 143 CX

Points are in parentheses. Show your work to receive partial credit; an answer like $[1-(6/7)^3]/4$ is worth more than 0.09257, because it displays your reasoning more clearly.

1. *(10 points)* Parapsychologists, who purport to make scientific studies of extra-sensory perception (ESP) including precognition ("foreknowing"), use special decks of 20 cards: four each of squares, circles, stars, triangles and lunes (crescent moons). Checking a subject for precognition, a parapsychologist asks the subject to pick one of the five symbols and then picks a card from the deck and notes whether the card's symbol matches the subject's choice. (Then the card is put back in the deck, and the deck is shuffled.) If this experiment is repeated 8 times, what is the probability that card and choice will agree at least 6 times, just by chance?

2. *(10 points)*. Select the histogram that best approximates the probabilities of the results of the following experiments. (A histogram can be used more than once or not at all.)

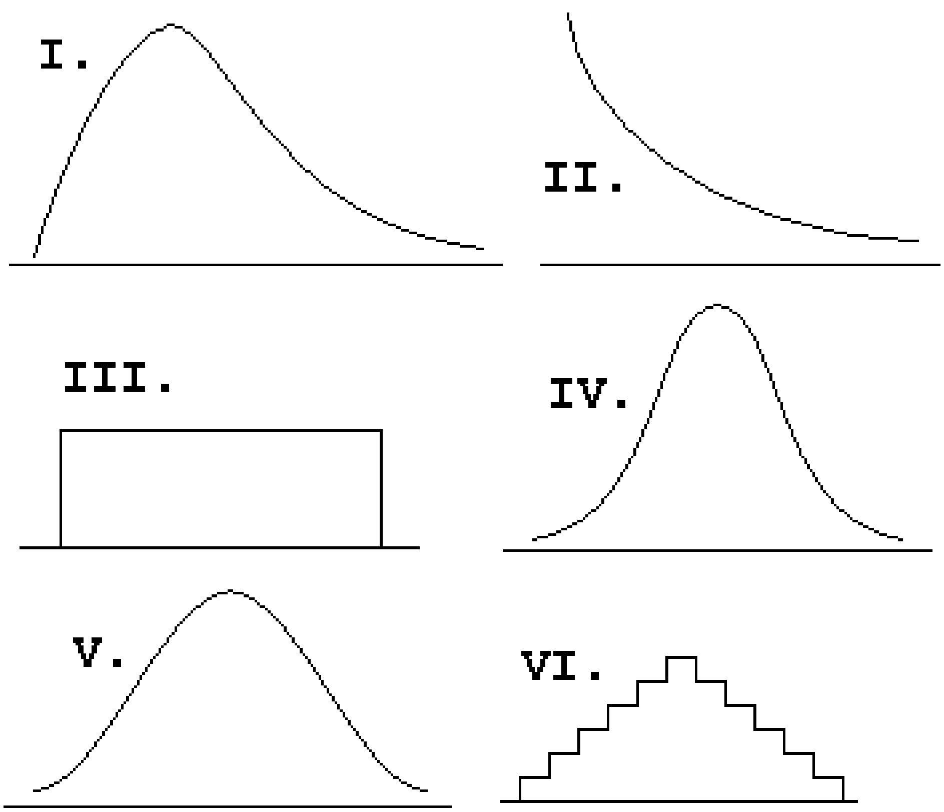
   (a) Results when fair die is rolled once.

   (b) Averages of the rolls when a fair die is rolled twice.

   (c) Averages of the rolls when a fair die is rolled 50 times.

   (d) Averages of the rolls when a fair die is rolled 500 times

   (e) Lengths of time until a certain kind of light bulb will burn before burning out.



3. *(20 points)* Suppose we have a large population whose incomes we know; they do not follow the normal curve (as with most income distributions, they are strongly skewed to the right), but they average \$35,000, with an SD of \$24,000. From this population we randomly choose 900 people and average their incomes. We repeat this 400 times.

   (a) Should we expect the distribution of these 400 averages to be close to a normal curve? Why or why not?

   (b) What should we expect the average of the 400 averages to be?

   (c) How much variability should we expect in the 400 averages? (Answer in the form "________ percent of them are likely to be in the range ________ to ________ .")

   (d) If instead of 400 times we did this only 100 times, how would the answers to (a), (b) and (c) change, if at all?

   (e) If instead of using samples of 900, we used samples of 1600 (but again 400 samples), how would the answers to (a), (b) and (c), if at all?

4. *(30 points)* A person stands in a shopping mall. She stops 400 people and asks each how many times per month they go out for hamburgers, and which of two national hamburger chains they prefer. For the 400, the average number of burger runs per month is 20, with an SD of 15, and 250 say they prefer MacTavish's hamburgers to Burger Empire's.

   (a) What is a 95% confidence interval for the average number of visits to a hamburger restaurant for a person in that city?

   (b) How many people would she have had to question to reduce the size of her confidence interval by a factor of 3 (assuming a similar average and SD)?

   (c) Burger Empire claims that 45% of the people in the city prefer their burgers to MacTavish's. Is the surveyor's result (statistically) significantly different from Burger Empire's claim?

   (d) Describe one possible source of error or bias in the data.

5. *(20 points)* A veterinary professor knows that the actual average weight of a house cat is 8.5 pounds, with a population standard deviation of 3 pounds; but to challenge a student's scientific accuracy, she tells him to prove that the average weight is 7 pounds. The student weighs 10 house cats and conducts a two-sided significance test with the null hypothesis of a 7-pound average and with an α-level of 5%.

   (a) What kind of error might he make, type I or II?

   (b) Which table would he use to apply his test?

   (c) Assuming that his 10-cat sample also has a standard deviation of 3 pounds, how far above and below 7 pounds could his 10-cat average go and he would still conclude that he does not reject the null hypothesis?

   (d) In view of the fact that the average weight is really 8.5 pounds, how likely is he to make an error? (You may need to use a z-table at some point in place of a t-table, because our t-table does not provide enough info.)

6. *(10 points)* Related to the article, "After the Bell Curve", by David L. Kirp: How does the paragraph on adoptions between different social classes give evidence for Kirp's argument? How does the same data give evidence against his argument?

Some possibly useful formulas:

$$SE \text{ for sample proportion} = \sqrt{\frac{p(1-p)}{n}} \qquad \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

$$SE \text{ for sample average} = \frac{s}{\sqrt{n}} \qquad z \text{ (or } t) = \frac{Observed - EV}{SE}$$

$$z = \frac{B-C}{\sqrt{B+C}} \qquad \overline{x} \pm t(SE) \qquad df = n-1$$

Solutions to Exam II

1. The probability of chance agreement on any one trial is 1/5, so the probability on exactly 6, exactly 7 or exactly 8 trials is $C(8,6)(1/5)^6(4/5)^2 + C(8,7)(1/5)^7(4/5)^1 + C(8,8)(1/5)^8 = (28(4^2) + 8(4) + 1)/5^8 = 481/390,625 = 0.00123136$.

2. (a) III. (b) VI. (c) V. (d) IV. (Another normal curve, but more closely huddled around the average.) (e) I.

3. (a) Yes: By the Central Limit Theorem, no matter what distribution you start with, the average of many draws from the distribution will have a distribution that is close to a normal curve.
   (b) The EV for the average, i.e., the average income of the population, \$35,000.
   (c) The SE for the average is the s of the population divided by the square root of the number in the sample: \$24,000/30 = \$800. So we should expect about 68% of the averages to be between \$34,200 and \$35,800. (We could also say that we expect 95% are between \$33,400 and \$36,600.)
   (d) None of the answers would change: The 400 did not enter into these calculations; we were just looking at 400 numbers from the distribution of averages.
   (e) The 900 <u>did</u> enter into the computation in (c): The new SE for the average is \$24,000/40 = \$600, so about 68% of the averages are between \$34,400 and \$35,600.

4. (a) Roughly $20 \pm 2(15/\sqrt{400}) = 20 \pm 1.5$ visits per month.
   (b) To reduce the size of the interval to 20 ± 0.5 visits per month, she would have had to interview 9 times as many people, or 3600.
   (c) Her result was that $250/400 = 62.5\%$ prefer "Mickey T's", more than the 55% that BE's figure would give. So if BE's claim were correct (the null hypothesis), the SE of % would be $\sqrt{(.55)(1-.55)}/\sqrt{400} \approx 2.49\%$, so the probability that she would have gotten her result was
$$P(\% \geq 62.5) = P(z \geq \frac{62.5 - 55}{2.49} \approx 3) < 5\% ;$$
   so her result is significantly different. (Even with a two-sided test, the result would have been significant at the 5% level.)
   (d) For one thing, she was interviewing in a shopping mall, so her subjects were probably more likely to eat out than the average person.

5. (a) Because the null hypothesis is false, he might make a type II error, failing to reject the null when it is false.
   (b) With the small sample, he would use a t-table (with $10 - 1 = 9$ degrees of freedom).
   (c) The standard error for sample average is $3/\sqrt{10} \approx .95$, and from the t-table we see that he should allow $2.262(.95) \approx 2.15$ pounds above and below 7 and still not reject the null.
   (d) $7 - 2.15 = 4.85$ pounds corresponds to $t = (4.85 - 8.5)/(3/\sqrt{10}) \approx 3.84$, so there is very little chance of getting a 10-cat sample with average weight lower than the low end of the "do not reject" interval. On the other hand, $7 + 2.15 = 9.5$ corresponds to $t = (9.15 - 8)/(3/\sqrt{10}) \approx .69$, and from the cumulative z-table (we should really use a t-table, but our version of that table doesn't give enough info) we see that the probability of getting a result within the "do not reject" interval — and therefore making a type II error — is about 75%.

6. In favor of Kirp's point that environment, and in particular socio-economic class, strongly affects I.Q. was the fact that, both for children with well-to-do biological parents and also for children born into impoverished families, as the class of the adopting parents went up, so did the children's I.Q. So more family money can raise a child's I.Q., regardless of the child's genetics. On the other hand, against his point is the fact that the lowest group I.Q's among children with well-to-do biological parents was still higher than the highest group I.Q.'s of children with impoverished parents. So family money cannot be the only, and perhaps not even the main, factor in determining a child's I.Q.