May 9, 2003

Math 102 / Core 143 Section AX and BX — Final Exam II

Show all work clearly for partial credit — an unevaluated expression is worth more than the numerical answer.

1. *(24 points)* A survey is taken of 1600 households in a large city; it is found that 900 of them heat with electricity, and that the average monthly electric bill (for all 1600) is \$250, with an SD of \$120.

   (a) Assuming that the households surveyed were simple random sample of the city, what is a 95% confidence interval for the average monthly electric bill in the city?

   (b) How many households would the survey have had include to get a 95% confidence interval only half as wide (assuming similar average and SD)?

   (c) The power company claims that 60% of the city's households use electric heat. Is the survey's result significant evidence that the company's estimate is too high?

   (d) Suppose the survey sample was conducted by e-mail, using the client list of an internet service provider. How might that have biased the results?

2. *(20 points)* A "Magic 8-Ball" is a toy that is almost spherical but with one flat window into its interior. A dodecahedron (12-sided solid) floats in the black fluid inside the 8-ball; when the 8-ball is held with the window up, one side of the dodecahedron floats up to the window, and the user can read what is printed on that side. Four sides of the dodecahedron say "Yes", four say "No", three say "Maybe", and one says "Ask again later". The manufacturer tests the the 8-ball by shaking it 240 times and noting results. She gets 85 "Yes", 70 "No", 63 "Maybe" and 22 "Ask again later". Is the 8-ball fair?

3. *(13 points)* From a new batch at the Utica Club Brewery, 16 bottles of Erie Canal Hard Water are sampled for levels of duclamine (an ingredient necessary for that authentic rock-strewn flavor). The specifications for ECHW say it should contain 3 mg of duclamine per ounce; the 16 bottles averaged 2.8 mg per ounce, with an SD of 0.7 mg. We are to decide whether the sample's higher level batch means (with 95% certainty) that the whole batch's duclamine level is too low. Complete as much of the decision process as you can.

4. *(16 points)* Recall that a "straight" deck of cards (i.e., one used for bridge or poker) has 52 cards, in 13 ranks and 4 suits.

   (a) If two cards are chosen at random <u>without</u> replacement, what is the probability that both of them are clubs?

   (b) If a card is chosen at random, what is the probability that it is either a king or a club?

   (c) What is the probability of getting a king at least once if a card is drawn 8 times <u>with</u> replacement?

   (d) What is the probability of getting a <u>club</u> exactly 5 times if a card is drawn 8 times <u>with</u> replacement?

5. *(15 points)* At Turning Stone Casino, you decide to play roulette 100 times, betting \$1 each time.

   (a) What are your expected total winnings, give or take how much, if you bet splits (2 winners out of 38 numbers, paying 17 to 1)?

   (b) What are your expected average winnings, give or take how much, if you bet sections (12 winners out of 38 numbers, paying 2 to 1)?

   (c) With which game are you more likely to lose more than \$10? Explain.

6. *(15 points)* The employment rate (fraction of the work force that is employed) averages 87% in New York villages, with a SD of 10%, and the welfare rate (fraction of the population on welfare) averages 8% with an SD of 6%. The correlation between the rates is -0.6.

   (a) What should we guess is the welfare rate in a village with an employment rate of 92%?

   (b) How far should we expect our guess in (a) to be off?

   (c) If we found that the welfare rate in that village was really 7%, what is the corresponding residual?

7. *(12 points)* In a small community, an unusually large number of people seem to have colds, and many of the sufferers are found to frequent the community's only restaurant. Answer each of the following in a sentence or two:

   (a) How might going to the restaurant have caused its customers to have colds?

   (b) How might the fact that they have colds caused people to eat at the restaurant?

   (c) What other factor may have caused people both to get colds and to eat at the restaurant?

8. *(10 points)* Relative to the article "Monitor after-school programs carefully" by Megan Beckett: Describe how this article calls for methods from the current section of this course [significance tests] and from the first section on experimental design. Why are both concepts needed?

Math 102 / Core 143 AX and BX— Solutions to Final Exam II

1. (a) $\$250 \pm 2(\$120/\sqrt{1600}) = \$250 \pm \$6$.
   (b) Four times as many households, or 6400.
   (c) Taking the power company's estimate as the null hypothesis, we perform a z-test: Because the SE for percent is (using the figure from the null hypothesis)
   $$(1-0)\sqrt{(.6)(.4)}/\sqrt{1600} \approx 1.2\% \ ,$$
   we have
   $$P(\% \le 900/1600 \approx 56\%) = P(z \le (56\% - 60\%)/1.2\% \approx -3.1) < 5\% \ ;$$
   so we reject the NH: the power company's estimate is too high.
   (d) One possible answer: People with e-mail addresses are more likely to be technologically inclined, and hence to prefer electric heat to, say, oil heat.

2. We expect counts of 80,80,60,20 of a fair 8-ball, and we got 85,70,63,22; so we can do a χ^2-test to decide whether there is a significant difference between the two.
   $$\begin{aligned} \chi^2 &= \frac{(85-80)^2}{80} + \frac{(70-80)^2}{80} + \frac{(63-60)^2}{60} + \frac{(22-20)^2}{20} \\ &= \frac{25}{80} + \frac{100}{80} + \frac{9}{60} + \frac{4}{20} = \frac{25+100+12+16}{80} \approx 1.91 \end{aligned}$$
   and with $4-1=3$ degrees of freedom, we see from the table that a χ^2-value that high occurs by chance between 70% and 50% of the time, not less than 5%; so we do not reject the null hypothesis: The 8-ball is fair.

3. With the small sample, we conduct a t-test, with the null hypothesis that the batch's duclamine level is the required 3 mg per ounce, and using the SD^+ of the sample to estimate the SD of the batch: The SE is $.7(\sqrt{\frac{16}{15}})/\sqrt{16} \approx .18$, so $t = (2.8-3)/.18$, which is slightly less than -1. The degrees of freedom is $16-1=15$. We don't have a t-table, but it would probably show that the probability of a t-value that low or lower is not less than 5%, so we would not reject the null hypothesis and would accept the batch.

4. (a) $(13/52)(12/51) = 1/17 \approx 6\%$.
   (b) $(4/52) + (13/52) - (1/52) = 16/52 = 4/13 \approx 31\%$.
   (c) $1 - (12/13)^8 \approx 47\%$
   (d) $C(8,5)(1/4)^5(3/4)^3 \approx 2\%$

5. (a) The EV of the sum is $(\frac{2}{38}(17) + \frac{36}{38}(-1))(100) = \frac{-100}{19} \approx -5.3$ dollars, and the SE of the sum is $(17-(-1))\sqrt{\frac{2}{38}\cdot\frac{36}{38}}(\sqrt{100}) \approx 40.2$ dollars.
   (b) The EV of the average is $(\frac{12}{38}(2) + \frac{26}{38}(-1)) = \frac{-1}{19} \approx -0.053$ dollars, and the SE of the average is $(2-(-1))\sqrt{\frac{12}{38}\cdot\frac{26}{38}}/\sqrt{100} \approx .139$ dollars.
   (c) Multiplying our answers to (b) by 100, to compare sums of winnings, we see that with both games we expect to lose 5.3 dollars, but the greater variability of playing splits (40 vs. 14 dollars) makes it more likely to lose more than 10 dollars with that game rather than playing sections.

6. (a) The employment rate in standard units is $(92\% - 87\%)/10\% = .5$, so we should expect the welfare rate in standard units to be $(-.6)(.5) = -.3$, or in percent, $8\% + (-.3)(6\%) = 6.2\%$.

   (b) By the RMS error for regression: $\sqrt{1 - (-.6)^2}(6\%) = 4.8\%$.

   (c) $7\% - 6.2\% = .8\%$. (I won't take off points for the sign, because we haven't talked about it much.)

7. (a) Someone in the restaurant, perhaps even an employee, might have spread the cold to those around him/her.

   (b) Cold sufferers might not feel up to cooking for themselves and decide to go out to eat.

   (c) One possibility: If it's a college town, students who are studying for finals late into the night are more susceptible to colds and less likely to feel like cooking for themselves.

8. Beckett points out that, although California schools have been given a large amount of money ($550 million, because I have the article beside me) for after-school programs, little research has been done on which ones give the best results. She urges testing, using random assignment to test and control groups (which she calls the "gold standard" of experimental design), to look for statistically significant improvements in measurements of the qualities that (she says) everyone wants such programs to have. For instance, one such quality is "a safe place to stay for youngsters who don't have a parent at home when school lets out"; so one might compare the accident rates for students in one such program versus another — probably <u>not</u> versus students who are not in any program, because that would not tell whether one program is better than another, but only whether a program was better than no program at all (though of course a badly run program may allow more accidents than a student would experience who simply goes home to watch TV). Both parts are needed because the test must be fair (requiring good experimental design) and the results interpreted unambiguously (in the standard scientific manner) for the results to gain general acceptance and be trustworthy bases for establishment of new programs.