May 4, 1999

Math 102 / Core 143, Sections B and C — Final Exam

Explain answers and show all work _clearly_. Be sure to complete both pages of the exam.

1. *(8 points)* Suppose a high correlation is found between the amount of welfare support an unwed mother receives and the number of children she has. Give 3 ways, in as many different directions as possible, for interpreting possible causation for this association.

2. *(20 points)* It is found that the longer a fetus is exposed to ultrasound (in the now-common prenatal examination procedure), the lower its birth weight is likely to be. Suppose that for a large collection of babies, total ultrasound exposure averaged 5 hours, with an SD of 1 hour; that birth weights averaged 8 lbs, with an SD of 2 lbs; and that the correlation coefficient was -0.6.
    (a) What is the equation of the regression line for predicting birth weight from ultrasound exposure?
    (b) If a fetus is exposed to 4.5 hours of ultrasound, what should you guess its birth weight will be?
    (c) Assuming the data is homoscedastic, how far wrong should you expect your answer in (b) to be?
    (d) If a fetus is exposed to 5 hours of ultrasound (the average amount) and weighs 8.7 lbs at birth, what is the value of the corresponding residual?

3. *(10 points)* A "big die" in the form of a decagon (10 pentagonal faces) has the numbers 1 through 10 on it, one on each face.
    (a) On one roll of a big die, are the events "getting an even number" and "getting a number at least 4" independent? Explain.
    (b) If a big die is rolled 6 times, what is the probability of getting exactly four 3's?

4. *(20 points)* A gambler proposes a game: You shuffle a deck, draw a card, note its rank (for example, king or five), return it to the deck, reshuffle, and pick a second card. If the cards you pick are the same rank (for example, both kings or both fives), he will pay you \$11, but if they are of different ranks, you must pay him \$1. Suppose you play 130 times.
    (a) How much money should you expect to win, give or take how much?
    (b) Approximately what is the probability of your at least breaking even?

5. *(15 points)* Ten percent of the households in a market survey of a simple random sample of 100 households across the country use Excrud dish detergent.
    (a) Give a 95% confidence interval for the percentage of all households in the country that use Excrud.
    (b) How many households should have been surveyed to give a 95% confidence interval only one-third as wide as in (a)?

6. *(20 points)* Two hundred pregnant women are paired off randomly. One of each pair, chosen by coin flip, has an ultrasound examination as part of her prenatal care, while the other is examined by traditional methods. When the babies are born, it is found that, in 62 of the 100 pairs, the baby who received ultrasound examination as a fetus was lighter than the one who did not. It is decided to apply a z-test for statistical significance.
   (a) State the Null Hypothesis.
   (b) What significance level results?
   (c) What do we conclude?

7. *(8 points)* Suppose two identical coins are flipped 100 times, and the flips are sorted by the number of heads. What should the expected frequencies be for use in a χ^2-test for the fairness of the two coins?

8. *(24 points)* Short answers. Explanations may yield extra credit, especially for the true/false questions.
   (a) One of the last articles we read concerned cancer risk for whom?
   (b) True or false: If, in each county in New York State, the percentage of Republicans who vote in a given election is greater than the percentage of Democrats who vote, then the percentage of Republicans in the state who vote is greater than the percentage of Democrats in the state who vote.
   (c) If we deal two cards from a (straight=poker=bridge) deck and put them side by side face down on the table, what is the probability that both are hearts?
   (d) The "Science Wars" article discussed what factor affecting interpretation of scientific findings?
   (e) For a significance test of data from an experimental sample of 15 subjects, what statistic is used if a "bootstrap" estimate of the SD of the population is needed?
   (f) True or false: Even if the null hypothesis in an experiment is true, the experiment may produce scientifically significant results that deny the null.
   (g) True or false: If we play a fair game (one with expected value $0) more and more times, we should expect our total winnings to get closer and closer to $0, though it may vary up and down as we play.
   (h) From a house plant that is particularly tall for its species, we take a cutting and root and plant it, but it fails to reach the height of the original. We conclude that we failed to provide for the cutting the same care as the original received. What error have we committed?

Solutions to final for Sections B and C of Math 102 / Core 143

1. (i) Simple: Welfare payments are alloted on the basis of the number of children, so more children cause more money. (ii) More indirect: Perhaps the welfare payments are more generous than necessary to support a child, so that having more children gives her more spending money; in this case, she would have more children in order to get more money, i.e., more money causes more children. (iii) Still more indirect: Both unwed motherhood and the need for welfare support reflect a failure of the educational system to prepare children for adult responsibilities; i.e., a separate common basis causes both effects.

2. (a) Using y for birth weight in pounds and x for ultrasound exposure in hours: $y - 8 = (-.6)(2/1)(x - 5)$, or equivalently $y = -1.2x + 14$.
   (b) $y = -1.2(4.5) + 14 = 8.6$ pounds.
   (c) The RMS error for regression, $\sqrt{1 - (-.6)^2} \cdot 2 = 1.6$ pounds.
   (d) Since the value on the regression line corresponding to 5 hours of ultrasound is the average birthweight, 8 pounds, the resulting residual is $8.7 - 8 = .7$ pounds.

3. (a) In general, the probability of getting an even number is 50%. If we are given that we get a roll of at least 4, then there are 7 equally likely possible outcomes, of which 4 are even, so the probability of getting an even number is $4/7$, greater than 50%. So the events are not independent.
   (b) $C(6, 4)(1/10)^4(9/10)^2 = 15 \cdot 81/(10^6) = .001215$

4. (a) Since the probability of matching the suit of the first card is $1/13$, you should expect to win EV of sum $= (130)(\frac{1}{13}(11) + \frac{12}{13}(-1)) = -10$, give or take SE of sum $= \sqrt{130}\,(11 - (-1))\sqrt{\frac{1}{13} \cdot \frac{12}{13}} \approx 36.5$
   (b) Breaking even means winning at least 0 dollars, which corresponds to a z-value of $(0 - (-10))/36.5 \approx .27$; and from the normal table, the probability of getting a z-value at least as large as 0.27 is about $(100 - 20)/2 = 40$ percent — not great, but not as bad as might be expected.

5. (a) Using a bootstrap procedure to find the SE of percentage of homes using Excrud, we find $SE = (1 - 0)\sqrt{(.1)(.9)}/\sqrt{100} = .03 = 3\%$. So the 95% confidence interval is $10\% \pm 2(3\%)$, i.e. from 4% to 16%.
   (b) We would need to the decrease the computed SE by a factor of 3, which we can only do by increasing the sample size by a factor of $3^2 = 9$; so we would need to survey 900 homes to decrease the interval by a factor of 3.

6. (a) The null hypothesis is that ultrasound has no effect on birthweight, so that, in each pair, it is equally likely that the baby who had received ultrasound treatment is lighter or that the one who did not is lighter.
   (b) With this null hypothesis, we expect that the number of pairs in which the ultrasound baby is lighter is 50 out of the 100 pairs; and the standard error of the count is $\sqrt{100}(1-0)\sqrt{(.5)(.5)} = 5$. So 62 corresponds to a z-value of $(62 - 50)/5 = 2.4$, and the probability of getting a z-value at least that large, according to the normal table, is about $(100 - 98)/2 = 1$ percent.
   (c) We reject the null hypothesis: Ultrasound does affect birthweight.

7. Since the probability of 0 heads is 1/4, of 1 head is 1/2, and of 2 heads is 1/4, the expected frequencies in 100 flips are 25, 50 and 25 respectively.

8. (a) Female spouses of smokers.
   (b) False: Simpson's paradox may be at work.
   (c) $(13/52)(12/51) = 1/17$
   (d) Bias, specifically the scientists' preconceptions.
   (e) Use the SD^+ (rather than the usual SD) of the sample.
   (f) True. (This is a "Type I error," and it should occur about 5% of the time. A "Type II error" is accepting the null hypothesis when it is false; its frequency cannot be projected because we don't know how far wrong the null hypothesis is.)
   (g) False: The Law of Averages states that, though our average winnings should approach 0, we should expect our total winnings to diverge from 0, at a rate approximately proportional to the square root of the number of times we play.
   (h) The Regression Fallacy.