The Shortcut Formula for the SD of a 2-Value Box

Suppose we have a box with tickets having only 2 different values them; denote the larger value by L and the smaller by s. Let p denote the fraction (or percentage) of the tickets in the box having the larger value; then $1 - p$ is the fraction having the smaller value. The average of the box is

$$A = pL + (1 - p)s \ ,$$

so the tickets with the larger value have deviation from the average

$$\begin{aligned} L - A \ &= L - (pL + (1 - p)s) = L - pL - (1 - p)s \\ &= (1 - p)L + (1 - p)s = (1 - p)(L - s) \ , \end{aligned}$$

and the tickets with the smaller value have deviation

$$\begin{aligned} s - A \ &= s - (pL + (1 - p)s) = s - pL - (1 - p)s \\ &= s - pL - s + ps = -p(L - s) \ . \end{aligned}$$

The RMS of these deviations gives the SD of the box:

$$\sqrt{p[(1 - p)(L - s)]^2 + (1 - p)[-p(L - s)]^2}$$

We can take the (positive) common factor $(L - s)$ out of the radical — it is squared under the radical, but taking it out removes the square — and have

$$(L - s)\sqrt{p(1 - p)^2 + (1 - p)(-p)^2}$$

Now $(-p)^2 = p^2$, so each term under the radical has a factor of $p(1 - p)$; taking it out leaves

$$\begin{aligned} (L - s)\sqrt{p(1 - p)((1 - p) + p)} \ &= (L - s)\sqrt{p(1 - p)(1)} \\ &= (L - s)\sqrt{p(1 - p)} \ . \end{aligned}$$

Thus, the formula for the SD of a 2-value box can be expressed as: the difference between the two values in the box times the square root of the product of the fraction of the box with the larger value times the fraction of the box with the smaller value.