Exam 2 Math 112 October 19, 2006

NAME_____

- PLEASE NOTE: You may use the table of integrals provided wherever it applies. You must state the number of the formula that you are using as well as values for each symbol used in the table entry (e.g. u and a).
- No calculator is allowed. You do not need to evaluate numerical expressions. For example, an answer such as $2(10.3 + (2)(11.1) + (3)(20.0))$ would be acceptable; you do not have to simplify.
- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, "I can't tell you."
- Do the problems that you find easiest first. Take deep breaths between questions.
- There are 100 points on this exam, and you have 120 minutes.
- I hope you all do well. Good luck!

Question	Points	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	6	
11	4	
TOTAL	100	

Error Formulas:

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

Trigonometric Product Formulas:

$$\sin(A)\sin(B) = \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos(A)\cos(B) = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin(A)\cos(B) = \tfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

1. (10 pts) A helicopter raises a bucket of water from the ground to help put out a forest fire. The bucket (without water) weighs 150 pounds. Initially, the bucket holds 1000 gallons of water, but as the bucket is lifted, it leaks water at the rate of 15 gallons every second. Find the work done by the helicopter to raise the bucket of water 500 feet in 25 seconds at a constant velocity. Assume that one gallon of water weighs 8 lbs. You may stop simplifying your answer when it is an expression that contains only numbers.

2. (10 pts) Set up, but DO NOT EVALUATE, an integral that gives the circumference of the ellipse defined by the equation

$$x^2 + \frac{y^2}{9} = 1$$

3. (10 pts) Use the method of partial fractions to convert each of the following rational functions into a form that could be integrated.

   Do not solve for the unknown coefficients in your expansion, and do not integrate anything!

   (a) $f(x) = \dfrac{x^3 + 3}{x^2 - x - 6}$

   (b) $f(x) = \dfrac{x + 1}{(x - 1)^2}$

   (c) $f(x) = \dfrac{1}{(x + 2)(x^2 + 4x + 5)}$

4. (10 pts) Evaluate the integral $\int \frac{x^3 - 2x}{(2x^2 + 4)^{1/3}}\, dx$

5. (10 pts) Evaluate the integral $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)\, dx$

6. (10 pts) Evaluate the integral $\int \frac{\cos^5(x)}{\sqrt{\sin\, x}}\, dx$

7. (10 pts) Evaluate the integral $\int \frac{x}{\sqrt{x^2+8x+9}}\, dx$

8. (10 pts) Evaluate the integral $\int \cos(\ln x)\, dx$

9. (10 pts) Evaluate the integral $\int \dfrac{x+5}{x^2+4x+5}\, dx$

10. (6 pts) The speed v of a car (in miles per hour) is measured each half second over a period of 4 seconds. The following table shows the measurements:

t (seconds)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
v (mph)	0.0	3.2	10.1	17.0	25.3	31.0	37.0	42.1	46.7

Approximate the distance traveled by the car by using two methods:

(a) the trapezoidal rule, and

(b) Simpson's rule.

11. (4 pts) Estimate the error in the approximation if the integral

$$\int_0^4 -\frac{x^3}{6} + \frac{x^2}{2} + 8\,dx$$

is approximated by using the trapezoidal rule with only four intervals.

(Do *not* find the approximate value of the integral!)