

NAME \_\_\_\_\_

- No calculator is allowed. Do not round off answers. If you get  $10/6$  as an answer, you should leave your answer as  $5/3$ . No decimal approximations, please.
- For problems with numerical approximation of integrals, write your answer as an expression involving only numbers (no symbols). You do not need to evaluate that expression.
- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, "I can't tell you."
- Do the problems that you find easiest first. Take deep breaths between questions.
- There are 100 points on this exam, and you have 120 minutes.
- I hope you all do well. Good luck!

Take three deep breaths.

| Question | Points | Points Earned |
|----------|--------|---------------|
| 1        | 10     |               |
| 2        | 8      |               |
| 3        | 8      |               |
| 4        | 7      |               |
| 5a       | 7      |               |
| 5b       | 7      |               |
| 5c       | 7      |               |
| 5d       | 7      |               |
| 5e       | 7      |               |
| 6        | 8      |               |
| 7        | 7      |               |
| 8        | 10     |               |
| 9        | 7      |               |
| TOTAL    | 100    |               |

What information have you been given?

What information do you need?

How can you get from one to the other?

Error Formulas:

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

Trigonometric Product Formulas:

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$\sin(A) \cos(B) = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

1. (10 points) Identify whether each series converges or diverges. Justify your answer and show your work.

(a)  $\sum_{n=0}^{\infty} (-1)^{n+1} \sin\left(\frac{\pi}{n}\right)$

(b)  $\sum_{n=1}^{\infty} \frac{2n+3}{n(n+1)}$

2. (8 points) Find the radius of convergence for the following series. You don't have to examine the endpoints—just give the radius of convergence  $R$ .

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n^6 3^n}$$

3. (8 points) Find a power series representation for the function  $f(x) = \frac{x^3}{(1-3x)^2}$  and determine the radius of convergence.

4. (7 points) Find the series representation for  $f(x) = \int e^{-(x^2)} dx$  about  $a = 0$ .

5. (35 points) Evaluate the following integrals:

(a)  $\int \frac{x^3 + 5x + 3}{x^2 + 4} dx$

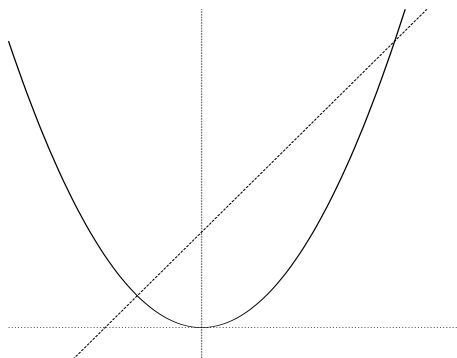
(b)  $\int \frac{x^4}{\sqrt{x^{10} + 4}} dx$

(c)  $\int \sin^3(x) \cos^2(x) dx$

(d)  $\int e^{2x} \cos(3x) dx$

(e)  $\int_0^2 x^3 e^{(x^2)} dx$  (Hint: use a  $w$ -substitution instead of a  $u$ -substitution to start.)

6. (8 points) Set up but DO NOT EVALUATE an integral representing the volume of the solid obtained by rotating the following region about the line  $x = -6$ . The region lies between  $y = 2x^2$  and  $y = 4x + 2$ . Stop when your integral contains only one variable and is ready for integration.



7. (7 points) An elevator weighs 800kg and is suspended 70m below its support by a cable. The cable weighs 10kg/m. Set up but DO NOT EVALUATE an integral for the work it takes to raise the elevator 10m (about three floors) from its current position?

8. (10 points) Find the area of the region that lies inside both curves.  $r = \cos(\theta)$ ,  $r = \sin(\theta)$

9. (7 points) Write the first three terms of the Taylor series for  $f(x) = \sqrt{x}$  about  $x = 4$ . (You don't have to find the expression for the general term or write it in summation form. Just write the first three terms.)

Use the first two terms to estimate  $\sqrt{5}$ . (FYI:  $\sqrt{5} \approx 2.236$ )