Chapter 2: Solving Linear Equations

2.7. Transposes and Permutations

`At this point, switch to the first part of the presentation for Unit 4.`

We mentioned earlier that a matrix that has one 1 in each row and column and 0's otherwise is a *permutation matrix*, because premultiplication by it permutes the rows of the other matrix:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ q & r & s \\ x & y & z \end{bmatrix} = \begin{bmatrix} q & r & s \\ x & y & z \\ a & b & c \end{bmatrix}.$$

Questions:

(a) What is the (i, j)-th entry of $A^T A$, in terms of A? [Answer: The dot product of the i-th and j-th columns of A.]

(b) In view of (a), what are the main diagonal entries of $A^T A$? [Answer: The squares of the norms of the columns of A.]

(c) If P is a permutation matrix, what is $P^T P$? [Answer: I, so $P^T = P^{-1}$. (We'll see later that this means permutation matrices are examples of "unitary matrices", but there are other unitary matrices.)]

The text also points out that, if A is invertible, then we could, if we wanted, rearrange the rows of A so that it has an LU-decomposition: $PA = LU$ where P is a permutation matrix. But how could we find that P? The author says that $PA = LU$ is useful in numerical analysis. I'll take his word for it.

`At this point, switch to the second part of the presentation for Unit 4.`