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Adjacency matrices

The Adjacency matrix of a "(di)(multi)graph (with loops)" is the $n \times n$ matrix A, where n is the number of "vertices", for which $a_{i,j}$ = the number of edges from vertex i to vertex j.

Example:

A man wants to take his pet wolf, a goat and his immense prize cabbage to market. He cannot leave the goat alone with the cabbage, or the wolf alone with the goat. He comes to a river, with a boat only large enough to accommodate him and <u>one</u> of his wares/companions. Can he cross without something being eaten?



Seeing the graph, it's not hard to come up with a solution. But is a shorter one possible?



0	0	0	0	0	1	0	0	0	0 -	1
0	0	0	0	0	0	1	1	0	0	
0	0	0	0	0	1	1	0	1	0	l
0	0	0	0	0	0	0	1	1	0	
0	0	0	0	0	0	0	1	0	1	
1	0	1	0	0	0	0	0	0	0	
0	1	1	0	0	0	0	0	0	0	
0	1	0	1	1	0	0	0	0	0	
0	0	1	1	0	0	0	0	0	0	
0	0	0	0	1	0	0	0	0	0	

According to R,

	Γ Ο	0	0	0	0	22	23	16	23	2	1
$A^7 =$	0	0	0	0	0	39	63	84	62	23	
	0	0	0	0	0	68	84	80	84	16	ĺ
	0	0	0	0	0	39	62	84	63	23	ĺ
	0	0	0	0	0	18	39	68	39	22	
	22	39	68	39	18	0	0	0	0	0	
	23	63	84	62	39	0	0	0	0	0	
	16	84	80	84	68	0	0	0	0	0	
	23	62	84	63	39	0	0	0	0	0	
	2	23	16	23	22	0	0	0	0	0	

but no smaller power of A has (1, 10)-th entry nonzero, so we have the two shortest solutions.