Practice Problems
Math 310
November 16, 2003

1. (Total 10 pts.) Consider the network in Figure 1.

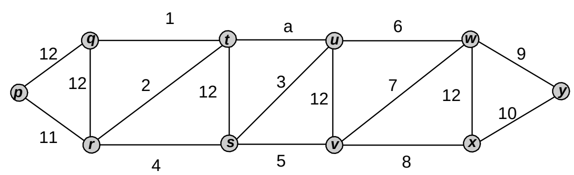

Figure 1: Network N.

(a) (3 pts.) Determine all values of integer (positve or negative) a for which N contains the edge labeled a in one of its minimal spanning trees.

(b) (3 pts.) Determine all values of a for which N contains a unique minimal spanning tree.

(c) (4 pts.) For $a = 4$ list the edges of the minimal spanning tree.

2. (Total 10 pts.) We form "words" (sequences of letters with repetition) composed of the vowels: `a, e, i, o , u.`

   (a) (2 pts.) How many six-letter "words" are there?

   (b) (2 pts.) How many six-letter words are ther with exactly one `a`?

   (c) (2 pts.) How many six-letter words are there that are the same when the order of their letters is inverted (e.g. `aeiiea`).

   (d) (4 pts.) Repeat all three subproblems for n-letter words.

3. (Total 10 pts.) How many ways are there to pick 2 different cards from a standard 52-card deck such that:

   (a) (3 pts.) The first card is not a King and the second card is not a Queen?

   (b) (3 pts.) The first card is a spade and the second card is not a spade?

   (c) (4 pts.) The first card is a spade and the second card is not an Ace?

4. (Total 10 pts.) On a 10-question test:

   (a) (3 pts.) How many ways are there to answer exactly 7 questions correctly?

   (b) (3 pts.) How many ways are there to answer exactly 7 questions correctly and either the first or the second question, but not both, are answered correctly.

   (c) (4 pts.) How many ways are there to answer exactly 7 questions correctly and the three out of the first six questions are answered correctly.

5. (Total 10 pts.) We are forming 8-digit sequences with digits 1,2,3,4,5,6.

   (a) (3 pts.) How many sequences can be formed?

   (b) (3 pts.) How many sequences can be formed with exactly two 3s, two 4s and two 6s. [The remaining two digits are arbitray.]

   (c) (4 pts.) How many sequences can be formed with exactly three different digits?

6. (Total 10 pts.) How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 26$ with:

   (a) (3 pts.) $x_i \geq 0$

   (b) (3 pts.) $x_i > 0$

   (c) (4 pts.) $x_i > i + 1, i = 1, 2, 3, 4, 5, 6$

7. (Total 10 pts.) How many integer solutions are there to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1234$$

with:

(a) (3 pts.) $x_i \geq 0$.

(b) (3 pts.) $x_i > 0$.

(c) (4 pts.) Write the generating function for a_r the number of integer solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = r$ with $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq x_3, x_5 \geq x_1 + x_2$.

8. (Total 10 pts.) Let B_r denote the frame network composed of $4r$ blocks, see the Figure 2 for the case $r = 4$. The lower left corner is denoted A and the upper right corner is denoted B. Let a_r denote the number of walks (shortest paths) from A to B in L_r.

Figure 2: Block-walking network B_4. See the page block.htm in the same directory

   (a) (2 pts.) Determine a_4, the number of shortest paths from corner A to corner B in B_4.

   (b) (2 pts.) Determine the values of a_r, for $r = 2, 3, 4, 5$.

   (c) (3 pts.) Determine the formula for a_r.

   (d) (4 pts.) Write the generating function for a_r.

9. (Total 10 pts.) Let $a_r = (-1/2)^r$ and let $s_n = 1 - 1/2 + ... \pm (1/2)^n$.

(a) (3 pts.) Find a generating function for a_r.

(b) (3 pts.) Find a generating function for s_n.

(c) (4 pts.) Evaluate the sum $s_n = 1 - 1/2 + ... \pm (1/2)^n$.