Definitions for Test 1 Math 310 October 1, 2003

- 1. Let G be a graph on n vertices. The sequence $[d_1, d_2, \ldots, d_n]$ is called the **degree sequence** of G if $d_1 \ge d_2 \ge \cdots \ge d_n$ and there is a labelling of vertices x_1, x_2, \ldots, x_n of G such that the vertex x_i has degree d_i .
- 2. A sequence $[d_1, d_2, \ldots, d_n]$ consisting of non-negative integers: $d_1 \ge d_2 \ge \ldots \ge d_n$ is called **graphical** if it is a degree sequence of some graph.
- 3. Consider the graphs determined by moves of chess pieces (chessmen): Queen (Q), King (K), Rook (R) and Bishop (B) on an $n \times n$ chessboard. The corresponding graphs are denoted by Q(n), K(n), R(n), B(n). So Q(n) has vertices squares of the chessboard and two squares s_1 and s_2 are adjacent if and only if a queen can move from s_1 to s_2 in a single move.
- 4. A path P_n with endvertices u and v is a graph with vertices $u = x_1, x_2, ..., x_n = v$ and the following and jacencies: $x_1 \sim x_2, x_2 \sim x_3, ..., x_{n-1} \sim x_n$.
- 5. A sun with k rays and range n_1 is a rooted tree $S(k; n_1, n_2, ..., n_k), n_1 \ge n_2 \ge ... \ge n_k$ formed out of k disjoint paths $P_{n_1+1}, P_{n_2+1}, ..., P_{n_k+1}$ by identifying a common endpoint, the root.



Figure 1: The grid graph G(5,5).

6. A grid graph G(m, n) is a graph on $m \times n$ vertices, obtained by interlocking *m* horizontal copies of P_n and *n* vertical copies of P_m . For instance, a circuit graph C_4 can be regarded as G(2, 2).

7. Graph G_3 .



Figure 2: Graph G_3 .

- 8. A generalized Petersen graph P(n,r) has 2n vertices: $u_0, u_1, ..., u_{n-1}, v_0, v_1, ..., v_{n-1}$ and edges $u_i u_{i+1}, u_i v_i, v_i v_{i+r}$ (where addition is taken mod n).
- 9. A triangle is a circuit of length 3.
- 10. A graph is **cubic** if each of its vertices has degree 3.



Figure 3: Truncation and triangle contraction.

- 11. A **truncation** of a vertex x in a cubic graph G is a process in which x is replaced by a triangle to which the edges a, b, c that were incident to x are reattached.
- 12. A triangle contraction in a cubic graph is the process opposite to a vertex truncation.
- 13. Let S be any finite set of binary strings: Let G(S) be the graph obtained from the set S in the following way: the vertices are the binary strings from S. Two strings are adjacent if and only if one can be obtained

from the other either by dropping the last digit or by adding a digit. This means, for instance, that "000" is adjacent to "00", "0000", and to "0001" but "0001" is not adjacent to "00" or "0000".