Math 312 Dynamic Models Part I
Big Picture Lessons Summary

For time dependent models with continuous time variable t:

1. Simple nonlinearities in systems with 3 (or more) variables lead to chaotic unpredictable behavior. That behavior may be restricted to a finite region where a "strange attractor" resides, but it is unpredictable where on the attractor the system will be.

2. We can expect to see one of the following behaviors in the long run:
   a) movement toward a steady state (exponential decay to equilibrium)
   b) movement toward a limit cycle (stable oscillating system)
   c) movement toward a strange attractor (the system is confined to a region of phase space, but not predictable within that region).
   d) some variables heading toward infinity (prices, income, population, crime)

3. It is rare that a steady state is stable. Every eigenvalue of the steady state must have a negative real part for the steady state to be stable. As the number of variables increases this becomes less and less likely (for arbitrarily chosen parameter values). There may be important situations where symmetries ensure that a steady state be stable, but we should not expect stable equilibria. Strange attractors are much more likely to be governing the behavior of most complicated systems.

4. We should try to find situations where equilibria occur because they often drive the behavior of the system, or create separatrices (tipping points) which distinguish one long term behavior from another.

5. We should probably not try to study equilibria without regard to stability because while the implication is that a system tends toward equilibrium, we know that most steady states will not be stable.

Modeling tips

Make sure the model's results are robust to small changes in the model.

Take dynamics into account when studying any system.
(Change is the only constant.)