



COMPOSITE NUMBERS THAT REMAIN COMPOSITE WHEN INSERTING, REPLACING, OR DELETING A DIGIT

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Abstract

Expanding on the work done by Filaseta, Southwick, and the second author on widely digitally stable composite numbers, we show that there are infinitely many composite numbers N , coprime to 10, with the property that if any digit is replaced, deleted, or a digit is inserted anywhere in its base 10 representation, including the infinitely many leading zeros of N , then the resulting number will remain composite.

1. Introduction

Answering a question posed by Klamkin [11], Erdős [3] in 1979 showed that there are infinitely many prime numbers N with the property that if any digit (in base 10) is changed to any other digit, then that prime becomes composite. The first such prime is 294001, meaning that every number in the set

$$\{d94001, 2d4001, 29d001, 294d01, 2940d1, 29400d : d \in \{0, 1, 2, \dots, 9\}\}$$

is either 294001 or composite. Such primes are called *digitally delicate*. The proof of Erdős utilized partial covering systems and a sieve argument. In 2011, Tao [16] refined the sieve argument used by Erdős to show that a positive lower asymptotic density of primes are digitally delicate.

In 2021, Filaseta and Southwick [8] considered *widely digitally delicate* prime numbers; that is, primes that become composite when changing any digit to any other digit, including any of the infinitely many leading zeros. They showed, in a similar argument to that of Tao, that a positive proportion of primes are widely digitally delicate. Observe that the digitally delicate prime 294001 is not widely digitally delicate since 10294001 is prime. Their result was mentioned in Quanta magazine [13] and Grantham [10] gives an explicit example of a widely digitally delicate prime with 4032 digits.

Later that year, Filaseta and the second author [4] gave an alternate proof that there are infinitely many widely digitally delicate primes using only covering systems, eliminating the need for a sieve argument. Combined with a result by Shiu [15], they proved that for every positive integer k , there exist k consecutive primes all of which are widely digitally delicate. See also [2] and [14].

The investigation into widely digitally delicate primes continued by combining the notion with Brier numbers. An odd number k is a *Sierpiński* number if $k \cdot 2^n + 1$ is composite for every positive integer n . Similarly, an odd number k is a *Riesel* number if $k \cdot 2^n - 1$ is composite for every positive integer n . A *Brier* number is an odd integer that is both a Sierpiński and Riesel number. In 2023, Filaseta, Luckner, and the second author [5] proved that for every positive integer k , there exist k consecutive primes each of which are both widely digitally delicate and a Brier number. Furthermore, they showed that a positive proportion of the primes are both widely digitally delicate and a Brier number. The authors of [5] also note that for every positive integer k , there exist k consecutive primes each of which are widely digitally delicate in both base 2 and base 10. Their proof utilizes the coverings from [4], as well as two new coverings; one to ensure the primes are Sierpiński numbers and another to ensure the primes are Riesel numbers.

Instead of considering prime numbers that lose their primality when manipulating their digits, this paper is concerned with composite numbers that preserve their composite nature when manipulating their digits. In 2013, Konyagin [12] proved that a positive lower asymptotic density of composite numbers, that are coprime to 10, remain composite when any base 10 digit is changed to another base 10 digit. For example, the number 212159 satisfies this property, meaning that every element in the set

$$\{d12159, 2d2159, 21d159, 212d59, 2121d9, 21215d : d \in \{0, 1, 2, \dots, 9\}\}$$

is composite. Observe that being coprime to 10 is a necessary requirement to get intriguing results. Without it, the numbers

$$N = (11 \cdot 13 \cdot 17 \cdot 19 \cdot 10)m + 15 \quad \text{for } m \in \mathbb{N}$$

will remain divisible by 5 for any digit change that is not the right-most digit. A similar result can be established by replacing the 15 with a 12.

In 2010, Filaseta, Kozek, Nicol, and Selfridge [7] considered inserting a digit between any two digits instead of replacing a digit. That is, they showed that there exist infinitely many composite numbers N , coprime to 10, that remain composite when inserting a base 10 digit between any two digits of N and to the right of the units digit of N . The first such composite number is 25011, meaning that each number in the set

$$\{d25011, 2d5011, 25d011, 250d11, 2501d1, 25011d : d \in \{0, 1, 2, \dots, 9\}\}$$

is composite. A similar result for prime numbers is currently not known to hold.

Following along the same path as widely digitally delicate primes, Filaseta, Southwick, and the second author [6] extended the above result to also look at inserting a digit between any two of the infinitely many leading zeros of N . They proved that there are infinitely many *widely digitally stable composite numbers*; that is, composite numbers N , coprime to 10, that remain composite when inserting a digit between any two digits of N , including between any two of the infinitely many leading zeros, or to the right of the right-most digit of N . Note that 25011 is not widely digitally stable since 10025011 is prime. Similar to widely digitally delicate primes, constructing a widely digitally stable composite number proves to be difficult and would likely also result in a number with a large number of digits.

In this paper, we use the methods from [6] and [7] to prove the following stronger result.

Theorem 1. *There exist infinitely many composite numbers N , coprime to 10, that remain composite when doing any of the following: (1) replacing a digit of N with another base 10 digit, including any of the infinitely many leading zeros, (2) deleting a digit of N , or (3) inserting a digit between any two digits of N , to the right of the right-most digit of N , or between any two of the infinitely many leading zeros.*

Observe that 25011 violates all three aspects of Theorem 1 since replacing the units digit with a 3 results in the prime number 25013, deleting the digit 2 results in the prime number 5011, and inserting the digit 1 between the second and third leading zero results in the prime number 10025011.

2. The First Steps of the Argument

To prove Theorem 1, we will rely heavily on constructing coverings of the integers.

Definition 1 (Covering System). A *covering system* (or *covering*) is a finite set of congruences

$$x \equiv a_1 \pmod{b_1}, \quad x \equiv a_2 \pmod{b_2}, \quad \dots, \quad x \equiv a_r \pmod{b_r},$$

where $r \in \mathbb{Z}^+$, each $a_i \in \mathbb{Z}$, and each $b_i \in \mathbb{Z}^+$, such that every integer satisfies at least one congruence in the set of congruences.

For our use of coverings, we will allow $b_i = 1$, and the use of repeated moduli. There will be restrictions on the moduli that will arise in the coverings we construct that will be made clear as we proceed. To discuss the specific way we will construct the coverings used in this paper, we begin by considering

$$N = \frac{10^n - 1}{9} + M,$$

where n and M are large natural numbers satisfying criteria imposed from the coverings that will be mentioned in the next section, and n is large enough that the left-most digit of N is 1. Then N can be written as

$$N = 00 \dots 00 \underbrace{11 \dots 11}_{n \text{ total 1's}} + M,$$

where $11 \dots 11$ is a string of n digits that are all 1's. Thus, N has three parts: the infinitely many leading zeros, a string of leading ones, and a finite number of unknown digits resulting from M .

For a nonnegative integer k , let $N^{(k)}(x)$ denote inserting the digit x to the right of the $(k+1)^{st}$ digit of N . For example, $N^{(2)}(x)$ represents inserting x to the right of the third digit of N , which looks like

$$N^{(2)}(x) = 00 \dots 00d_{n-1}d_{n-2} \dots d_2xd_1d_0.$$

Set K to be one more than the number of digits of M ; thus, initially, K is unknown to us. Observe that $d_k = 1$ for $K \leq k \leq n-1$. We can formulate $N^{(k)}(x)$ above nicely based on the target location of the insertion. Specifically, for $k \geq n$, the value of $N^{(k)}(x)$ takes the form

$$N_0^{(k)}(x) = \frac{10^n - 1}{9} + M + x \cdot 10^k.$$

If $x \in \{0, 1, \dots, 9\}$ and $k \geq n$, then $N_0^{(k)}(x)$ represents inserting a digit to the right of one of the infinitely many leading zeros of N , which is the same as replacing one of the infinitely many leading zeros with the digit x . Also note that if $x \in \{-1, 0, \dots, 8\}$ and $K \leq k < n$, then $N_0^{(k)}(x)$ represents replacing one of the leading ones of N with $x \in \{0, 1, \dots, 9\}$. While $N_0^{(k)}(x)$ is written specifically for $k \geq n$, we do not know n until the very end; thus, we will ensure that $N_0^{(k)}(x)$ is composite for each integer $k \geq 1$ and $x \in \{-1, 0, \dots, 9\}$. To do so, we will construct a covering for the exponents k on 10 for each value of $x \in \{-1, 0, \dots, 9\}$ and impose criteria on n and M for each congruence in each covering. Details on these criteria will be presented in the next section.

For $K \leq k < n$, the value of $N^{(k)}(x)$ takes the form

$$N_1^{(k)}(d) = 10^n + \frac{10^n - 1}{9} + M + (d - 1) \cdot 10^k,$$

and represents inserting the digit d to the right of a leading one. To avoid confusion, we will use x when referring to inserting the digit x using $N_0^{(k)}(x)$, and d when referring to inserting the digit d using $N_1^{(k)}(d)$. To ensure N remains composite when inserting a digit to the right of a leading one, we will construct a covering for the exponents k on 10 for each value of $d \in \{0, 1, \dots, 9\}$.

Once we construct coverings for k for $x \in \{-1, 0, \dots, 9\}$ and $d \in \{0, 1, \dots, 9\}$, the number N will be composite when inserting a digit in the leading zeros, inserting a digit in the leading ones, replacing a leading zero, and when replacing a leading one with another digit. Observe that deleting a leading zero does not change N , and deleting a leading one is the same as replacing the left-most one with a zero, which corresponds to $N_0^{(n)}(-1)$. Thus, after constructing the 21 coverings, and explaining the correlation between the congruences in the coverings and the conditions on n and M that ensure the changes on N result in a composite number, to finish the proof of Theorem 1 we need only ensure that inserting, replacing, or deleting a digit in the K right-most digits of N , coming from M , results in a composite number.

We clarify that the work on widely digitally stable composite numbers in [6] looked at N having a string of leading sevens and only looked at inserting a digit. To include replacing a digit, one would have to look at $x \in \{-7, \dots, 9\}$ with a modified $N_0^{(k)}(x)$, resulting in needing 6 more coverings. By changing N to having a string of leading ones we only need to look at $x \in \{-1, \dots, 9\}$, reducing the coverings needed by 5. This is the motivation behind looking at the value of N chosen; however, it does mean that we cannot use some of the coverings in [6] as they appear there.

Before discussing the more complicated coverings, we consider the unique primes 3 and 7. The prime 3 is used to complete seven of the coverings, namely $x \in \{-1, 2, 5, 8\}$ and $d \in \{2, 5, 8\}$. Note that $10 \equiv 1 \pmod{3}$, so using the covering $k \equiv 0 \pmod{1}$ we can impose the restrictions $M \equiv 1 \pmod{3}$ and $n \equiv 0 \pmod{3}$. Then for $x \equiv 2 \pmod{3}$, the value $N_0^{(k)}(x)$ is divisible by 3 and hence composite, and for $d \equiv 2 \pmod{3}$, the value $N_1^{(k)}(d)$ is divisible by 3 and hence composite. Thus, we now need coverings for $x \in \{0, 1, 3, 4, 6, 7, 9\}$ and $d \in \{0, 1, 3, 4, 6, 7, 9\}$.

We can also use the prime 7 in a similar way since 7 appears as a value for both x and d . Taking $n \equiv 0 \pmod{6}$ and $x \equiv M \equiv 0 \pmod{7}$, then $N_0^{(k)}(x)$ is divisible by 7 for $x = 0$ and 7. Thus, the covering $k \equiv 0 \pmod{1}$ suffices for $x = 0$ and 7; moreover, notice that $N_0^{(k)}(0) = N$. This means that N is guaranteed to be composite since it is divisible by 7. We now need coverings for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 1, 3, 4, 6, 7, 9\}$.

Observe that for $d = 1$, we have $N_1^{(k)}(1) = 10^n + N$ for all integers $k \geq 1$. Take a

prime $p > 10$ not used in any other covering and set $n \equiv 0 \pmod{p-1}$ and $M \equiv -1 \pmod{p}$. Then $N_1^{(k)}(1)$ is divisible by p for all values of k , and hence composite. Thus, the covering $k \equiv 0 \pmod{1}$ suffices for $d = 1$. Putting this all together, we need a total of 11 coverings, namely for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$.

3. The Coverings

Before discussing the construction of the coverings, we explain more precisely how the coverings for a specific x or d value are used to ensure that $N_0^{(k)}(x)$ or $N_1^{(k)}(d)$ is composite. Let $k \equiv a_i \pmod{b_i}$ be a congruence in the covering for a specific x value. Take $n \equiv 0 \pmod{b_i}$, and

$$M \equiv -x \cdot 10^{a_i} \pmod{p_i}$$

where p_i is a prime number such that 10 has multiplicative order b_i . Under these congruence conditions it is not hard to see that $N_0^{(k)}(x)$ is divisible by p_i whenever $k \equiv a_i \pmod{b_i}$ (see Lemma 1 in [6] and Lemma 1 in [7]). Similarly, for a fixed d , taking

$$M \equiv -1 - (d-1) \cdot 10^{a_i} \pmod{p_i}$$

and $n \equiv 0 \pmod{b_i}$ results in $N_1^{(k)}(d)$ divisible by p_i for $k \equiv a_i \pmod{b_i}$ (see Lemma 2 in [6]).

Once all the coverings have been constructed, we use the Chinese Remainder Theorem to get a single congruence condition on M and n . We will ensure such an M exists by limiting the number of times the modulus b_i appears in all coverings based on the number of unique primes p_i we can find such that 10 has order b_i modulo p_i . To find these primes p_i , we make use of the following Lemma which is discussed in [1, III C 1].

Lemma 1. *Let p be a prime dividing $\Phi_b(10)$ where $\Phi_b(x)$ is the b^{th} cyclotomic polynomial. If $p > b$, then the multiplicative order of 10 modulo p is b . If $p \leq b$ and $b > 2$, then p is the largest prime dividing b and p^2 does not divide $\Phi_b(10)$.*

Thus, we can use the modulus b a number of times equal to the number prime divisors of $\Phi_b(10)$ that do not divide b . This is what complicates the covering systems as we proceed; the primes considered in general cannot be used in multiple coverings. The number of congruences used in the coverings for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$ are found in Table 1.

To further illustrate the use of the coverings, consider the covering used for $x = 4$ in Table 8, which contains 4 congruences. The congruences for $x = 4$ are

$$k \equiv 1 \pmod{2}, \quad k \equiv 2 \pmod{4}, \quad k \equiv 0 \pmod{8}, \quad \text{and} \quad k \equiv 4 \pmod{8}.$$

x	# cong.	x	# cong.	d	# cong.	d	# cong.
-1	1	5	1	0	453	5	1
0	1	6	41	1	1	6	5
1	157	7	1	2	1	7	136
2	1	8	1	3	375	8	1
3	19	9	14	4	240	9	368
4	4						

Table 1: Number of congruences for each covering

One can easily verify that every integer k satisfies one of these congruences, so it is indeed a covering. Each congruence corresponds to a different prime p with 10 having order modulo p equal to the modulus, b_i . We used the prime 11 with 10 of order 2, the prime 101 with 10 of order 4, and the primes 73 and 137 with 10 of order 8. For each congruence, take $n \equiv 0 \pmod{b_i}$, and take $M \equiv -4 \cdot 10^{b_i} \pmod{p_i}$; that is, $M \equiv 4 \pmod{11}$, $M \equiv 4 \pmod{101}$, $M \equiv 69 \pmod{73}$, and $M \equiv 4 \pmod{137}$. Thus, for $k \equiv 1 \pmod{2}$, the value $N_0^{(k)}(4)$ is divisible by 11; for $k \equiv 2 \pmod{4}$, the value $N_0^{(k)}(4)$ is divisible by 101; for $k \equiv 0 \pmod{8}$, the value $N_0^{(k)}(4)$ is divisible by 73; and for $k \equiv 4 \pmod{8}$, the value $N_0^{(k)}(4)$ is divisible by 137.

We note that the choices made for the congruences are, in general, arbitrary. That is, the decision to start the covering with $k \equiv 1 \pmod{2}$ instead of $k \equiv 0 \pmod{2}$, and then changing the other congruences to ensure the four congruences still form a covering, was made arbitrarily. Moreover, the choice of using the primes 73 for the congruence $k \equiv 0 \pmod{8}$ and 137 for $k \equiv 4 \pmod{8}$ was also arbitrary. We could have interchanged the roles of these primes since 10 has order 8 for both the primes 73 and 137. While these decisions are generally made arbitrarily, some congruences were specifically chosen to help make later coverings easier to construct.

Consider the prime 7. As discussed in Section 2, we take $n \equiv 0 \pmod{6}$ and $M \equiv 0 \pmod{7}$. Then $N_0^{(k)}(x)$ is divisible by 7 for $x = 0$ and 7. In general, we cannot use the prime 7 again in a different covering because it could result in a different congruence condition on M ; however, observe that for $d = 4$ taking $k \equiv 2 \pmod{6}$ also results in $M \equiv 0 \pmod{7}$. Thus, we can use the prime 7 in the covering for $d = 4$ as long as it corresponds to the congruence $k \equiv 2 \pmod{6}$. Thus, we can use a prime in multiple coverings as long as the congruence condition on M remains the same. Table 2 gives a list of all the primes that are used in multiple coverings. The function $\rho(b, p)$ will be explained in the next section.

Looking back at the covering used for $x = 4$, the choice of starting with $k \equiv 1 \pmod{2}$ instead of $k \equiv 0 \pmod{2}$ for the prime 11 was made to maximize the number of times we can reuse the prime 11. From Table 2 we see that the prime 11, corresponding to half a covering, is also used in the coverings for $d = 6$ and $d = 7$.

p	congruence on M	coverings used	$\rho(b, p)$
7	0 (mod 7)	$d = 0, 4$	2
11	4 (mod 11)	$x = 4, d = 6, 7$	1
19	8 (mod 19)	$x = 9, d = 0$	1
31	4 (mod 31)	$x = 3, d = 0$	1
73	69 (mod 73)	$x = 4, d = 4, 9$	1
101	4 (mod 101)	$x = 4, d = 6$	1
137	4 (mod 137)	$x = 4, d = 6$	2
271	268 (mod 271)	$x = 3, d = 3$	1
333667	233566 (mod 333667)	$x = 9, d = 0$	1

Table 2: Primes used for more than one covering

The same is true when choosing to associate the prime 73 with the congruence $k \equiv 0 \pmod{8}$, and the prime 137 with the congruence $k \equiv 4 \pmod{8}$.

Recall that the modulus of each congruence, b , is such that 10 has multiplicative order b modulo a unique prime. A modulus b can only be used a number of times equal to the number of primes with 10 of order b except for the cases presented in Table 2. For the covering used for $x = 4$, the modulus 8 is used twice because there are only two primes with 10 of order 8, namely 73 and 137. To obtain these primes, we looked for prime factors of $\Phi_b(10)$ where $\Phi_b(x)$ is the b -th cyclotomic polynomial. While most of the primes used in our coverings can be found in [4] and [6], our coverings are different because of the primes we were able to reuse in Table 2; thus, we had to find more primes corresponding to moduli not used in either [4] or [6]. Using Magma V2.25-7 we were able to find the new prime factors needed to complete our coverings. So that the reader can check the data, we have made available the list of these new primes associated with each modulus b available through [9]. We did not always get a complete factorization of $\Phi_b(10)$ in a reasonable time, but were able to use the remaining unfactored part as follows. If the remaining unfactored part is composite, relatively prime to the factored part of $\Phi_b(10)$ and b , and not a prime power then there must be at least two more distinct prime factors of $\Phi_b(10)$. This allowed us to get a lower bound for the number of distinct primes of a given order b , denoted $L(b)$. A complete list of the moduli used, b , with a lower bound on the prime divisors of $\Phi_b(10)$, corresponding to a lower bound on the number of primes for which 10 has order b , is found in Table 4.

Tables 5–16 give the congruences $k \equiv a_i \pmod{b_i}$ that form the coverings for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$.

4. Verifying the Coverings

To finish the argument on the leading zeros and the string of leading ones of N , we must provide a covering for each $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$. To keep the data as concise as possible, for each b in Table 4 take the $L(b)$ primes that have 10 of order b and order them in some way. As discussed with the covering for $x = 4$, the order is not important to us. With the primes ordered in some way, let $\rho(b, p_i) = i$ represent the i -th prime with 10 of order b . Observe that from Table 2 the primes used in multiple coverings have a specified $\rho(b, p)$ assigned to them. Tables 5–16 give the congruences $k \equiv a_i \pmod{b_i}$ that form the coverings for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$, along with the prime p_i associated with that particular congruence. When the primes in Table 2 appear in Tables 5–16 an $(*)$ will be used to notify the reader.

The remainder of this section deals with verifying the coverings and the associated information on the primes.

The most direct way to check that a system of congruences is a covering is by setting ℓ to be the least common multiple of the moduli in the set of congruences and then to check if every integer in the interval $[0, \ell - 1]$ satisfies at least one congruence in $\mathcal{C}$. When ℓ is large this computation can be very time consuming. Alternative methods can be found in [4], [5], and [6]. We will discuss the specific method we used to verify each covering here. Table 3 contains the least common multiple ℓ for each covering that has more than one congruence. The fourth column contains the maximum prime divisor of ℓ .

x	Table	ℓ	max p
1	5	14636160	11
3	6	360	5
4	7	8	2
6	8	5040	7
9	9	216	3

d	Table	ℓ	max p
0	11	11690842363200	31
3	12	36054018000	29
4	13	281080800	13
6	14	16	2
7	15	6126120	17
9	16	12738408883200	19

Table 3: Least common multiple of the moduli for the coverings in each table

The coverings for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{6, 7\}$ were verified directly by checking if every integer in the interval $[0, \ell - 1]$ satisfies at least one congruence. The value of ℓ proved to be too large for $d \in \{0, 3, 4, 9\}$ to verify directly. To verify these four coverings we used the method discussed in Section 4 of [4] which we briefly outline here.

Consider the covering used for $d = 0$. We know that the congruences

$$k \equiv 0 \pmod{3}, \quad k \equiv 1 \pmod{3}, \quad \text{and} \quad k \equiv 2 \pmod{3}$$

form a covering. The covering for $d = 0$ was constructed by considering each of

these three congruences separately. To verify this covering, we can instead verify that each of these three congruences is properly covered. When considering $k \equiv 0 \pmod{3}$, we used the congruence $k \equiv 0 \pmod{6}$ with the prime 7 in Table 2, and constructed $k \equiv 3 \pmod{6}$ by looking at moduli that were divisible by 31. To break it down even further, one can check that the set of congruences

$$A := \{k \equiv a_i \pmod{186} : a_i = 6j + 3 \text{ for } j \in \{0, \dots, 30\}\}$$

covers $k \equiv 3 \pmod{6}$. Thus, to verify that $k \equiv 0 \pmod{3}$ is covered, one needs to verify that each of the congruences in A is covered. Some of these congruences are covered by a single congruence, while some take up to 21 congruences. The largest of these collections of congruences has 21 congruences, with least common multiple of the moduli being 12648. Checking each collection of congruences that correspond to $k \equiv 1 \pmod{3}$ and $k \equiv 2 \pmod{3}$ individually also reduces ℓ to 89812800 and 272160, respectively.

A similar method can be used to verify the covering for $d = 3$ by looking at each of the residue classes modulo 5 separately; for $d = 4$ one can look at the residue classes modulo 13, and for $d = 9$ one can look at the residue classes for 2.

To finish verification of the coverings found in the appendix and made available in [9], the following checks should be and were completed. For each congruence $k \equiv a \pmod{b}$, the modulus b corresponds to a prime p with 10 of order b . To verify this, we checked that p divides $\Phi_b(10)$ and is coprime to b . Furthermore, throughout all coverings, no prime is repeated unless it appears in Table 2. For the primes appearing in Table 2, we verified that the congruence condition on M , found either by $M \equiv -x \cdot 10^b$ or $M \equiv -1 - (d - 1) \cdot 10^b$, depending on whether the covering is for a value of x or d , is that which appears in Table 2. For each composite factor of $\Phi_b(10)$ used, we verified that the composite number was used at most twice and was not a prime power. Then we made sure the composite number is coprime to each other factor of $\Phi_b(10)$ and b . These verifications guarantee that we get two unused and distinct primes from the composite number.

With the verifications of the coverings completed, we have congruence conditions on n and M that guarantee that

$$N = \frac{10^n - 1}{9} + M$$

remains composite when inserting, replacing, or deleting a digit in the leading zeros or the leading ones.

5. Proof of Theorem 1

We now provide a proof of Theorem 1 utilizing the coverings discussed in the previous sections.

Proof of Theorem 1. Let

$$N = \frac{10^n - 1}{9} + M$$

where n and M are large natural numbers. As discussed in Sections 2 and 3, if we use the coverings outlined in Tables 5–16, then we have congruence conditions on n and M that will guarantee that N remains composite when inserting, replacing, or deleting a digit anywhere in the leading zeroes or leading ones. Apply those congruence conditions on n and M . We now turn our attention to the right-most digits corresponding to M .

Define B to be the least common multiple of the moduli appearing in the congruences $k \equiv a \pmod{b}$ in each covering. Recall that the primes 3 and 7 were considered separately in Section 2 corresponding to their own coverings, namely $k \equiv 0 \pmod{1}$, so 6 divides B ; in particular, the order of 10 modulo 3 and 7 divides B . Moreover, recall that for the covering for $d = 1$, an unused prime $p_d > 10$ is taken, so $p_d - 1$ also divides B . Then we take n to be such that $n \equiv 0 \pmod{B}$. For each $x \in \{1, 3, 4, 6, 9\}$, let $\mathcal{P}_x$ be the set of primes appearing in the last column of the table given for x ; that is, $\mathcal{P}_x$ is the set of primes used in the covering for x . For example, looking at the covering for $x = 4$ discussed in Section 3, we have $\mathcal{P}_4 = \{11, 73, 101, 137\}$. Likewise, for $d \in \{0, 3, 4, 6, 7, 9\}$, let $\mathcal{P}'_d$ be the set of primes used appearing in the last column for the table given for d . Set

$$\mathcal{P} = \{3, 7, p_d\} \cup \mathcal{P}_1 \cup \mathcal{P}_3 \cup \mathcal{P}_4 \cup \mathcal{P}_6 \cup \mathcal{P}_9 \cup \mathcal{P}'_0 \cup \mathcal{P}'_3 \cup \mathcal{P}'_4 \cup \mathcal{P}'_6 \cup \mathcal{P}'_7 \cup \mathcal{P}'_9.$$

Then $\mathcal{P}$ contains every prime that appears in at least one of the coverings. Recall that each prime $p \in \mathcal{P}$ results in a congruence condition on M . Further impose $M \equiv 0 \pmod{10}$. From the definition of N we know that $N \equiv 1 \pmod{10}$, hence N is coprime to 10.

To handle the right-most digits, we fix M above and let n vary. The following argument is similar to that found in [4] and [7]. The basic strategy is to consider every possible insertion, replacement, and deletion in the right-most digits individually. For each digit modification, take a prime dividing the number resulting from inserting, replacing, or deleting a right-most digit and add it to $\mathcal{P}$. Since M is fixed, there are only finitely many possible digit modifications, and hence only finitely many primes to consider.

Let K be the number of digits on the fixed M as above. We are now interested in $k \in \{0, \dots, K - 1\}$. Fix $n_0 \equiv 0 \pmod{B}$ satisfying $10^{n_0-2} > M$. We let n vary with $n \equiv 0 \pmod{B}$ and $n \geq n_0$. Since M is fixed, there are only finitely many digit insertions, replacements, and deletions in the range $k \in \{0, \dots, K - 1\}$ independent of n . Let $N_\delta^{(k)}(x)$ denote the number obtained by replacing the $(k+1)^{\text{st}}$ digit of N with $x \in \{0, \dots, 9\}$, deleting the $(k+1)^{\text{st}}$ digit of N , or inserting a digit $x \in \{0, \dots, 9\}$ to the right of the $(k+1)^{\text{st}}$ digit of N . In Section 2, we considered $N^{(k)}(x)$ using two separate cases depending on the location of the insertion, and showed that it includes the cases that we are replacing or deleting a digit for $k \geq K$.

Momentarily fix $k \in \{0, \dots, K-1\}$ and $x \in \{0, \dots, 9\}$. Set

$$\overline{N} = \frac{10^{n_0} - 1}{9} + M.$$

Since $M \equiv 0 \pmod{10}$, we still have that $\overline{N}$ is coprime to 10. Let $\overline{N}_\delta^{(k)}(x)$ denote the number obtained by inserting $x \in \{0, 1, \dots, 9\}$ to the right of the $(k+1)^{\text{st}}$ digit of $\overline{N}$, or by replacing or deleting the $(k+1)^{\text{st}}$ digit of $\overline{N}$. Since $k \in \{0, \dots, K-1\}$ is fixed, these digit changes only affect M . Let $q = q(k, x)$ denote the least prime $q = q(k, x)$ dividing $\overline{N}_\delta^{(k)}(x)$. If q is not 2 or 5, and $n \equiv n_0 \pmod{c(q)}$ where $c(q)$ is the order of 10 modulo q , then

$$N_\delta^{(k)}(x) - \overline{N}_\delta^{(k)}(x) = 10^{n_0} \cdot \frac{10^{n-n_0} - 1}{9} \equiv 0 \pmod{q}.$$

Thus, we have that q divides $N_\delta^{(k)}(x)$ whenever $n \equiv n_0 \pmod{c(q)}$. If q is 2 or 5, then the units digit of $\overline{N}_\delta^{(k)}(x)$ will be even or 5, which implies that, for $n \geq n_0$, the units digit of $N_\delta^{(k)}(x)$ remains even or 5. In either case, $N_\delta^{(k)}(x)$ is composite.

Let $k \in \{0, 1, \dots, K-1\}$ and $x \in \{0, 1, \dots, 9\}$ vary. We can take $n \equiv n_0 \pmod{q(k, x)}$ for all such k and x and $n \equiv 0 \pmod{B}$. In fact, since $n_0 \equiv 0 \pmod{B}$, we see that these simultaneous congruences have the solution $n = n_0$ and hence infinitely many solutions. Thus, for such n , after inserting, replacing, or deleting a digit anywhere in the decimal representation of N , including the infinitely many leading zeros, the resulting number will be divisible by at least one of the primes $q(k, x)$ above or one of the primes $p \in \mathcal{P}$. Taking n sufficiently large satisfying $n \equiv n_0 \pmod{q(k, x)}$ for all such k and x and $n \equiv 0 \pmod{B}$, we obtain that Theorem 1 holds. □

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Appendix

To aid in the verification of the computations in this paper, all the data in this appendix can be found in [9] as lists suitable for computations. Table 4 gives a lower bound $L(b)$ on the number of distinct primes dividing $\Phi_b(10)$ that are relatively prime to b . Note that the $(*)$ in Table 4 means that at least one of the primes associated with the modulus b appears in Table 2 and appears in more than one covering.

The remaining tables, Tables 5–16, give the congruences $k \equiv a \pmod{b}$ that form the covering systems obtained for $x \in \{1, 3, 4, 6, 9\}$ and $d \in \{0, 3, 4, 6, 7, 9\}$. Recall that for a fixed modulus b , the prime associated with a given congruence class does not matter. We simply put a number in the second column to refer to a prime with 10 of order b , and a different number the next time the modulus b appears to indicate a different prime is used. For these tables, an $(*)$ is used when the prime appears in Table 2 and appears in more than one covering. For these congruences, the choice of prime does matter and is identified in Table 2.

b	L	b	L	b	L	b	L	b	L	b	L
2*	1	66	2	165	3	297	6	468	3	720	2
3	1	68	3	168	3	300	6	480	3	722	3
4*	1	69	3	169	3	304	3	484	6	725	4
5*	2	70	2	170	3	306	6	486	4	726	4
6*	2	72	3	171	3	310	4	494	4	728	2
7	2	75	3	174	3	312	5	495	5	729	3
8*	2	76	2	175	3	319	4	496	4	741	4
9*	1	77	4	176	2	322	4	507	7	744	7
10	1	78	4	180	3	323	3	510	5	748	4
11	2	80	2	182	2	324	2	512	5	754	2
12	1	81	5	184	2	330	3	513	2	760	2
13	3	84	2	186	4	336	2	520	2	765	2
14	1	85	3	187	3	338	4	522	4	768	6
15*	2	87	3	189	3	340	5	525	3	780	4
16	2	88	2	190	3	341	5	527	4	783	3
17	2	90	2	192	4	342	5	528	3	792	2
18*	2	91	7	195	3	345	6	532	5	798	3
19	1	92	3	198	2	348	4	540	7	800	5
20	2	93	1	200	4	350	2	546	2	805	4
21	3	95	5	203	4	351	4	550	3	806	3
22	3	96	4	204	6	352	4	552	4	810	5
23	1	99	4	207	3	360	3	558	3	812	6
24	1	100	4	208	6	361	7	560	5	819	2
25	3	102	2	209	5	363	3	561	3	825	3
26	2	104	2	210	3	364	3	567	3	828	2
27	2	105	3	216	2	372	3	572	5	836	3
28	3	108	3	217	3	374	5	575	6	837	4
29	5	110	4	220	6	377	4	580	4	840	5
30	3	114	2	221	3	378	4	585	3	845	2
31	3	115	5	224	4	380	8	589	2	864	4
32	2	116	4	228	7	384	5	594	5	868	6
33	2	117	4	230	2	390	3	595	1	870	7
34	3	119	4	231	4	396	3	605	2	880	4
35	3	120	1	232	5	399	5	608	3	884	4
36	1	121	4	234	3	400	3	609	3	900	7
38	1	124	2	238	3	403	5	620	2	912	2
39	1	125	4	240	3	405	5	624	2	920	1
40	2	128	3	242	5	406	5	630	5	930	4
42	3	130	2	243	2	414	5	638	5	936	5
44	2	133	3	247	5	416	4	646	7	960	2
45	2	135	5	248	5	418	4	648	3	969	4
46	4	136	2	250	2	420	2	650	5	972	4
50	3	138	3	255	4	429	4	651	3	988	3
51	4	140	4	256	4	432	2	660	6	990	4
52	2	143	4	260	5	434	5	663	3	992	4
54	2	145	4	261	8	435	4	665	2	1001	4
55	4	150	1	264	6	440	2	676	3	1014	4
56	2	152	4	266	3	442	5	680	4	1015	7
57	3	153	6	270	4	448	3	682	6	1020	7
58	2	154	6	273	5	450	2	684	4	1023	4
60	3	155	3	275	5	456	5	690	5	1024	4
62	1	156	3	276	5	460	8	700	5	1026	4
63	3	160	4	279	3	462	3	702	4	1035	3
64	4	161	5	286	5	464	6	704	4	1044	4
65	2	162	4	290	6	465	5	715	3	1054	2

Table 4: Number of primes used, $L = L(b)$, with 10 of order b , Part I

b	L	b	L	b	L	b	L	b	L
1056	3	1395	5	1824	5	2464	6	3876	3
1064	3	1400	6	1859	5	2535	3	3888	2
1080	3	1408	4	1860	8	2584	8	3960	6
1083	7	1430	4	1872	2	2604	5	4004	2
1085	1	1444	4	1885	3	2610	2	4060	4
1100	7	1450	2	1890	4	2640	3	4092	2
1105	4	1452	1	1938	5	2700	6	4104	2
1120	4	1482	2	1944	8	2704	4	4216	4
1131	3	1488	2	1953	6	2888	1	4332	3
1134	3	1530	6	1976	2	2904	2	4356	3
1140	11	1536	3	1980	8	2970	3	4400	2
1150	2	1539	3	2002	5	3045	5	4536	2
1152	3	1581	6	2028	3	3060	4	4560	4
1170	4	1595	3	2030	4	3078	2	4600	3
1178	5	1596	4	2040	4	3162	3	4788	4
1183	4	1600	3	2046	3	3190	2	5220	3
1190	6	1610	6	2052	1	3192	3	5280	2
1200	3	1620	9	2070	6	3200	2	5577	3
1210	3	1638	2	2128	4	3255	4	5655	3
1216	9	1650	6	2160	2	3300	3	5776	2
1232	4	1672	2	2166	5	3380	4	5808	4
1260	7	1674	4	2170	5	3420	3	5940	3
1276	2	1680	4	2200	3	3450	3	6090	2
1292	3	1690	4	2210	2	3480	3	6156	2
1296	6	1716	3	2268	5	3534	4	6324	2
1302	4	1725	3	2280	4	3549	3	6760	2
1305	2	1740	9	2300	4	3600	4	6960	2
1320	8	1755	3	2356	3	3610	5	7068	5
1330	3	1767	4	2366	4	3630	1	7436	5
1352	8	1768	4	2394	5	3648	2	14872	2
1364	6	1782	6	2418	5	3718	2		
1368	4	1800	4	2420	6	3770	6		
1380	9	1805	5	2432	2	3780	4		

Table 4: Number of primes used, $L = L(b)$, with 10 of order b (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 9 \pmod{11}$	1	$k \equiv 248 \pmod{264}$	6	$k \equiv 446 \pmod{1056}$	2
$k \equiv 8 \pmod{11}$	2	$k \equiv 50 \pmod{352}$	1	$k \equiv 798 \pmod{1056}$	3
$k \equiv 7 \pmod{22}$	1	$k \equiv 138 \pmod{352}$	2	$k \equiv 6 \pmod{1408}$	1
$k \equiv 18 \pmod{22}$	2	$k \equiv 226 \pmod{352}$	3	$k \equiv 358 \pmod{1408}$	2
$k \equiv 17 \pmod{22}$	3	$k \equiv 314 \pmod{352}$	4	$k \equiv 710 \pmod{1408}$	3
$k \equiv 28 \pmod{264}$	1	$k \equiv 182 \pmod{704}$	1	$k \equiv 1062 \pmod{1408}$	4
$k \equiv 72 \pmod{264}$	2	$k \equiv 534 \pmod{704}$	2	$k \equiv 16 \pmod{33}$	1
$k \equiv 116 \pmod{264}$	3	$k \equiv 270 \pmod{704}$	3	$k \equiv 27 \pmod{33}$	2
$k \equiv 160 \pmod{264}$	4	$k \equiv 622 \pmod{704}$	4	$k \equiv 5 \pmod{66}$	1
$k \equiv 204 \pmod{264}$	5	$k \equiv 94 \pmod{1056}$	1	$k \equiv 38 \pmod{66}$	2

Table 5: Covering information for $x = 1$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 47 \pmod{55}$	1	$k \equiv 23 \pmod{220}$	1	$k \equiv 1837 \pmod{2420}$	3
$k \equiv 36 \pmod{55}$	2	$k \equiv 78 \pmod{220}$	2	$k \equiv 1232 \pmod{2420}$	4
$k \equiv 25 \pmod{55}$	3	$k \equiv 133 \pmod{220}$	3	$k \equiv 627 \pmod{2420}$	5
$k \equiv 14 \pmod{55}$	4	$k \equiv 188 \pmod{220}$	4	$k \equiv 22 \pmod{2420}$	6
$k \equiv 3 \pmod{110}$	1	$k \equiv 177 \pmod{220}$	5	$k \equiv 2178 \pmod{3630}$	1
$k \equiv 58 \pmod{110}$	2	$k \equiv 122 \pmod{220}$	6	$k \equiv 253 \pmod{363}$	1
$k \equiv 98 \pmod{99}$	1	$k \equiv 67 \pmod{440}$	1	$k \equiv 132 \pmod{363}$	2
$k \equiv 87 \pmod{99}$	2	$k \equiv 287 \pmod{440}$	2	$k \equiv 11 \pmod{363}$	3
$k \equiv 76 \pmod{99}$	3	$k \equiv 12 \pmod{880}$	1	$k \equiv 605 \pmod{726}$	1
$k \equiv 65 \pmod{99}$	4	$k \equiv 232 \pmod{880}$	2	$k \equiv 484 \pmod{726}$	2
$k \equiv 153 \pmod{198}$	1	$k \equiv 452 \pmod{880}$	3	$k \equiv 363 \pmod{726}$	3
$k \equiv 54 \pmod{198}$	2	$k \equiv 672 \pmod{880}$	4	$k \equiv 242 \pmod{726}$	4
$k \equiv 32 \pmod{297}$	1	$k \equiv 276 \pmod{330}$	1	$k \equiv 847 \pmod{1452}$	1
$k \equiv 43 \pmod{297}$	2	$k \equiv 221 \pmod{330}$	2	$k \equiv 3630 \pmod{4356}$	1
$k \equiv 131 \pmod{297}$	3	$k \equiv 166 \pmod{330}$	3	$k \equiv 726 \pmod{4356}$	2
$k \equiv 142 \pmod{297}$	4	$k \equiv 111 \pmod{660}$	1	$k \equiv 2178 \pmod{4356}$	3
$k \equiv 241 \pmod{297}$	5	$k \equiv 441 \pmod{660}$	2	$k \equiv 1573 \pmod{2904}$	1
$k \equiv 230 \pmod{297}$	6	$k \equiv 56 \pmod{660}$	3	$k \equiv 121 \pmod{2904}$	2
$k \equiv 318 \pmod{396}$	1	$k \equiv 386 \pmod{660}$	4	$k \equiv 4356 \pmod{5808}$	1
$k \equiv 120 \pmod{396}$	2	$k \equiv 1 \pmod{660}$	5	$k \equiv 2904 \pmod{5808}$	2
$k \equiv 219 \pmod{396}$	3	$k \equiv 331 \pmod{660}$	6	$k \equiv 1452 \pmod{5808}$	3
$k \equiv 21 \pmod{792}$	1	$k \equiv 110 \pmod{121}$	1	$k \equiv 0 \pmod{5808}$	4
$k \equiv 417 \pmod{792}$	2	$k \equiv 99 \pmod{121}$	2	$k \equiv 37 \pmod{44}$	1
$k \equiv 10 \pmod{495}$	1	$k \equiv 88 \pmod{121}$	3	$k \equiv 26 \pmod{44}$	2
$k \equiv 109 \pmod{495}$	2	$k \equiv 77 \pmod{121}$	4	$k \equiv 59 \pmod{88}$	1
$k \equiv 208 \pmod{495}$	3	$k \equiv 66 \pmod{242}$	1	$k \equiv 15 \pmod{88}$	2
$k \equiv 307 \pmod{495}$	4	$k \equiv 55 \pmod{242}$	2	$k \equiv 136 \pmod{176}$	1
$k \equiv 406 \pmod{495}$	5	$k \equiv 187 \pmod{242}$	3	$k \equiv 48 \pmod{176}$	2
$k \equiv 68 \pmod{77}$	1	$k \equiv 176 \pmod{242}$	4	$k \equiv 92 \pmod{528}$	1
$k \equiv 57 \pmod{77}$	2	$k \equiv 44 \pmod{484}$	1	$k \equiv 268 \pmod{528}$	2
$k \equiv 46 \pmod{77}$	3	$k \equiv 165 \pmod{484}$	2	$k \equiv 444 \pmod{528}$	3
$k \equiv 35 \pmod{77}$	4	$k \equiv 286 \pmod{484}$	3	$k \equiv 1060 \pmod{1232}$	1
$k \equiv 2 \pmod{154}$	1	$k \equiv 407 \pmod{484}$	4	$k \equiv 884 \pmod{1232}$	2
$k \equiv 13 \pmod{154}$	2	$k \equiv 275 \pmod{484}$	5	$k \equiv 708 \pmod{1232}$	3
$k \equiv 24 \pmod{154}$	3	$k \equiv 33 \pmod{484}$	6	$k \equiv 532 \pmod{1232}$	4
$k \equiv 79 \pmod{154}$	4	$k \equiv 154 \pmod{242}$	5	$k \equiv 1588 \pmod{2464}$	1
$k \equiv 90 \pmod{154}$	5	$k \equiv 506 \pmod{605}$	1	$k \equiv 356 \pmod{2464}$	2
$k \equiv 101 \pmod{154}$	6	$k \equiv 385 \pmod{605}$	2	$k \equiv 1412 \pmod{2464}$	3
$k \equiv 45 \pmod{110}$	3	$k \equiv 264 \pmod{1210}$	1	$k \equiv 180 \pmod{2464}$	4
$k \equiv 100 \pmod{110}$	4	$k \equiv 869 \pmod{1210}$	2	$k \equiv 1236 \pmod{2464}$	5
$k \equiv 34 \pmod{165}$	1	$k \equiv 748 \pmod{1210}$	3	$k \equiv 4 \pmod{2464}$	6
$k \equiv 89 \pmod{165}$	2	$k \equiv 143 \pmod{2420}$	1		
$k \equiv 144 \pmod{165}$	3	$k \equiv 1353 \pmod{2420}$	2		

Table 6: Covering information for $x = 1$ (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 1 \pmod{5}$	2	$k \equiv 33 \pmod{45}$	1	$k \equiv 34 \pmod{40}$	1
$k \equiv 0 \pmod{5}$	1*	$k \equiv 18 \pmod{45}$	2	$k \equiv 14 \pmod{40}$	2
$k \equiv 7 \pmod{10}$	1	$k \equiv 48 \pmod{90}$	1	$k \equiv 44 \pmod{60}$	1
$k \equiv 2 \pmod{20}$	1	$k \equiv 3 \pmod{90}$	2	$k \equiv 24 \pmod{60}$	2
$k \equiv 12 \pmod{20}$	2	$k \equiv 29 \pmod{30}$	1	$k \equiv 4 \pmod{60}$	3
$k \equiv 13 \pmod{15}$	1*	$k \equiv 19 \pmod{30}$	2		
$k \equiv 8 \pmod{15}$	2	$k \equiv 9 \pmod{30}$	3		

Table 7: Covering information for $x = 3$

congruence	p	congruence	p	congruence	p
$k \equiv 1 \pmod{2}$	1*	$k \equiv 0 \pmod{8}$	1*	$k \equiv 4 \pmod{8}$	2*
$k \equiv 2 \pmod{4}$	1*				

Table 8: Covering information for $x = 4$

congruence	p	congruence	p	congruence	p
$k \equiv 0 \pmod{7}$	1	$k \equiv 16 \pmod{63}$	1	$k \equiv 23 \pmod{210}$	2
$k \equiv 6 \pmod{7}$	2	$k \equiv 37 \pmod{63}$	2	$k \equiv 107 \pmod{210}$	3
$k \equiv 12 \pmod{14}$	1	$k \equiv 58 \pmod{63}$	3	$k \equiv 212 \pmod{420}$	1
$k \equiv 19 \pmod{28}$	1	$k \equiv 51 \pmod{84}$	1	$k \equiv 2 \pmod{420}$	2
$k \equiv 5 \pmod{28}$	2	$k \equiv 72 \pmod{84}$	2	$k \equiv 29 \pmod{35}$	1
$k \equiv 3 \pmod{21}$	1	$k \equiv 114 \pmod{168}$	1	$k \equiv 22 \pmod{35}$	2
$k \equiv 10 \pmod{21}$	2	$k \equiv 30 \pmod{168}$	2	$k \equiv 15 \pmod{35}$	3
$k \equiv 17 \pmod{21}$	3	$k \equiv 93 \pmod{168}$	3	$k \equiv 43 \pmod{70}$	1
$k \equiv 25 \pmod{28}$	3	$k \equiv 9 \pmod{336}$	1	$k \equiv 8 \pmod{70}$	2
$k \equiv 32 \pmod{42}$	1	$k \equiv 177 \pmod{336}$	2	$k \equiv 1 \pmod{140}$	1
$k \equiv 18 \pmod{42}$	2	$k \equiv 86 \pmod{105}$	1	$k \equiv 36 \pmod{140}$	2
$k \equiv 4 \pmod{42}$	3	$k \equiv 65 \pmod{105}$	2	$k \equiv 71 \pmod{140}$	3
$k \equiv 39 \pmod{56}$	1	$k \equiv 44 \pmod{105}$	3	$k \equiv 106 \pmod{140}$	4
$k \equiv 11 \pmod{56}$	2	$k \equiv 128 \pmod{210}$	1		

Table 9: Covering information for $x = 6$

congruence	p	congruence	p	congruence	p
$k \equiv 0 \pmod{3}$	1	$k \equiv 46 \pmod{72}$	2	$k \equiv 20 \pmod{27}$	1
$k \equiv 1 \pmod{6}$	1	$k \equiv 70 \pmod{72}$	3	$k \equiv 11 \pmod{27}$	2
$k \equiv 4 \pmod{12}$	1	$k \equiv 8 \pmod{9}$	1*	$k \equiv 29 \pmod{54}$	1
$k \equiv 10 \pmod{24}$	1	$k \equiv 14 \pmod{18}$	1*	$k \equiv 2 \pmod{54}$	2
$k \equiv 22 \pmod{72}$	1	$k \equiv 5 \pmod{18}$	2		

Table 10: Covering information for $x = 9$

congruence	p	congruence	p	congruence	p
$k \equiv 0 \pmod{6}$	2*	$k \equiv 168 \pmod{1674}$	4	$k \equiv 2358 \pmod{3255}$	2
$k \equiv 28 \pmod{31}$	1	$k \equiv 1191 \pmod{1395}$	1	$k \equiv 1707 \pmod{3255}$	3
$k \equiv 22 \pmod{31}$	2	$k \equiv 912 \pmod{1395}$	2	$k \equiv 1056 \pmod{3255}$	4
$k \equiv 16 \pmod{31}$	3	$k \equiv 633 \pmod{1395}$	3	$k \equiv 405 \pmod{1085}$	1
$k \equiv 41 \pmod{62}$	1	$k \equiv 354 \pmod{1395}$	4	$k \equiv 1521 \pmod{1953}$	1
$k \equiv 66 \pmod{93}$	1	$k \equiv 75 \pmod{1395}$	5	$k \equiv 870 \pmod{1953}$	2
$k \equiv 29 \pmod{124}$	1	$k \equiv 813 \pmod{930}$	1	$k \equiv 219 \pmod{1953}$	3
$k \equiv 91 \pmod{124}$	2	$k \equiv 627 \pmod{930}$	2	$k \equiv 1335 \pmod{1953}$	4
$k \equiv 147 \pmod{186}$	1	$k \equiv 441 \pmod{930}$	3	$k \equiv 684 \pmod{1953}$	5
$k \equiv 141 \pmod{186}$	2	$k \equiv 255 \pmod{930}$	4	$k \equiv 33 \pmod{1953}$	6
$k \equiv 135 \pmod{186}$	3	$k \equiv 69 \pmod{155}$	1	$k \equiv 647 \pmod{682}$	1
$k \equiv 129 \pmod{186}$	4	$k \equiv 1737 \pmod{1860}$	1	$k \equiv 585 \pmod{682}$	2
$k \equiv 309 \pmod{372}$	1	$k \equiv 1551 \pmod{1860}$	2	$k \equiv 523 \pmod{682}$	3
$k \equiv 123 \pmod{372}$	2	$k \equiv 1365 \pmod{1860}$	3	$k \equiv 461 \pmod{682}$	4
$k \equiv 303 \pmod{372}$	3	$k \equiv 1179 \pmod{1860}$	4	$k \equiv 399 \pmod{682}$	5
$k \equiv 2349 \pmod{2604}$	1	$k \equiv 993 \pmod{1860}$	5	$k \equiv 337 \pmod{682}$	6
$k \equiv 1977 \pmod{2604}$	2	$k \equiv 807 \pmod{1860}$	6	$k \equiv 275 \pmod{341}$	1
$k \equiv 1605 \pmod{2604}$	3	$k \equiv 621 \pmod{1860}$	7	$k \equiv 213 \pmod{341}$	2
$k \equiv 1233 \pmod{2604}$	4	$k \equiv 435 \pmod{1860}$	8	$k \equiv 151 \pmod{341}$	3
$k \equiv 861 \pmod{2604}$	5	$k \equiv 94 \pmod{155}$	2	$k \equiv 89 \pmod{341}$	4
$k \equiv 489 \pmod{868}$	1	$k \equiv 63 \pmod{155}$	3	$k \equiv 27 \pmod{341}$	5
$k \equiv 117 \pmod{868}$	2	$k \equiv 305 \pmod{310}$	1	$k \equiv 1881 \pmod{2046}$	1
$k \equiv 669 \pmod{744}$	1	$k \equiv 243 \pmod{310}$	2	$k \equiv 1695 \pmod{2046}$	2
$k \equiv 483 \pmod{744}$	2	$k \equiv 181 \pmod{310}$	3	$k \equiv 1509 \pmod{2046}$	3
$k \equiv 297 \pmod{744}$	3	$k \equiv 119 \pmod{310}$	4	$k \equiv 300 \pmod{1023}$	1
$k \equiv 111 \pmod{744}$	4	$k \equiv 367 \pmod{620}$	1	$k \equiv 114 \pmod{1023}$	2
$k \equiv 663 \pmod{744}$	5	$k \equiv 57 \pmod{620}$	2	$k \equiv 951 \pmod{1023}$	3
$k \equiv 477 \pmod{744}$	6	$k \equiv 423 \pmod{465}$	1	$k \equiv 765 \pmod{1023}$	4
$k \equiv 291 \pmod{744}$	7	$k \equiv 330 \pmod{465}$	2	$k \equiv 1261 \pmod{1364}$	1
$k \equiv 849 \pmod{1488}$	1	$k \equiv 237 \pmod{465}$	3	$k \equiv 579 \pmod{1364}$	2
$k \equiv 105 \pmod{1488}$	2	$k \equiv 144 \pmod{465}$	4	$k \equiv 1075 \pmod{1364}$	3
$k \equiv 471 \pmod{558}$	1	$k \equiv 51 \pmod{465}$	5	$k \equiv 393 \pmod{1364}$	4
$k \equiv 285 \pmod{558}$	2	$k \equiv 1161 \pmod{1302}$	1	$k \equiv 889 \pmod{1364}$	5
$k \equiv 99 \pmod{558}$	3	$k \equiv 975 \pmod{1302}$	2	$k \equiv 207 \pmod{1364}$	6
$k \equiv 186 \pmod{279}$	1	$k \equiv 789 \pmod{1302}$	3	$k \equiv 2067 \pmod{4092}$	1
$k \equiv 93 \pmod{279}$	2	$k \equiv 603 \pmod{1302}$	4	$k \equiv 21 \pmod{4092}$	2
$k \equiv 0 \pmod{279}$	3	$k \equiv 200 \pmod{217}$	1	$k \equiv 2247 \pmod{2418}$	1
$k \equiv 211 \pmod{248}$	1	$k \equiv 14 \pmod{217}$	2	$k \equiv 2061 \pmod{2418}$	2
$k \equiv 149 \pmod{248}$	2	$k \equiv 45 \pmod{217}$	3	$k \equiv 1875 \pmod{2418}$	3
$k \equiv 87 \pmod{248}$	3	$k \equiv 411 \pmod{434}$	1	$k \equiv 1689 \pmod{2418}$	4
$k \equiv 25 \pmod{248}$	4	$k \equiv 349 \pmod{434}$	2	$k \equiv 1503 \pmod{2418}$	5
$k \equiv 19 \pmod{248}$	5	$k \equiv 287 \pmod{434}$	3	$k \equiv 511 \pmod{806}$	1
$k \equiv 453 \pmod{496}$	1	$k \equiv 225 \pmod{434}$	4	$k \equiv 325 \pmod{806}$	2
$k \equiv 205 \pmod{496}$	2	$k \equiv 163 \pmod{434}$	5	$k \equiv 139 \pmod{806}$	3
$k \equiv 391 \pmod{496}$	3	$k \equiv 535 \pmod{868}$	3	$k \equiv 356 \pmod{403}$	1
$k \equiv 143 \pmod{496}$	4	$k \equiv 101 \pmod{868}$	4	$k \equiv 170 \pmod{403}$	2
$k \equiv 825 \pmod{992}$	1	$k \equiv 473 \pmod{868}$	5	$k \equiv 387 \pmod{403}$	3
$k \equiv 577 \pmod{992}$	2	$k \equiv 39 \pmod{868}$	6	$k \equiv 201 \pmod{403}$	4
$k \equiv 329 \pmod{992}$	3	$k \equiv 498 \pmod{651}$	1	$k \equiv 15 \pmod{403}$	5
$k \equiv 81 \pmod{992}$	4	$k \equiv 312 \pmod{651}$	2	$k \equiv 3357 \pmod{3534}$	1
$k \equiv 819 \pmod{837}$	1	$k \equiv 126 \pmod{651}$	3	$k \equiv 3171 \pmod{3534}$	2
$k \equiv 540 \pmod{837}$	2	$k \equiv 1893 \pmod{2170}$	1	$k \equiv 2985 \pmod{3534}$	3
$k \equiv 261 \pmod{837}$	3	$k \equiv 1459 \pmod{2170}$	2	$k \equiv 2799 \pmod{3534}$	4
$k \equiv 726 \pmod{837}$	4	$k \equiv 1025 \pmod{2170}$	3	$k \equiv 6147 \pmod{7068}$	1
$k \equiv 1284 \pmod{1674}$	1	$k \equiv 591 \pmod{2170}$	4	$k \equiv 2613 \pmod{7068}$	2
$k \equiv 447 \pmod{1674}$	2	$k \equiv 157 \pmod{2170}$	5	$k \equiv 5961 \pmod{7068}$	3
$k \equiv 1005 \pmod{1674}$	3	$k \equiv 3009 \pmod{3255}$	1	$k \equiv 2427 \pmod{7068}$	4

Table 11: Covering information for $d = 0$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 5775 \pmod{7068}$	5	$k \equiv 1480 \pmod{1782}$	2	$k \equiv 37 \pmod{2640}$	2
$k \equiv 2241 \pmod{2356}$	1	$k \equiv 1318 \pmod{1782}$	3	$k \equiv 1597 \pmod{2640}$	3
$k \equiv 1187 \pmod{2356}$	2	$k \equiv 1156 \pmod{1782}$	4	$k \equiv 2917 \pmod{5280}$	1
$k \equiv 9 \pmod{2356}$	3	$k \equiv 994 \pmod{1782}$	5	$k \equiv 277 \pmod{5280}$	2
$k \equiv 288 \pmod{1767}$	1	$k \equiv 832 \pmod{1782}$	6	$k \equiv 67 \pmod{120}$	1
$k \equiv 102 \pmod{1767}$	2	$k \equiv 76 \pmod{594}$	1	$k \equiv 367 \pmod{480}$	1
$k \equiv 1683 \pmod{1767}$	3	$k \equiv 508 \pmod{594}$	2	$k \equiv 247 \pmod{480}$	2
$k \equiv 1497 \pmod{1767}$	4	$k \equiv 346 \pmod{594}$	3	$k \equiv 127 \pmod{480}$	3
$k \equiv 381 \pmod{589}$	1	$k \equiv 184 \pmod{594}$	4	$k \equiv 487 \pmod{960}$	1
$k \equiv 195 \pmod{589}$	2	$k \equiv 22 \pmod{594}$	5	$k \equiv 7 \pmod{960}$	2
$k \equiv 133 \pmod{1178}$	1	$k \equiv 166 \pmod{243}$	1	$k \equiv 601 \pmod{630}$	1
$k \equiv 1125 \pmod{1178}$	2	$k \equiv 85 \pmod{243}$	2	$k \equiv 511 \pmod{630}$	2
$k \equiv 939 \pmod{1178}$	3	$k \equiv 490 \pmod{729}$	1	$k \equiv 421 \pmod{630}$	3
$k \equiv 753 \pmod{1178}$	4	$k \equiv 247 \pmod{729}$	2	$k \equiv 331 \pmod{630}$	4
$k \equiv 567 \pmod{1178}$	5	$k \equiv 4 \pmod{729}$	3	$k \equiv 241 \pmod{630}$	5
$k \equiv 2979 \pmod{3162}$	1	$k \equiv 169 \pmod{180}$	1	$k \equiv 1411 \pmod{1890}$	1
$k \equiv 2793 \pmod{3162}$	2	$k \equiv 139 \pmod{180}$	2	$k \equiv 781 \pmod{1890}$	2
$k \equiv 2607 \pmod{3162}$	3	$k \equiv 109 \pmod{180}$	3	$k \equiv 151 \pmod{1890}$	3
$k \equiv 840 \pmod{1581}$	1	$k \equiv 259 \pmod{360}$	1	$k \equiv 1321 \pmod{1890}$	4
$k \equiv 654 \pmod{1581}$	2	$k \equiv 79 \pmod{360}$	2	$k \equiv 2581 \pmod{3780}$	1
$k \equiv 468 \pmod{1581}$	3	$k \equiv 229 \pmod{360}$	3	$k \equiv 691 \pmod{3780}$	2
$k \equiv 282 \pmod{1581}$	4	$k \equiv 409 \pmod{720}$	1	$k \equiv 1951 \pmod{3780}$	3
$k \equiv 96 \pmod{1581}$	5	$k \equiv 49 \pmod{720}$	2	$k \equiv 61 \pmod{3780}$	4
$k \equiv 1491 \pmod{1581}$	6	$k \equiv 1099 \pmod{1260}$	1	$k \equiv 841 \pmod{900}$	1
$k \equiv 406 \pmod{527}$	1	$k \equiv 919 \pmod{1260}$	2	$k \equiv 751 \pmod{900}$	2
$k \equiv 220 \pmod{527}$	2	$k \equiv 739 \pmod{1260}$	3	$k \equiv 661 \pmod{900}$	3
$k \equiv 34 \pmod{527}$	3	$k \equiv 559 \pmod{1260}$	4	$k \equiv 571 \pmod{900}$	4
$k \equiv 375 \pmod{527}$	4	$k \equiv 379 \pmod{1260}$	5	$k \equiv 481 \pmod{900}$	5
$k \equiv 251 \pmod{1054}$	1	$k \equiv 199 \pmod{1260}$	6	$k \equiv 463 \pmod{900}$	6
$k \equiv 65 \pmod{1054}$	2	$k \equiv 19 \pmod{1260}$	7	$k \equiv 13 \pmod{900}$	7
$k \equiv 3351 \pmod{4216}$	1	$k \equiv 133 \pmod{150}$	1	$k \equiv 1291 \pmod{1800}$	1
$k \equiv 2297 \pmod{4216}$	2	$k \equiv 253 \pmod{300}$	1	$k \equiv 391 \pmod{1800}$	2
$k \equiv 189 \pmod{4216}$	3	$k \equiv 103 \pmod{300}$	2	$k \equiv 1201 \pmod{1800}$	3
$k \equiv 1243 \pmod{4216}$	4	$k \equiv 223 \pmod{300}$	3	$k \equiv 301 \pmod{1800}$	4
$k \equiv 3165 \pmod{6324}$	1	$k \equiv 73 \pmod{300}$	4	$k \equiv 2011 \pmod{2700}$	1
$k \equiv 3 \pmod{6324}$	2	$k \equiv 193 \pmod{300}$	5	$k \equiv 1111 \pmod{2700}$	2
$k \equiv 10 \pmod{18}$	1*	$k \equiv 43 \pmod{300}$	6	$k \equiv 211 \pmod{2700}$	3
$k \equiv 10 \pmod{15}$	1*	$k \equiv 313 \pmod{450}$	1	$k \equiv 1921 \pmod{2700}$	4
$k \equiv 34 \pmod{36}$	1	$k \equiv 163 \pmod{450}$	2	$k \equiv 1021 \pmod{2700}$	5
$k \equiv 88 \pmod{108}$	1	$k \equiv 817 \pmod{840}$	1	$k \equiv 121 \pmod{2700}$	6
$k \equiv 52 \pmod{108}$	2	$k \equiv 697 \pmod{840}$	2	$k \equiv 2731 \pmod{3600}$	1
$k \equiv 16 \pmod{108}$	3	$k \equiv 577 \pmod{840}$	3	$k \equiv 1831 \pmod{3600}$	2
$k \equiv 148 \pmod{162}$	1	$k \equiv 457 \pmod{840}$	4	$k \equiv 931 \pmod{3600}$	3
$k \equiv 130 \pmod{162}$	2	$k \equiv 337 \pmod{840}$	5	$k \equiv 31 \pmod{3600}$	4
$k \equiv 112 \pmod{162}$	3	$k \equiv 1057 \pmod{1680}$	1	$k \equiv 901 \pmod{990}$	1
$k \equiv 94 \pmod{162}$	4	$k \equiv 217 \pmod{1680}$	2	$k \equiv 811 \pmod{990}$	2
$k \equiv 238 \pmod{324}$	1	$k \equiv 937 \pmod{1680}$	3	$k \equiv 721 \pmod{990}$	3
$k \equiv 76 \pmod{324}$	2	$k \equiv 97 \pmod{1680}$	4	$k \equiv 631 \pmod{990}$	4
$k \equiv 382 \pmod{486}$	1	$k \equiv 1237 \pmod{1320}$	1	$k \equiv 1531 \pmod{1980}$	1
$k \equiv 220 \pmod{486}$	2	$k \equiv 1117 \pmod{1320}$	2	$k \equiv 541 \pmod{1980}$	2
$k \equiv 58 \pmod{486}$	3	$k \equiv 997 \pmod{1320}$	3	$k \equiv 1441 \pmod{1980}$	3
$k \equiv 364 \pmod{486}$	4	$k \equiv 877 \pmod{1320}$	4	$k \equiv 451 \pmod{1980}$	4
$k \equiv 688 \pmod{972}$	1	$k \equiv 757 \pmod{1320}$	5	$k \equiv 1351 \pmod{1980}$	5
$k \equiv 202 \pmod{972}$	2	$k \equiv 637 \pmod{1320}$	6	$k \equiv 361 \pmod{1980}$	6
$k \equiv 526 \pmod{972}$	3	$k \equiv 517 \pmod{1320}$	7	$k \equiv 1261 \pmod{1980}$	7
$k \equiv 40 \pmod{972}$	4	$k \equiv 397 \pmod{1320}$	8	$k \equiv 271 \pmod{1980}$	8
$k \equiv 1642 \pmod{1782}$	1	$k \equiv 1357 \pmod{2640}$	1	$k \equiv 2161 \pmod{2970}$	1

Table 11: Covering information for $d = 0$ (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 1171 \pmod{2970}$	2	$k \equiv 290 \pmod{648}$	2	$k \equiv 11 \pmod{4536}$	2
$k \equiv 181 \pmod{2970}$	3	$k \equiv 74 \pmod{648}$	3	$k \equiv 245 \pmod{270}$	1
$k \equiv 3061 \pmod{3960}$	1	$k \equiv 1127 \pmod{1296}$	1	$k \equiv 191 \pmod{270}$	2
$k \equiv 2071 \pmod{3960}$	2	$k \equiv 911 \pmod{1296}$	2	$k \equiv 137 \pmod{270}$	3
$k \equiv 1081 \pmod{3960}$	3	$k \equiv 695 \pmod{1296}$	3	$k \equiv 83 \pmod{270}$	4
$k \equiv 91 \pmod{3960}$	4	$k \equiv 479 \pmod{1296}$	4	$k \equiv 29 \pmod{540}$	1
$k \equiv 2971 \pmod{3960}$	5	$k \equiv 263 \pmod{1296}$	5	$k \equiv 299 \pmod{540}$	2
$k \equiv 991 \pmod{3960}$	6	$k \equiv 47 \pmod{1296}$	6	$k \equiv 218 \pmod{540}$	3
$k \equiv 3961 \pmod{5940}$	1	$k \equiv 1748 \pmod{1944}$	1	$k \equiv 488 \pmod{540}$	4
$k \equiv 1981 \pmod{5940}$	2	$k \equiv 1532 \pmod{1944}$	2	$k \equiv 164 \pmod{540}$	5
$k \equiv 1 \pmod{5940}$	3	$k \equiv 1316 \pmod{1944}$	3	$k \equiv 434 \pmod{540}$	6
$k \equiv 223 \pmod{462}$	1	$k \equiv 1100 \pmod{1944}$	4	$k \equiv 110 \pmod{540}$	7
$k \equiv 289 \pmod{462}$	2	$k \equiv 884 \pmod{1944}$	5	$k \equiv 596 \pmod{810}$	1
$k \equiv 355 \pmod{462}$	3	$k \equiv 668 \pmod{1944}$	6	$k \equiv 326 \pmod{810}$	2
$k \equiv 157 \pmod{231}$	1	$k \equiv 452 \pmod{1944}$	7	$k \equiv 56 \pmod{810}$	3
$k \equiv 91 \pmod{231}$	2	$k \equiv 236 \pmod{1944}$	8	$k \equiv 542 \pmod{810}$	4
$k \equiv 25 \pmod{231}$	3	$k \equiv 1964 \pmod{3888}$	1	$k \equiv 272 \pmod{810}$	5
$k \equiv 190 \pmod{231}$	4	$k \equiv 20 \pmod{3888}$	2	$k \equiv 812 \pmod{1620}$	1
$k \equiv 8 \pmod{9}$	1*	$k \equiv 173 \pmod{189}$	1	$k \equiv 2 \pmod{1620}$	2
$k \equiv 23 \pmod{135}$	1	$k \equiv 146 \pmod{189}$	2	$k \equiv 920 \pmod{1620}$	3
$k \equiv 50 \pmod{135}$	2	$k \equiv 119 \pmod{189}$	3	$k \equiv 1460 \pmod{1620}$	4
$k \equiv 77 \pmod{135}$	3	$k \equiv 92 \pmod{378}$	1	$k \equiv 380 \pmod{1620}$	5
$k \equiv 104 \pmod{135}$	4	$k \equiv 281 \pmod{378}$	2	$k \equiv 371 \pmod{432}$	1
$k \equiv 131 \pmod{135}$	5	$k \equiv 254 \pmod{378}$	3	$k \equiv 155 \pmod{432}$	2
$k \equiv 14 \pmod{81}$	1	$k \equiv 65 \pmod{378}$	4	$k \equiv 776 \pmod{864}$	1
$k \equiv 41 \pmod{81}$	2	$k \equiv 416 \pmod{567}$	1	$k \equiv 560 \pmod{864}$	2
$k \equiv 68 \pmod{81}$	3	$k \equiv 227 \pmod{567}$	2	$k \equiv 344 \pmod{864}$	3
$k \equiv 59 \pmod{81}$	4	$k \equiv 38 \pmod{567}$	3	$k \equiv 128 \pmod{864}$	4
$k \equiv 32 \pmod{81}$	5	$k \equiv 956 \pmod{1134}$	1	$k \equiv 965 \pmod{1080}$	1
$k \equiv 329 \pmod{405}$	1	$k \equiv 767 \pmod{1134}$	2	$k \equiv 749 \pmod{1080}$	2
$k \equiv 248 \pmod{405}$	2	$k \equiv 578 \pmod{1134}$	3	$k \equiv 533 \pmod{1080}$	3
$k \equiv 167 \pmod{405}$	3	$k \equiv 1523 \pmod{2268}$	1	$k \equiv 1397 \pmod{2160}$	1
$k \equiv 86 \pmod{405}$	4	$k \equiv 389 \pmod{2268}$	2	$k \equiv 317 \pmod{2160}$	2
$k \equiv 5 \pmod{405}$	5	$k \equiv 1334 \pmod{2268}$	3	$k \equiv 1721 \pmod{1620}$	6
$k \equiv 209 \pmod{216}$	1	$k \equiv 200 \pmod{2268}$	4	$k \equiv 1181 \pmod{1620}$	7
$k \equiv 182 \pmod{216}$	2	$k \equiv 1145 \pmod{2268}$	5	$k \equiv 641 \pmod{1620}$	8
$k \equiv 506 \pmod{648}$	1	$k \equiv 2279 \pmod{4536}$	1	$k \equiv 101 \pmod{1620}$	9

Table 11: Covering information for $d = 0$ (Part III)

congruence	p	congruence	p	congruence	p
$k \equiv 0 \pmod{5}$	1*	$k \equiv 51 \pmod{1100}$	6	$k \equiv 64 \pmod{435}$	3
$k \equiv 22 \pmod{25}$	1	$k \equiv 626 \pmod{825}$	1	$k \equiv 494 \pmod{580}$	1
$k \equiv 17 \pmod{25}$	2	$k \equiv 351 \pmod{825}$	2	$k \equiv 349 \pmod{580}$	2
$k \equiv 12 \pmod{25}$	3	$k \equiv 76 \pmod{825}$	3	$k \equiv 204 \pmod{580}$	3
$k \equiv 32 \pmod{50}$	1	$k \equiv 1401 \pmod{1650}$	1	$k \equiv 59 \pmod{580}$	4
$k \equiv 7 \pmod{50}$	2	$k \equiv 1126 \pmod{1650}$	2	$k \equiv 634 \pmod{725}$	1
$k \equiv 27 \pmod{50}$	3	$k \equiv 851 \pmod{1650}$	3	$k \equiv 489 \pmod{725}$	2
$k \equiv 52 \pmod{100}$	1	$k \equiv 576 \pmod{1650}$	4	$k \equiv 344 \pmod{725}$	3
$k \equiv 2 \pmod{100}$	2	$k \equiv 301 \pmod{1650}$	5	$k \equiv 199 \pmod{725}$	4
$k \equiv 71 \pmod{75}$	1	$k \equiv 26 \pmod{1650}$	6	$k \equiv 779 \pmod{1450}$	1
$k \equiv 46 \pmod{75}$	2	$k \equiv 826 \pmod{1100}$	7	$k \equiv 54 \pmod{1450}$	2
$k \equiv 21 \pmod{75}$	3	$k \equiv 1651 \pmod{2200}$	1	$k \equiv 774 \pmod{870}$	1
$k \equiv 41 \pmod{100}$	3	$k \equiv 551 \pmod{2200}$	2	$k \equiv 629 \pmod{870}$	2
$k \equiv 91 \pmod{100}$	4	$k \equiv 1376 \pmod{2200}$	3	$k \equiv 484 \pmod{870}$	3
$k \equiv 166 \pmod{200}$	1	$k \equiv 2476 \pmod{4400}$	1	$k \equiv 339 \pmod{870}$	4
$k \equiv 116 \pmod{200}$	2	$k \equiv 276 \pmod{4400}$	2	$k \equiv 194 \pmod{870}$	5
$k \equiv 66 \pmod{200}$	3	$k \equiv 2201 \pmod{3300}$	1	$k \equiv 49 \pmod{870}$	6
$k \equiv 16 \pmod{200}$	4	$k \equiv 1101 \pmod{3300}$	2	$k \equiv 769 \pmod{870}$	7
$k \equiv 111 \pmod{125}$	1	$k \equiv 1 \pmod{3300}$	3	$k \equiv 102 \pmod{174}$	1
$k \equiv 86 \pmod{125}$	2	$k \equiv 28 \pmod{29}$	1	$k \equiv 131 \pmod{174}$	2
$k \equiv 61 \pmod{125}$	3	$k \equiv 23 \pmod{29}$	2	$k \equiv 160 \pmod{174}$	3
$k \equiv 36 \pmod{125}$	4	$k \equiv 18 \pmod{29}$	3	$k \equiv 189 \pmod{348}$	1
$k \equiv 136 \pmod{250}$	1	$k \equiv 13 \pmod{29}$	4	$k \equiv 15 \pmod{348}$	2
$k \equiv 11 \pmod{250}$	2	$k \equiv 8 \pmod{29}$	5	$k \equiv 218 \pmod{348}$	3
$k \equiv 156 \pmod{175}$	1	$k \equiv 32 \pmod{58}$	1	$k \equiv 44 \pmod{348}$	4
$k \equiv 131 \pmod{175}$	2	$k \equiv 3 \pmod{58}$	2	$k \equiv 909 \pmod{1015}$	1
$k \equiv 106 \pmod{175}$	3	$k \equiv 85 \pmod{87}$	1	$k \equiv 764 \pmod{1015}$	2
$k \equiv 256 \pmod{350}$	1	$k \equiv 56 \pmod{87}$	2	$k \equiv 619 \pmod{1015}$	3
$k \equiv 81 \pmod{350}$	2	$k \equiv 27 \pmod{87}$	3	$k \equiv 474 \pmod{1015}$	4
$k \equiv 581 \pmod{700}$	1	$k \equiv 109 \pmod{116}$	1	$k \equiv 329 \pmod{1015}$	5
$k \equiv 406 \pmod{700}$	2	$k \equiv 80 \pmod{116}$	2	$k \equiv 184 \pmod{1015}$	6
$k \equiv 231 \pmod{700}$	3	$k \equiv 51 \pmod{116}$	3	$k \equiv 39 \pmod{1015}$	7
$k \equiv 56 \pmod{700}$	4	$k \equiv 22 \pmod{116}$	4	$k \equiv 92 \pmod{203}$	1
$k \equiv 556 \pmod{700}$	5	$k \equiv 104 \pmod{145}$	1	$k \equiv 150 \pmod{203}$	2
$k \equiv 1081 \pmod{1400}$	1	$k \equiv 99 \pmod{145}$	2	$k \equiv 5 \pmod{203}$	3
$k \equiv 381 \pmod{1400}$	2	$k \equiv 94 \pmod{145}$	3	$k \equiv 63 \pmod{203}$	4
$k \equiv 906 \pmod{1400}$	3	$k \equiv 89 \pmod{145}$	4	$k \equiv 324 \pmod{406}$	1
$k \equiv 206 \pmod{1400}$	4	$k \equiv 229 \pmod{290}$	1	$k \equiv 121 \pmod{406}$	2
$k \equiv 731 \pmod{1400}$	5	$k \equiv 84 \pmod{290}$	2	$k \equiv 382 \pmod{406}$	3
$k \equiv 31 \pmod{1400}$	6	$k \equiv 224 \pmod{290}$	3	$k \equiv 179 \pmod{406}$	4
$k \equiv 356 \pmod{525}$	1	$k \equiv 79 \pmod{290}$	4	$k \equiv 237 \pmod{406}$	5
$k \equiv 181 \pmod{525}$	2	$k \equiv 219 \pmod{290}$	5	$k \equiv 440 \pmod{812}$	1
$k \equiv 6 \pmod{525}$	3	$k \equiv 74 \pmod{290}$	6	$k \equiv 34 \pmod{812}$	2
$k \equiv 251 \pmod{275}$	1	$k \equiv 214 \pmod{232}$	1	$k \equiv 1624 \pmod{1740}$	1
$k \equiv 226 \pmod{275}$	2	$k \equiv 185 \pmod{232}$	2	$k \equiv 1479 \pmod{1740}$	2
$k \equiv 201 \pmod{275}$	3	$k \equiv 156 \pmod{232}$	3	$k \equiv 1334 \pmod{1740}$	3
$k \equiv 176 \pmod{275}$	4	$k \equiv 127 \pmod{232}$	4	$k \equiv 1189 \pmod{1740}$	4
$k \equiv 151 \pmod{275}$	5	$k \equiv 98 \pmod{232}$	5	$k \equiv 1044 \pmod{1740}$	5
$k \equiv 401 \pmod{550}$	1	$k \equiv 301 \pmod{464}$	1	$k \equiv 899 \pmod{1740}$	6
$k \equiv 126 \pmod{550}$	2	$k \equiv 69 \pmod{464}$	2	$k \equiv 754 \pmod{1740}$	7
$k \equiv 326 \pmod{550}$	3	$k \equiv 272 \pmod{464}$	3	$k \equiv 609 \pmod{1740}$	8
$k \equiv 926 \pmod{1100}$	1	$k \equiv 40 \pmod{464}$	4	$k \equiv 464 \pmod{1740}$	9
$k \equiv 651 \pmod{1100}$	2	$k \equiv 243 \pmod{464}$	5	$k \equiv 2059 \pmod{3480}$	1
$k \equiv 376 \pmod{1100}$	3	$k \equiv 11 \pmod{464}$	6	$k \equiv 319 \pmod{3480}$	2
$k \equiv 101 \pmod{1100}$	4	$k \equiv 354 \pmod{435}$	1	$k \equiv 1914 \pmod{3480}$	3
$k \equiv 601 \pmod{1100}$	5	$k \equiv 209 \pmod{435}$	2	$k \equiv 3654 \pmod{6960}$	1

Table 12: Covering information for $d = 3$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 174 \pmod{6960}$	2	$k \equiv 565 \pmod{609}$	1	$k \equiv 53 \pmod{345}$	6
$k \equiv 3509 \pmod{5220}$	1	$k \equiv 362 \pmod{609}$	2	$k \equiv 393 \pmod{460}$	1
$k \equiv 1769 \pmod{5220}$	2	$k \equiv 159 \pmod{609}$	3	$k \equiv 278 \pmod{460}$	2
$k \equiv 29 \pmod{5220}$	3	$k \equiv 623 \pmod{812}$	3	$k \equiv 163 \pmod{460}$	3
$k \equiv 198 \pmod{319}$	1	$k \equiv 420 \pmod{812}$	4	$k \equiv 48 \pmod{460}$	4
$k \equiv 53 \pmod{319}$	2	$k \equiv 217 \pmod{812}$	5	$k \equiv 388 \pmod{460}$	5
$k \equiv 227 \pmod{319}$	3	$k \equiv 14 \pmod{812}$	6	$k \equiv 273 \pmod{460}$	6
$k \equiv 82 \pmod{319}$	4	$k \equiv 241 \pmod{261}$	1	$k \equiv 158 \pmod{460}$	7
$k \equiv 894 \pmod{1595}$	1	$k \equiv 212 \pmod{261}$	2	$k \equiv 43 \pmod{460}$	8
$k \equiv 749 \pmod{1595}$	2	$k \equiv 183 \pmod{261}$	3	$k \equiv 578 \pmod{920}$	1
$k \equiv 604 \pmod{1595}$	3	$k \equiv 154 \pmod{261}$	4	$k \equiv 498 \pmod{575}$	1
$k \equiv 459 \pmod{638}$	1	$k \equiv 125 \pmod{261}$	5	$k \equiv 383 \pmod{575}$	2
$k \equiv 140 \pmod{638}$	2	$k \equiv 96 \pmod{261}$	6	$k \equiv 268 \pmod{575}$	3
$k \equiv 633 \pmod{638}$	3	$k \equiv 67 \pmod{261}$	7	$k \equiv 153 \pmod{575}$	4
$k \equiv 314 \pmod{638}$	4	$k \equiv 38 \pmod{261}$	8	$k \equiv 38 \pmod{575}$	5
$k \equiv 488 \pmod{638}$	5	$k \equiv 531 \pmod{783}$	1	$k \equiv 608 \pmod{690}$	1
$k \equiv 807 \pmod{1276}$	1	$k \equiv 270 \pmod{783}$	2	$k \equiv 493 \pmod{690}$	2
$k \equiv 169 \pmod{1276}$	2	$k \equiv 9 \pmod{783}$	3	$k \equiv 378 \pmod{690}$	3
$k \equiv 1619 \pmod{3190}$	1	$k \equiv 294 \pmod{435}$	4	$k \equiv 263 \pmod{690}$	4
$k \equiv 24 \pmod{3190}$	2	$k \equiv 1019 \pmod{1305}$	1	$k \equiv 148 \pmod{690}$	5
$k \equiv 251 \pmod{377}$	1	$k \equiv 584 \pmod{1305}$	2	$k \equiv 1413 \pmod{2070}$	1
$k \equiv 106 \pmod{377}$	2	$k \equiv 1454 \pmod{2610}$	1	$k \equiv 723 \pmod{2070}$	2
$k \equiv 338 \pmod{377}$	3	$k \equiv 149 \pmod{2610}$	2	$k \equiv 33 \pmod{2070}$	3
$k \equiv 193 \pmod{377}$	4	$k \equiv 439 \pmod{522}$	1	$k \equiv 120 \pmod{138}$	1
$k \equiv 686 \pmod{754}$	1	$k \equiv 352 \pmod{522}$	2	$k \equiv 97 \pmod{138}$	2
$k \equiv 309 \pmod{754}$	2	$k \equiv 265 \pmod{522}$	3	$k \equiv 74 \pmod{138}$	3
$k \equiv 1179 \pmod{1885}$	1	$k \equiv 178 \pmod{522}$	4	$k \equiv 189 \pmod{276}$	1
$k \equiv 1034 \pmod{1885}$	2	$k \equiv 613 \pmod{1044}$	1	$k \equiv 51 \pmod{276}$	2
$k \equiv 889 \pmod{1885}$	3	$k \equiv 91 \pmod{1044}$	2	$k \equiv 166 \pmod{276}$	3
$k \equiv 2629 \pmod{3770}$	1	$k \equiv 526 \pmod{1044}$	3	$k \equiv 28 \pmod{276}$	4
$k \equiv 744 \pmod{3770}$	2	$k \equiv 4 \pmod{1044}$	4	$k \equiv 143 \pmod{276}$	5
$k \equiv 2484 \pmod{3770}$	3	$k \equiv 21 \pmod{23}$	1	$k \equiv 281 \pmod{552}$	1
$k \equiv 599 \pmod{3770}$	4	$k \equiv 16 \pmod{46}$	1	$k \equiv 5 \pmod{552}$	2
$k \equiv 2339 \pmod{3770}$	5	$k \equiv 39 \pmod{46}$	2	$k \equiv 138 \pmod{161}$	1
$k \equiv 454 \pmod{3770}$	6	$k \equiv 34 \pmod{46}$	3	$k \equiv 115 \pmod{161}$	2
$k \equiv 918 \pmod{1131}$	1	$k \equiv 11 \pmod{46}$	4	$k \equiv 92 \pmod{161}$	3
$k \equiv 541 \pmod{1131}$	2	$k \equiv 52 \pmod{69}$	1	$k \equiv 69 \pmod{161}$	4
$k \equiv 164 \pmod{1131}$	3	$k \equiv 29 \pmod{69}$	2	$k \equiv 46 \pmod{161}$	5
$k \equiv 3789 \pmod{5655}$	1	$k \equiv 6 \pmod{69}$	3	$k \equiv 184 \pmod{322}$	1
$k \equiv 1904 \pmod{5655}$	2	$k \equiv 70 \pmod{92}$	1	$k \equiv 23 \pmod{322}$	2
$k \equiv 19 \pmod{5655}$	3	$k \equiv 47 \pmod{92}$	2	$k \equiv 161 \pmod{322}$	3
$k \equiv 1899 \pmod{2030}$	1	$k \equiv 24 \pmod{92}$	3	$k \equiv 0 \pmod{322}$	4
$k \equiv 884 \pmod{2030}$	2	$k \equiv 93 \pmod{184}$	1	$k \equiv 110 \pmod{207}$	1
$k \equiv 1754 \pmod{2030}$	3	$k \equiv 1 \pmod{184}$	2	$k \equiv 202 \pmod{207}$	2
$k \equiv 739 \pmod{2030}$	4	$k \equiv 88 \pmod{115}$	1	$k \equiv 87 \pmod{207}$	3
$k \equiv 3639 \pmod{4060}$	1	$k \equiv 83 \pmod{115}$	2	$k \equiv 386 \pmod{414}$	1
$k \equiv 2624 \pmod{4060}$	2	$k \equiv 78 \pmod{115}$	3	$k \equiv 179 \pmod{414}$	2
$k \equiv 1609 \pmod{4060}$	3	$k \equiv 73 \pmod{115}$	4	$k \equiv 271 \pmod{414}$	3
$k \equiv 594 \pmod{4060}$	4	$k \equiv 68 \pmod{115}$	5	$k \equiv 64 \pmod{414}$	4
$k \equiv 2479 \pmod{3045}$	1	$k \equiv 178 \pmod{230}$	1	$k \equiv 363 \pmod{414}$	5
$k \equiv 1464 \pmod{3045}$	2	$k \equiv 63 \pmod{230}$	2	$k \equiv 570 \pmod{828}$	1
$k \equiv 449 \pmod{3045}$	3	$k \equiv 288 \pmod{345}$	1	$k \equiv 156 \pmod{828}$	2
$k \equiv 2334 \pmod{3045}$	4	$k \equiv 173 \pmod{345}$	2	$k \equiv 248 \pmod{1035}$	1
$k \equiv 1319 \pmod{3045}$	5	$k \equiv 58 \pmod{345}$	3	$k \equiv 133 \pmod{1035}$	2
$k \equiv 3349 \pmod{6090}$	1	$k \equiv 283 \pmod{345}$	4	$k \equiv 18 \pmod{1035}$	3
$k \equiv 304 \pmod{6090}$	2	$k \equiv 168 \pmod{345}$	5	$k \equiv 1278 \pmod{1380}$	1

Table 12: Covering information for $d = 3$ (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 588 \pmod{1380}$	2	$k \equiv 698 \pmod{805}$	1	$k \equiv 1958 \pmod{2300}$	1
$k \equiv 1163 \pmod{1380}$	3	$k \equiv 583 \pmod{805}$	2	$k \equiv 1383 \pmod{2300}$	2
$k \equiv 473 \pmod{1380}$	4	$k \equiv 468 \pmod{805}$	3	$k \equiv 808 \pmod{2300}$	3
$k \equiv 1048 \pmod{1380}$	5	$k \equiv 353 \pmod{805}$	4	$k \equiv 233 \pmod{2300}$	4
$k \equiv 358 \pmod{1380}$	6	$k \equiv 1043 \pmod{1610}$	1	$k \equiv 1268 \pmod{1725}$	1
$k \equiv 933 \pmod{1380}$	7	$k \equiv 238 \pmod{1610}$	2	$k \equiv 693 \pmod{1725}$	2
$k \equiv 243 \pmod{1380}$	8	$k \equiv 928 \pmod{1610}$	3	$k \equiv 118 \pmod{1725}$	3
$k \equiv 818 \pmod{1380}$	9	$k \equiv 123 \pmod{1610}$	4	$k \equiv 4028 \pmod{4600}$	1
$k \equiv 404 \pmod{552}$	3	$k \equiv 813 \pmod{1610}$	5	$k \equiv 2878 \pmod{4600}$	2
$k \equiv 128 \pmod{552}$	4	$k \equiv 8 \pmod{1610}$	6	$k \equiv 1728 \pmod{4600}$	3
$k \equiv 1393 \pmod{2070}$	4	$k \equiv 463 \pmod{575}$	6	$k \equiv 2303 \pmod{3450}$	1
$k \equiv 13 \pmod{2070}$	5	$k \equiv 923 \pmod{1150}$	1	$k \equiv 1153 \pmod{3450}$	2
$k \equiv 703 \pmod{2070}$	6	$k \equiv 348 \pmod{1150}$	2	$k \equiv 3 \pmod{3450}$	3

Table 12: Covering information for $d = 3$ (Part III)

congruence	p	congruence	p	congruence	p
$k \equiv 12 \pmod{13}$	1	$k \equiv 228 \pmod{312}$	2	$k \equiv 95 \pmod{117}$	2
$k \equiv 11 \pmod{13}$	2	$k \equiv 176 \pmod{312}$	3	$k \equiv 82 \pmod{117}$	3
$k \equiv 10 \pmod{13}$	3	$k \equiv 124 \pmod{312}$	4	$k \equiv 69 \pmod{117}$	4
$k \equiv 9 \pmod{26}$	1	$k \equiv 72 \pmod{312}$	5	$k \equiv 56 \pmod{234}$	1
$k \equiv 22 \pmod{26}$	2	$k \equiv 20 \pmod{624}$	1	$k \equiv 160 \pmod{234}$	2
$k \equiv 34 \pmod{39}$	1	$k \equiv 332 \pmod{624}$	2	$k \equiv 173 \pmod{234}$	3
$k \equiv 99 \pmod{195}$	1	$k \equiv 111 \pmod{156}$	1	$k \equiv 43 \pmod{468}$	1
$k \equiv 138 \pmod{195}$	2	$k \equiv 59 \pmod{156}$	2	$k \equiv 277 \pmod{468}$	2
$k \equiv 177 \pmod{195}$	3	$k \equiv 7 \pmod{156}$	3	$k \equiv 381 \pmod{468}$	3
$k \equiv 60 \pmod{585}$	1	$k \equiv 45 \pmod{65}$	1	$k \equiv 147 \pmod{936}$	1
$k \equiv 255 \pmod{585}$	2	$k \equiv 58 \pmod{65}$	2	$k \equiv 615 \pmod{936}$	2
$k \equiv 450 \pmod{585}$	3	$k \equiv 32 \pmod{130}$	1	$k \equiv 264 \pmod{936}$	3
$k \equiv 216 \pmod{1170}$	1	$k \equiv 97 \pmod{130}$	2	$k \equiv 732 \pmod{936}$	4
$k \equiv 801 \pmod{1170}$	2	$k \equiv 19 \pmod{260}$	1	$k \equiv 498 \pmod{936}$	5
$k \equiv 411 \pmod{1170}$	3	$k \equiv 84 \pmod{260}$	2	$k \equiv 30 \pmod{1872}$	1
$k \equiv 996 \pmod{1170}$	4	$k \equiv 149 \pmod{260}$	3	$k \equiv 966 \pmod{1872}$	2
$k \equiv 21 \pmod{1755}$	1	$k \equiv 214 \pmod{260}$	4	$k \equiv 17 \pmod{351}$	1
$k \equiv 606 \pmod{1755}$	2	$k \equiv 201 \pmod{260}$	5	$k \equiv 134 \pmod{351}$	2
$k \equiv 1191 \pmod{1755}$	3	$k \equiv 71 \pmod{520}$	1	$k \equiv 251 \pmod{351}$	3
$k \equiv 86 \pmod{273}$	1	$k \equiv 331 \pmod{520}$	2	$k \equiv 238 \pmod{351}$	4
$k \equiv 125 \pmod{273}$	2	$k \equiv 526 \pmod{650}$	1	$k \equiv 4 \pmod{702}$	1
$k \equiv 164 \pmod{273}$	3	$k \equiv 396 \pmod{650}$	2	$k \equiv 355 \pmod{702}$	2
$k \equiv 203 \pmod{273}$	4	$k \equiv 266 \pmod{650}$	3	$k \equiv 121 \pmod{702}$	3
$k \equiv 242 \pmod{273}$	5	$k \equiv 136 \pmod{650}$	4	$k \equiv 472 \pmod{702}$	4
$k \equiv 47 \pmod{546}$	1	$k \equiv 6 \pmod{650}$	5	$k \equiv 67 \pmod{78}$	1
$k \equiv 320 \pmod{546}$	2	$k \equiv 5 \pmod{91}$	1	$k \equiv 54 \pmod{78}$	2
$k \equiv 281 \pmod{819}$	1	$k \equiv 18 \pmod{91}$	2	$k \equiv 41 \pmod{78}$	3
$k \equiv 554 \pmod{819}$	2	$k \equiv 31 \pmod{91}$	3	$k \equiv 28 \pmod{78}$	4
$k \equiv 8 \pmod{1638}$	1	$k \equiv 44 \pmod{91}$	4	$k \equiv 327 \pmod{390}$	1
$k \equiv 827 \pmod{1638}$	2	$k \equiv 57 \pmod{91}$	5	$k \equiv 249 \pmod{390}$	2
$k \equiv 46 \pmod{52}$	1	$k \equiv 70 \pmod{91}$	6	$k \equiv 171 \pmod{390}$	3
$k \equiv 33 \pmod{52}$	2	$k \equiv 83 \pmod{91}$	7	$k \equiv 15 \pmod{780}$	1
$k \equiv 280 \pmod{312}$	1	$k \equiv 108 \pmod{117}$	1	$k \equiv 405 \pmod{780}$	2

Table 13: Covering information for $d = 4$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 93 \pmod{780}$	3	$k \equiv 159 \pmod{1430}$	1	$k \equiv 1730 \pmod{2535}$	3
$k \equiv 483 \pmod{780}$	4	$k \equiv 874 \pmod{1430}$	2	$k \equiv 27 \pmod{1352}$	1
$k \equiv 2 \pmod{6}$	2*	$k \equiv 16 \pmod{1430}$	3	$k \equiv 196 \pmod{1352}$	2
$k \equiv 0 \pmod{8}$	1*	$k \equiv 731 \pmod{1430}$	4	$k \equiv 365 \pmod{1352}$	3
$k \equiv 78 \pmod{104}$	1	$k \equiv 861 \pmod{1001}$	1	$k \equiv 534 \pmod{1352}$	4
$k \equiv 91 \pmod{104}$	2	$k \equiv 718 \pmod{1001}$	2	$k \equiv 703 \pmod{1352}$	5
$k \equiv 65 \pmod{208}$	1	$k \equiv 575 \pmod{1001}$	3	$k \equiv 872 \pmod{1352}$	6
$k \equiv 169 \pmod{208}$	2	$k \equiv 432 \pmod{1001}$	4	$k \equiv 1041 \pmod{1352}$	7
$k \equiv 52 \pmod{208}$	3	$k \equiv 289 \pmod{2002}$	1	$k \equiv 1210 \pmod{1352}$	8
$k \equiv 156 \pmod{208}$	4	$k \equiv 1290 \pmod{2002}$	2	$k \equiv 1028 \pmod{1183}$	1
$k \equiv 39 \pmod{208}$	5	$k \equiv 146 \pmod{2002}$	3	$k \equiv 859 \pmod{1183}$	2
$k \equiv 143 \pmod{208}$	6	$k \equiv 1147 \pmod{2002}$	4	$k \equiv 690 \pmod{1183}$	3
$k \equiv 26 \pmod{416}$	1	$k \equiv 1004 \pmod{2002}$	5	$k \equiv 521 \pmod{1183}$	4
$k \equiv 130 \pmod{416}$	2	$k \equiv 3 \pmod{4004}$	1	$k \equiv 183 \pmod{2366}$	1
$k \equiv 234 \pmod{416}$	3	$k \equiv 2005 \pmod{4004}$	2	$k \equiv 1366 \pmod{2366}$	2
$k \equiv 338 \pmod{416}$	4	$k \equiv 131 \pmod{169}$	1	$k \equiv 352 \pmod{2366}$	3
$k \equiv 533 \pmod{728}$	1	$k \equiv 144 \pmod{169}$	2	$k \equiv 1535 \pmod{2366}$	4
$k \equiv 637 \pmod{728}$	2	$k \equiv 157 \pmod{169}$	3	$k \equiv 14 \pmod{3549}$	1
$k \equiv 65 \pmod{364}$	1	$k \equiv 118 \pmod{338}$	1	$k \equiv 1197 \pmod{3549}$	2
$k \equiv 325 \pmod{364}$	2	$k \equiv 287 \pmod{338}$	2	$k \equiv 2380 \pmod{3549}$	3
$k \equiv 221 \pmod{364}$	3	$k \equiv 105 \pmod{338}$	3	$k \equiv 508 \pmod{2704}$	1
$k \equiv 117 \pmod{182}$	1	$k \equiv 274 \pmod{338}$	4	$k \equiv 1184 \pmod{2704}$	2
$k \equiv 13 \pmod{182}$	2	$k \equiv 92 \pmod{507}$	1	$k \equiv 1860 \pmod{2704}$	3
$k \equiv 133 \pmod{143}$	1	$k \equiv 261 \pmod{507}$	2	$k \equiv 2536 \pmod{2704}$	4
$k \equiv 120 \pmod{143}$	2	$k \equiv 430 \pmod{507}$	3	$k \equiv 846 \pmod{3380}$	1
$k \equiv 107 \pmod{143}$	3	$k \equiv 79 \pmod{507}$	4	$k \equiv 1522 \pmod{3380}$	2
$k \equiv 94 \pmod{143}$	4	$k \equiv 248 \pmod{507}$	5	$k \equiv 2198 \pmod{3380}$	3
$k \equiv 81 \pmod{286}$	1	$k \equiv 417 \pmod{507}$	6	$k \equiv 2874 \pmod{3380}$	4
$k \equiv 224 \pmod{286}$	2	$k \equiv 404 \pmod{507}$	7	$k \equiv 170 \pmod{6760}$	1
$k \equiv 68 \pmod{286}$	3	$k \equiv 742 \pmod{1014}$	1	$k \equiv 3550 \pmod{6760}$	2
$k \equiv 211 \pmod{286}$	4	$k \equiv 235 \pmod{1014}$	2	$k \equiv 1691 \pmod{1859}$	1
$k \equiv 185 \pmod{286}$	5	$k \equiv 573 \pmod{1014}$	3	$k \equiv 1015 \pmod{1859}$	2
$k \equiv 55 \pmod{572}$	1	$k \equiv 66 \pmod{1014}$	4	$k \equiv 1184 \pmod{1859}$	3
$k \equiv 198 \pmod{572}$	2	$k \equiv 222 \pmod{676}$	1	$k \equiv 1353 \pmod{1859}$	4
$k \equiv 341 \pmod{572}$	3	$k \equiv 391 \pmod{676}$	2	$k \equiv 1522 \pmod{1859}$	5
$k \equiv 484 \pmod{572}$	4	$k \equiv 560 \pmod{676}$	3	$k \equiv 2367 \pmod{3718}$	1
$k \equiv 328 \pmod{572}$	5	$k \equiv 53 \pmod{2028}$	1	$k \equiv 2705 \pmod{3718}$	2
$k \equiv 42 \pmod{1716}$	1	$k \equiv 729 \pmod{2028}$	2	$k \equiv 677 \pmod{7436}$	1
$k \equiv 614 \pmod{1716}$	2	$k \equiv 1405 \pmod{2028}$	3	$k \equiv 4395 \pmod{7436}$	2
$k \equiv 1186 \pmod{1716}$	3	$k \equiv 547 \pmod{845}$	1	$k \equiv 2029 \pmod{7436}$	3
$k \equiv 29 \pmod{429}$	1	$k \equiv 716 \pmod{845}$	2	$k \equiv 5747 \pmod{7436}$	4
$k \equiv 172 \pmod{429}$	2	$k \equiv 378 \pmod{1690}$	1	$k \equiv 4057 \pmod{7436}$	5
$k \equiv 315 \pmod{429}$	3	$k \equiv 1223 \pmod{1690}$	2	$k \equiv 7775 \pmod{14872}$	1
$k \equiv 315 \pmod{429}$	4	$k \equiv 209 \pmod{1690}$	3	$k \equiv 339 \pmod{14872}$	2
$k \equiv 302 \pmod{715}$	1	$k \equiv 1054 \pmod{1690}$	4	$k \equiv 1 \pmod{5577}$	1
$k \equiv 445 \pmod{715}$	2	$k \equiv 40 \pmod{2535}$	1	$k \equiv 1860 \pmod{5577}$	2
$k \equiv 588 \pmod{715}$	3	$k \equiv 885 \pmod{2535}$	2	$k \equiv 3719 \pmod{5577}$	3

Table 13: Covering information for $d = 4$ (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 1 \pmod{2}$	1*	$k \equiv 4 \pmod{8}$	2*	$k \equiv 0 \pmod{16}$	2
$k \equiv 2 \pmod{4}$	1*	$k \equiv 8 \pmod{16}$	1		

Table 14: Covering information for $d = 6$

congruence	p	congruence	p	congruence	p
$k \equiv 0 \pmod{2}$	1*	$k \equiv 77 \pmod{119}$	2	$k \equiv 309 \pmod{442}$	4
$k \equiv 16 \pmod{17}$	1	$k \equiv 43 \pmod{119}$	3	$k \equiv 275 \pmod{442}$	5
$k \equiv 14 \pmod{17}$	2	$k \equiv 9 \pmod{119}$	4	$k \equiv 20 \pmod{221}$	1
$k \equiv 29 \pmod{34}$	1	$k \equiv 313 \pmod{340}$	1	$k \equiv 207 \pmod{221}$	2
$k \equiv 27 \pmod{34}$	2	$k \equiv 279 \pmod{340}$	2	$k \equiv 173 \pmod{221}$	3
$k \equiv 25 \pmod{34}$	3	$k \equiv 245 \pmod{340}$	3	$k \equiv 581 \pmod{884}$	1
$k \equiv 57 \pmod{68}$	1	$k \equiv 211 \pmod{340}$	4	$k \equiv 139 \pmod{884}$	2
$k \equiv 23 \pmod{68}$	2	$k \equiv 177 \pmod{340}$	5	$k \equiv 547 \pmod{884}$	3
$k \equiv 55 \pmod{68}$	3	$k \equiv 483 \pmod{680}$	1	$k \equiv 105 \pmod{884}$	4
$k \equiv 89 \pmod{136}$	1	$k \equiv 143 \pmod{680}$	2	$k \equiv 513 \pmod{663}$	1
$k \equiv 21 \pmod{136}$	2	$k \equiv 449 \pmod{680}$	3	$k \equiv 292 \pmod{663}$	2
$k \equiv 36 \pmod{51}$	1	$k \equiv 109 \pmod{680}$	4	$k \equiv 71 \pmod{663}$	3
$k \equiv 19 \pmod{51}$	2	$k \equiv 755 \pmod{1020}$	1	$k \equiv 1363 \pmod{1768}$	1
$k \equiv 2 \pmod{51}$	3	$k \equiv 415 \pmod{1020}$	2	$k \equiv 921 \pmod{1768}$	2
$k \equiv 17 \pmod{51}$	4	$k \equiv 75 \pmod{1020}$	3	$k \equiv 479 \pmod{1768}$	3
$k \equiv 85 \pmod{102}$	1	$k \equiv 721 \pmod{1020}$	4	$k \equiv 37 \pmod{1768}$	4
$k \equiv 51 \pmod{102}$	2	$k \equiv 381 \pmod{1020}$	5	$k \equiv 887 \pmod{1105}$	1
$k \equiv 287 \pmod{306}$	1	$k \equiv 41 \pmod{1020}$	6	$k \equiv 666 \pmod{1105}$	2
$k \equiv 253 \pmod{306}$	2	$k \equiv 687 \pmod{1020}$	7	$k \equiv 445 \pmod{1105}$	3
$k \equiv 219 \pmod{306}$	3	$k \equiv 1367 \pmod{2040}$	1	$k \equiv 224 \pmod{1105}$	4
$k \equiv 185 \pmod{306}$	4	$k \equiv 347 \pmod{2040}$	2	$k \equiv 1108 \pmod{2210}$	1
$k \equiv 151 \pmod{306}$	5	$k \equiv 1027 \pmod{2040}$	3	$k \equiv 3 \pmod{2210}$	2
$k \equiv 117 \pmod{306}$	6	$k \equiv 7 \pmod{2040}$	4	$k \equiv 477 \pmod{510}$	1
$k \equiv 83 \pmod{153}$	1	$k \equiv 345 \pmod{374}$	1	$k \equiv 443 \pmod{510}$	2
$k \equiv 49 \pmod{153}$	2	$k \equiv 311 \pmod{374}$	2	$k \equiv 409 \pmod{510}$	3
$k \equiv 15 \pmod{153}$	3	$k \equiv 277 \pmod{374}$	3	$k \equiv 375 \pmod{510}$	4
$k \equiv 183 \pmod{204}$	1	$k \equiv 243 \pmod{374}$	4	$k \equiv 341 \pmod{510}$	5
$k \equiv 149 \pmod{204}$	2	$k \equiv 209 \pmod{374}$	5	$k \equiv 52 \pmod{85}$	3
$k \equiv 115 \pmod{204}$	3	$k \equiv 175 \pmod{187}$	1	$k \equiv 18 \pmod{255}$	1
$k \equiv 81 \pmod{204}$	4	$k \equiv 141 \pmod{187}$	2	$k \equiv 239 \pmod{255}$	2
$k \equiv 47 \pmod{204}$	5	$k \equiv 107 \pmod{187}$	3	$k \equiv 205 \pmod{255}$	3
$k \equiv 13 \pmod{204}$	6	$k \equiv 447 \pmod{748}$	1	$k \equiv 171 \pmod{255}$	4
$k \equiv 147 \pmod{170}$	1	$k \equiv 73 \pmod{748}$	2	$k \equiv 579 \pmod{765}$	1
$k \equiv 113 \pmod{170}$	2	$k \equiv 413 \pmod{748}$	3	$k \equiv 324 \pmod{765}$	2
$k \equiv 79 \pmod{170}$	3	$k \equiv 39 \pmod{748}$	4	$k \equiv 1157 \pmod{1530}$	1
$k \equiv 45 \pmod{85}$	1	$k \equiv 379 \pmod{561}$	1	$k \equiv 647 \pmod{1530}$	2
$k \equiv 11 \pmod{85}$	2	$k \equiv 192 \pmod{561}$	2	$k \equiv 137 \pmod{1530}$	3
$k \equiv 213 \pmod{238}$	1	$k \equiv 5 \pmod{561}$	3	$k \equiv 1123 \pmod{1530}$	4
$k \equiv 179 \pmod{238}$	2	$k \equiv 411 \pmod{442}$	1	$k \equiv 613 \pmod{1530}$	5
$k \equiv 145 \pmod{238}$	3	$k \equiv 377 \pmod{442}$	2	$k \equiv 103 \pmod{1530}$	6
$k \equiv 111 \pmod{119}$	1	$k \equiv 343 \pmod{442}$	3	$k \equiv 2364 \pmod{3060}$	1

Table 15: Covering information for $d = 7$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 1599 \pmod{3060}$	2	$k \equiv 35 \pmod{153}$	6	$k \equiv 341 \pmod{1190}$	5
$k \equiv 834 \pmod{3060}$	3	$k \equiv 1021 \pmod{1190}$	1	$k \equiv 171 \pmod{1190}$	6
$k \equiv 69 \pmod{3060}$	4	$k \equiv 851 \pmod{1190}$	2	$k \equiv 1 \pmod{595}$	1
$k \equiv 137 \pmod{153}$	4	$k \equiv 681 \pmod{1190}$	3		
$k \equiv 86 \pmod{153}$	5	$k \equiv 511 \pmod{1190}$	4		

Table 15: Covering information for $d = 7$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 6 \pmod{8}$	1*	$k \equiv 28 \pmod{32}$	1	$k \equiv 600 \pmod{1120}$	3
$k \equiv 90 \pmod{96}$	1	$k \equiv 20 \pmod{32}$	2	$k \equiv 40 \pmod{1120}$	4
$k \equiv 74 \pmod{96}$	2	$k \equiv 44 \pmod{64}$	1	$k \equiv 344 \pmod{400}$	1
$k \equiv 58 \pmod{96}$	3	$k \equiv 12 \pmod{64}$	2	$k \equiv 264 \pmod{400}$	2
$k \equiv 42 \pmod{96}$	4	$k \equiv 36 \pmod{64}$	3	$k \equiv 184 \pmod{400}$	3
$k \equiv 122 \pmod{192}$	1	$k \equiv 4 \pmod{64}$	4	$k \equiv 904 \pmod{1200}$	1
$k \equiv 26 \pmod{192}$	2	$k \equiv 72 \pmod{80}$	1	$k \equiv 504 \pmod{1200}$	2
$k \equiv 106 \pmod{192}$	3	$k \equiv 56 \pmod{80}$	2	$k \equiv 104 \pmod{1200}$	3
$k \equiv 10 \pmod{192}$	4	$k \equiv 168 \pmod{240}$	1	$k \equiv 1224 \pmod{1600}$	1
$k \equiv 146 \pmod{160}$	1	$k \equiv 88 \pmod{240}$	2	$k \equiv 824 \pmod{1600}$	2
$k \equiv 114 \pmod{160}$	2	$k \equiv 8 \pmod{240}$	3	$k \equiv 424 \pmod{1600}$	3
$k \equiv 82 \pmod{160}$	3	$k \equiv 112 \pmod{128}$	1	$k \equiv 1624 \pmod{3200}$	1
$k \equiv 50 \pmod{160}$	4	$k \equiv 80 \pmod{128}$	2	$k \equiv 24 \pmod{3200}$	2
$k \equiv 658 \pmod{800}$	1	$k \equiv 48 \pmod{128}$	3	$k \equiv 18 \pmod{19}$	1
$k \equiv 498 \pmod{800}$	2	$k \equiv 272 \pmod{384}$	3	$k \equiv 35 \pmod{38}$	1
$k \equiv 338 \pmod{800}$	3	$k \equiv 144 \pmod{384}$	4	$k \equiv 52 \pmod{57}$	1
$k \equiv 178 \pmod{800}$	4	$k \equiv 16 \pmod{384}$	5	$k \equiv 33 \pmod{57}$	2
$k \equiv 18 \pmod{800}$	5	$k \equiv 224 \pmod{256}$	1	$k \equiv 14 \pmod{57}$	3
$k \equiv 418 \pmod{448}$	1	$k \equiv 192 \pmod{256}$	2	$k \equiv 69 \pmod{76}$	1
$k \equiv 354 \pmod{448}$	2	$k \equiv 160 \pmod{256}$	3	$k \equiv 31 \pmod{76}$	2
$k \equiv 290 \pmod{448}$	3	$k \equiv 128 \pmod{256}$	4	$k \equiv 105 \pmod{114}$	1
$k \equiv 2 \pmod{224}$	1	$k \equiv 352 \pmod{512}$	2	$k \equiv 67 \pmod{114}$	2
$k \equiv 162 \pmod{224}$	2	$k \equiv 96 \pmod{512}$	3	$k \equiv 257 \pmod{342}$	1
$k \equiv 98 \pmod{224}$	3	$k \equiv 320 \pmod{512}$	4	$k \equiv 143 \pmod{342}$	2
$k \equiv 34 \pmod{224}$	4	$k \equiv 64 \pmod{512}$	5	$k \equiv 29 \pmod{342}$	3
$k \equiv 322 \pmod{384}$	1	$k \equiv 544 \pmod{768}$	1	$k \equiv 141 \pmod{152}$	1
$k \equiv 194 \pmod{384}$	2	$k \equiv 288 \pmod{768}$	2	$k \equiv 103 \pmod{152}$	2
$k \equiv 834 \pmod{1152}$	1	$k \equiv 32 \pmod{768}$	3	$k \equiv 65 \pmod{152}$	3
$k \equiv 450 \pmod{1152}$	2	$k \equiv 512 \pmod{768}$	4	$k \equiv 27 \pmod{152}$	4
$k \equiv 66 \pmod{1152}$	3	$k \equiv 256 \pmod{768}$	5	$k \equiv 215 \pmod{228}$	1
$k \equiv 898 \pmod{1024}$	1	$k \equiv 0 \pmod{768}$	6	$k \equiv 177 \pmod{228}$	2
$k \equiv 642 \pmod{1024}$	2	$k \equiv 520 \pmod{560}$	1	$k \equiv 139 \pmod{228}$	3
$k \equiv 386 \pmod{1024}$	3	$k \equiv 440 \pmod{560}$	2	$k \equiv 101 \pmod{228}$	4
$k \equiv 130 \pmod{1024}$	4	$k \equiv 360 \pmod{560}$	3	$k \equiv 63 \pmod{228}$	5
$k \equiv 258 \pmod{512}$	1	$k \equiv 280 \pmod{560}$	4	$k \equiv 25 \pmod{228}$	6
$k \equiv 1026 \pmod{1536}$	1	$k \equiv 200 \pmod{560}$	5	$k \equiv 213 \pmod{228}$	7
$k \equiv 514 \pmod{1536}$	2	$k \equiv 680 \pmod{1120}$	1	$k \equiv 403 \pmod{456}$	1
$k \equiv 2 \pmod{1536}$	3	$k \equiv 120 \pmod{1120}$	2	$k \equiv 175 \pmod{456}$	2

Table 16: Covering information for $d = 9$ (Part I)

congruence	p	congruence	p	congruence	p
$k \equiv 365 \pmod{456}$	3	$k \equiv 661 \pmod{1140}$	3	$k \equiv 1075 \pmod{1330}$	1
$k \equiv 137 \pmod{456}$	4	$k \equiv 471 \pmod{1140}$	4	$k \equiv 809 \pmod{1330}$	2
$k \equiv 327 \pmod{456}$	5	$k \equiv 281 \pmod{1140}$	5	$k \equiv 543 \pmod{1330}$	3
$k \equiv 555 \pmod{912}$	1	$k \equiv 91 \pmod{1140}$	6	$k \equiv 277 \pmod{665}$	1
$k \equiv 99 \pmod{912}$	2	$k \equiv 1003 \pmod{1140}$	7	$k \equiv 11 \pmod{665}$	2
$k \equiv 517 \pmod{684}$	1	$k \equiv 813 \pmod{1140}$	8	$k \equiv 313 \pmod{342}$	4
$k \equiv 289 \pmod{684}$	2	$k \equiv 623 \pmod{1140}$	9	$k \equiv 275 \pmod{342}$	5
$k \equiv 61 \pmod{684}$	3	$k \equiv 433 \pmod{1140}$	10	$k \equiv 66 \pmod{171}$	1
$k \equiv 479 \pmod{684}$	4	$k \equiv 243 \pmod{1140}$	11	$k \equiv 28 \pmod{171}$	2
$k \equiv 935 \pmod{1368}$	1	$k \equiv 2333 \pmod{3420}$	1	$k \equiv 161 \pmod{171}$	3
$k \equiv 251 \pmod{1368}$	2	$k \equiv 1193 \pmod{3420}$	2	$k \equiv 465 \pmod{513}$	1
$k \equiv 707 \pmod{1368}$	3	$k \equiv 53 \pmod{3420}$	3	$k \equiv 294 \pmod{513}$	2
$k \equiv 23 \pmod{1368}$	4	$k \equiv 585 \pmod{760}$	1	$k \equiv 1149 \pmod{1539}$	1
$k \equiv 287 \pmod{304}$	1	$k \equiv 395 \pmod{760}$	2	$k \equiv 636 \pmod{1539}$	2
$k \equiv 249 \pmod{304}$	2	$k \equiv 1725 \pmod{2280}$	1	$k \equiv 123 \pmod{1539}$	3
$k \equiv 211 \pmod{304}$	3	$k \equiv 965 \pmod{2280}$	2	$k \equiv 2099 \pmod{3078}$	1
$k \equiv 477 \pmod{608}$	1	$k \equiv 205 \pmod{2280}$	3	$k \equiv 1073 \pmod{3078}$	2
$k \equiv 173 \pmod{608}$	2	$k \equiv 1535 \pmod{2280}$	4	$k \equiv 769 \pmod{1026}$	1
$k \equiv 439 \pmod{608}$	3	$k \equiv 3055 \pmod{4560}$	1	$k \equiv 427 \pmod{1026}$	2
$k \equiv 743 \pmod{1216}$	1	$k \equiv 775 \pmod{4560}$	2	$k \equiv 85 \pmod{1026}$	3
$k \equiv 135 \pmod{1216}$	2	$k \equiv 2295 \pmod{4560}$	3	$k \equiv 731 \pmod{1026}$	4
$k \equiv 1009 \pmod{1216}$	3	$k \equiv 15 \pmod{4560}$	4	$k \equiv 1415 \pmod{2052}$	1
$k \equiv 705 \pmod{1216}$	4	$k \equiv 108 \pmod{133}$	1	$k \equiv 2441 \pmod{4104}$	1
$k \equiv 401 \pmod{1216}$	5	$k \equiv 70 \pmod{133}$	2	$k \equiv 389 \pmod{4104}$	2
$k \equiv 97 \pmod{1216}$	6	$k \equiv 32 \pmod{133}$	3	$k \equiv 3125 \pmod{6156}$	1
$k \equiv 971 \pmod{1216}$	7	$k \equiv 127 \pmod{266}$	1	$k \equiv 47 \pmod{6156}$	2
$k \equiv 667 \pmod{1216}$	8	$k \equiv 89 \pmod{266}$	2	$k \equiv 2061 \pmod{2394}$	1
$k \equiv 363 \pmod{1216}$	9	$k \equiv 51 \pmod{266}$	3	$k \equiv 1719 \pmod{2394}$	2
$k \equiv 1275 \pmod{2432}$	1	$k \equiv 279 \pmod{532}$	1	$k \equiv 1377 \pmod{2394}$	3
$k \equiv 59 \pmod{2432}$	2	$k \equiv 13 \pmod{532}$	2	$k \equiv 1035 \pmod{2394}$	4
$k \equiv 1541 \pmod{1824}$	1	$k \equiv 505 \pmod{532}$	3	$k \equiv 693 \pmod{2394}$	5
$k \equiv 1237 \pmod{1824}$	2	$k \equiv 239 \pmod{532}$	4	$k \equiv 2745 \pmod{4788}$	1
$k \equiv 933 \pmod{1824}$	3	$k \equiv 467 \pmod{532}$	5	$k \equiv 351 \pmod{4788}$	2
$k \equiv 629 \pmod{1824}$	4	$k \equiv 733 \pmod{1064}$	1	$k \equiv 2403 \pmod{4788}$	3
$k \equiv 325 \pmod{1824}$	5	$k \equiv 201 \pmod{1064}$	2	$k \equiv 9 \pmod{4788}$	4
$k \equiv 1845 \pmod{3648}$	1	$k \equiv 961 \pmod{1064}$	3	$k \equiv 178 \pmod{209}$	1
$k \equiv 21 \pmod{3648}$	2	$k \equiv 1759 \pmod{2128}$	1	$k \equiv 140 \pmod{209}$	2
$k \equiv 76 \pmod{95}$	1	$k \equiv 695 \pmod{2128}$	2	$k \equiv 102 \pmod{209}$	3
$k \equiv 57 \pmod{95}$	2	$k \equiv 1493 \pmod{2128}$	3	$k \equiv 64 \pmod{209}$	4
$k \equiv 38 \pmod{95}$	3	$k \equiv 429 \pmod{2128}$	4	$k \equiv 26 \pmod{209}$	5
$k \equiv 19 \pmod{95}$	4	$k \equiv 2291 \pmod{3192}$	1	$k \equiv 197 \pmod{418}$	1
$k \equiv 0 \pmod{95}$	5	$k \equiv 1227 \pmod{3192}$	2	$k \equiv 159 \pmod{418}$	2
$k \equiv 169 \pmod{190}$	1	$k \equiv 163 \pmod{3192}$	3	$k \equiv 121 \pmod{418}$	3
$k \equiv 131 \pmod{190}$	2	$k \equiv 657 \pmod{798}$	1	$k \equiv 83 \pmod{418}$	4
$k \equiv 93 \pmod{190}$	3	$k \equiv 391 \pmod{798}$	2	$k \equiv 463 \pmod{836}$	1
$k \equiv 245 \pmod{380}$	1	$k \equiv 125 \pmod{798}$	3	$k \equiv 45 \pmod{836}$	2
$k \equiv 55 \pmod{380}$	2	$k \equiv 220 \pmod{399}$	1	$k \equiv 425 \pmod{836}$	3
$k \equiv 207 \pmod{380}$	3	$k \equiv 353 \pmod{399}$	2	$k \equiv 843 \pmod{1672}$	1
$k \equiv 17 \pmod{380}$	4	$k \equiv 87 \pmod{399}$	3	$k \equiv 7 \pmod{1672}$	2
$k \equiv 357 \pmod{380}$	5	$k \equiv 182 \pmod{399}$	4	$k \equiv 214 \pmod{247}$	1
$k \equiv 167 \pmod{380}$	6	$k \equiv 315 \pmod{399}$	5	$k \equiv 176 \pmod{247}$	2
$k \equiv 319 \pmod{380}$	7	$k \equiv 1246 \pmod{1596}$	1	$k \equiv 138 \pmod{247}$	3
$k \equiv 129 \pmod{380}$	8	$k \equiv 847 \pmod{1596}$	2	$k \equiv 100 \pmod{247}$	4
$k \equiv 1041 \pmod{1140}$	1	$k \equiv 448 \pmod{1596}$	3	$k \equiv 62 \pmod{247}$	5
$k \equiv 851 \pmod{1140}$	2	$k \equiv 49 \pmod{1596}$	4	$k \equiv 271 \pmod{494}$	1

Table 16: Covering information for $d = 9$ (Part II)

congruence	p	congruence	p	congruence	p
$k \equiv 233 \pmod{494}$	2	$k \equiv 2093 \pmod{2584}$	1	$k \equiv 267 \pmod{1444}$	4
$k \equiv 195 \pmod{494}$	3	$k \equiv 1447 \pmod{2584}$	2	$k \equiv 1673 \pmod{2166}$	1
$k \equiv 157 \pmod{494}$	4	$k \equiv 801 \pmod{2584}$	3	$k \equiv 951 \pmod{2166}$	2
$k \equiv 613 \pmod{988}$	1	$k \equiv 155 \pmod{2584}$	4	$k \equiv 229 \pmod{2166}$	3
$k \equiv 119 \pmod{988}$	2	$k \equiv 2055 \pmod{2584}$	5	$k \equiv 1635 \pmod{2166}$	4
$k \equiv 575 \pmod{988}$	3	$k \equiv 1409 \pmod{2584}$	6	$k \equiv 913 \pmod{2166}$	5
$k \equiv 1069 \pmod{1976}$	1	$k \equiv 763 \pmod{2584}$	7	$k \equiv 191 \pmod{1083}$	1
$k \equiv 81 \pmod{1976}$	2	$k \equiv 117 \pmod{2584}$	8	$k \equiv 875 \pmod{1083}$	2
$k \equiv 1031 \pmod{1482}$	1	$k \equiv 1371 \pmod{1938}$	1	$k \equiv 514 \pmod{1083}$	3
$k \equiv 537 \pmod{1482}$	2	$k \equiv 725 \pmod{1938}$	2	$k \equiv 153 \pmod{1083}$	4
$k \equiv 43 \pmod{741}$	1	$k \equiv 79 \pmod{1938}$	3	$k \equiv 837 \pmod{1083}$	5
$k \equiv 252 \pmod{741}$	2	$k \equiv 1333 \pmod{1938}$	4	$k \equiv 476 \pmod{1083}$	6
$k \equiv 499 \pmod{741}$	3	$k \equiv 687 \pmod{1938}$	5	$k \equiv 115 \pmod{1083}$	7
$k \equiv 5 \pmod{741}$	4	$k \equiv 41 \pmod{969}$	1	$k \equiv 1521 \pmod{1805}$	1
$k \equiv 288 \pmod{323}$	1	$k \equiv 326 \pmod{969}$	2	$k \equiv 1160 \pmod{1805}$	2
$k \equiv 250 \pmod{323}$	2	$k \equiv 649 \pmod{969}$	3	$k \equiv 799 \pmod{1805}$	3
$k \equiv 212 \pmod{323}$	3	$k \equiv 3 \pmod{969}$	4	$k \equiv 438 \pmod{1805}$	4
$k \equiv 497 \pmod{646}$	1	$k \equiv 324 \pmod{361}$	1	$k \equiv 77 \pmod{1805}$	5
$k \equiv 459 \pmod{646}$	2	$k \equiv 286 \pmod{361}$	2	$k \equiv 3649 \pmod{4332}$	1
$k \equiv 421 \pmod{646}$	3	$k \equiv 248 \pmod{361}$	3	$k \equiv 2205 \pmod{4332}$	2
$k \equiv 383 \pmod{646}$	4	$k \equiv 210 \pmod{361}$	4	$k \equiv 761 \pmod{4332}$	3
$k \equiv 345 \pmod{646}$	5	$k \equiv 172 \pmod{361}$	5	$k \equiv 1483 \pmod{2888}$	1
$k \equiv 307 \pmod{646}$	6	$k \equiv 134 \pmod{361}$	6	$k \equiv 2927 \pmod{5776}$	1
$k \equiv 269 \pmod{646}$	7	$k \equiv 96 \pmod{361}$	7	$k \equiv 39 \pmod{5776}$	2
$k \equiv 877 \pmod{1292}$	1	$k \equiv 419 \pmod{722}$	1	$k \equiv 2889 \pmod{3610}$	1
$k \equiv 231 \pmod{1292}$	2	$k \equiv 381 \pmod{722}$	2	$k \equiv 2167 \pmod{3610}$	2
$k \equiv 839 \pmod{1292}$	3	$k \equiv 343 \pmod{722}$	3	$k \equiv 1445 \pmod{3610}$	3
$k \equiv 2777 \pmod{3876}$	1	$k \equiv 1027 \pmod{1444}$	1	$k \equiv 723 \pmod{3610}$	4
$k \equiv 1485 \pmod{3876}$	2	$k \equiv 305 \pmod{1444}$	2	$k \equiv 1 \pmod{3610}$	5
$k \equiv 193 \pmod{3876}$	3	$k \equiv 989 \pmod{1444}$	3		

Table 16: Covering information for $d = 9$ (Part III)