



HIGH TEMPERATURE DOMINEERING POSITIONS

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Abstract

DOMINEERING is a partizan game where two players have a collection of dominoes which they place on the grid in turn, covering up squares. One player places tiles vertically, while the other places them horizontally; the first player who cannot move loses. It has been conjectured that the highest temperature possible in DOMINEERING is 2. We have developed a program that enables a parallel exhaustive search of DOMINEERING positions with temperatures close to or equal to 2 to allow for analysis of such positions.

1. Introduction

DOMINEERING is a partizan game where Left places blue, vertical dominoes, and Right places red, horizontal dominoes. We will assume that if a player cannot make a move on their turn, then they lose.

Traditionally, DOMINEERING is played on a rectangular board. As a game progresses, it naturally breaks into smaller components though that can have different shapes (see Figure 1). On their turn, the players now choose a component and make their move in it. The decomposition into such components is called the *disjunctive sum*.

We consider each of the components as a game in its own right, with possibly non-alternating play. To determine which player wins in a sum and how, we can

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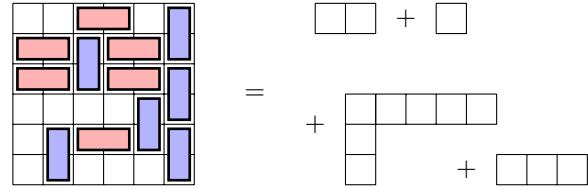


Figure 1: A possible position in DOMINEERING.

assign to each game a *game value* which indicates which player wins it and how much of an advantage they have.

Similarly, we can assign to each component a numerical value, called the *temperature*, that indicates the urgency of making a move in it, or, equivalently, how much the first player to go gains by making their move in that component. Note however that in general the best move is not always in the hottest component, although it is unknown whether such situations occur in DOMINEERING.

For more details about combinatorial game theory, which is the study of games similar to DOMINEERING, including the formal definitions of value and temperature, see, for example, [2, 11].

An open question is what temperatures are possible in DOMINEERING (see, for example, [2, 4, 1]) and it has been conjectured that the maximum possible temperature is 2. This conjecture is referred to as Berlekamp's conjecture, although it is unknown whether he was actually the first to state this conjecture.

Drummond–Cole has found the first DOMINEERING position with temperature 2 [3], which we show in Figure 2. In [9], Shankar and Sridharan list DOMINEERING positions with various temperatures found from a search of 5×6 , 4×8 , 3×10 , and 2×16 grids, including temperatures close to 2. At a Virtual CGT Workshop in 2020, several more positions with temperature 2 were found by trial and error. All positions of temperature 2 or close to 2 found thus far have a common structure, which we call a *hook* (see Figure 3).

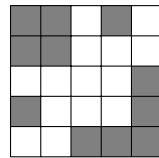


Figure 2: The Drummond–Cole position, which has temperature 2.

We are interested in whether the hook is a necessary condition for a high-temperature position in DOMINEERING since this might open new avenues towards a proof of Berlekamp's conjecture. Note that the hook is not sufficient. For example,

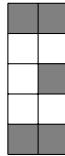


Figure 3: The DOMINEERING hook.

the position shown in Figure 4 has temperature 0.

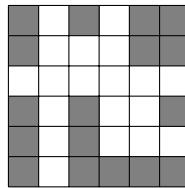


Figure 4: A DOMINEERING position with a hook and temperature 0.

2. Search for High Temperature Positions

2.1. Exhaustive Search

As a first step towards identifying whether the hook is necessary for a high temperature position, we started an exhaustive search for positions with temperature above a given threshold.

We developed a program [7] that allows for parallel, exhaustive search of DOMINEERING positions to compute game values, thermographs (used to find temperature), and temperatures. The implementation approach has been heavily inspired by Aaron Siegel's `cgsuite` [10] but implemented from scratch in Rust for improved performance.

For easier data storage, we will think of each game as embedded into a rectangle, with some spaces already occupied. For example, the game shown in Figure 5 indicates a disjunctive sum of two games, from which we can then remove several completely occupied rows and columns to reach the smallest possible rectangles in which to embed the components.

It is known that the temperature of a disjunctive sum of games is no larger than at least one of the components, i.e., $t(G+H) \leq \max \{t(G), t(H)\}$, for any two games G and H (see for example [11]). Due to this, in the search for high temperature

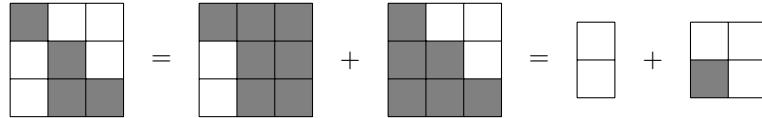


Figure 5: Using disjunctive sum and empty row/column removal for the smallest possible rectangles.

positions, it is enough if we consider only positions that do not decompose into a disjunctive sum.

Also note that we did not only consider positions that can be reached during actual game play from a rectangle, but any possible subgrid of rectangular grids.

2.2. Genetic Algorithm

Due to the exponential growth of the number of DOMINEERING positions as the board size increases, we are unable to run an exhaustive search for larger boards. For 7×7 boards, we ran a genetic algorithm to discover additional temperature 2 positions.

Genetic algorithms enable the search for objects that optimize a given fitness function through an iterative process where candidates are modified and evaluated. Candidates are represented as a string of “genes” that can be a bit string or a more complex list of instructions. The process starts with an initial population and assigns a numerical score to each candidate. After scoring, a new generation of candidates is created. The best candidates from the previous generation are selected, mutated by randomly changing parts of their genes, and crossed over with each other. This process involves taking parts of the genes from one candidate and combining them with parts of the genes from another to create a new candidate. The process terminates when a satisfactory solution is found.

Genetic algorithms were first applied to combinatorial games by Huggan and Tennenhouse in 2021 [5]. Since then, several more applications to games have been published (see for example [6, 8, 12]).

In genetic algorithm mode, our program starts with an initial population of DOMINEERING positions of a given grid size that is random or pre-set by the user, e.g., a population that is known to have high temperature. Each position is assigned a fitness score equal to its temperature, but to consider only positions without decompositions, the score is decreased if the board is not connected to eliminate these positions from the population. After the scoring step, the top positions from the generation are crossed over with each other by taking part of the grid from one position and the rest from the other, and mutated by flipping random grid cells, similar to the CROSSOVER-MUTATION game [5].

3. Infinite Family of Positions with Temperature 2

During our search we have found an infinite family of positions which have temperature 2. A position in this family is built by appending 2×2 L -shapes to the bottom of the Drummond–Cole position shown in Figure 2. We will denote the position with n such L -shapes added by DCL_n (see Figure 6).

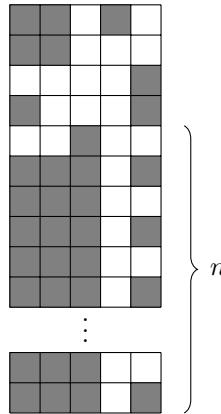


Figure 6: The position DCL_n , which has temperature 2.

We claim that this position is a disjunctive sum of the Drummond–Cole position and the chain of L s. The former has value $\pm(2*)$, while the chain has value 0 or $*$. Thus, the overall position has canonical form $\pm(2*)$ or ± 2 depending on n . To show this, we will determine the values of several related positions first. We will denote by L_n a chain of n of the 2×2 L -shapes. Then L_n^+ has one additional square in the path, while L_n^- has one removed. Finally, L_n^\cup is L_n^+ with an additional square and a hook on one end added. For all four types of positions, see Figure 7.

Lemma 1. *For all positive integers n , the positions L_n , L_n^+ , and L_n^- have value 0 if n is even and $*$ if n is odd.*

Technically, we do not need all three of these families to determine the value of DCL_n . But since some moves in one family result in positions in another family, it is easier to prove them all at once.

Proof of Lemma 1. Let $c(n) = 0$ when n is even and $c(n) = *$ when n is odd. We will show that $L_n = L_n^+ = L_n^- = c(n)$ by strong induction.

For the base case, L_1 and L_1^+ have value $*$ since for both players the only moves are to two single spaces, which is 0. Now assume that the result holds for $n \leq k-1$.

Since single squares have value 0, we will ignore them in the sums representing an option.

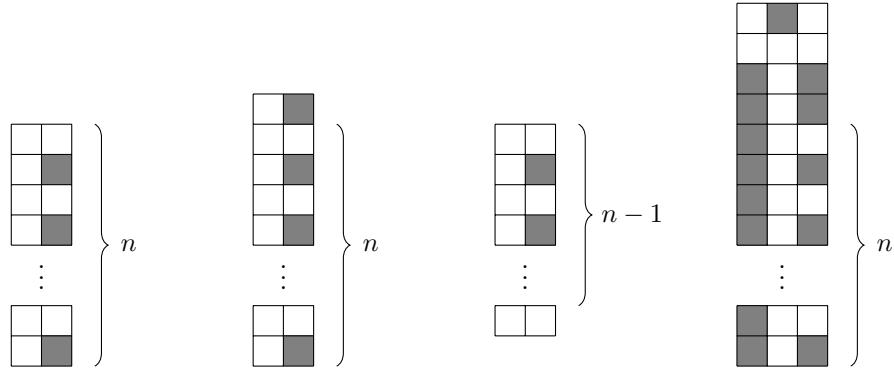


Figure 7: The positions L_n (left), L_n^+ (centre left), L_n^- (centre right), and L_n^U (right).

For L_n , any move by Left results in the sum $L_i + L_j$ or $L_i^- + L_j^+$ with $i + j = n - 1$, or L_{n-1} . By induction, all of these positions have value $c(n - 1)$. Any Right move is to a sum $L_i + L_j^+$ with $i + j = n - 1$, or to L_{n-1}^+ . Again, all of these positions have value $c(n - 1)$. Thus, L_n has value $c(n)$.

For L_n^+ , any move by Left results in the sum $L_i^+ + L_j$ with $i + j = n - 1$, L_{n-1} , or L_{n-1}^+ . By induction, all of these positions have value $c(n - 1)$. Any Right move is to a sum $L_i^+ + L_j^+$ with $i + j = n - 1$, or to L_{n-1}^+ . Again, all of these positions have value $c(n - 1)$. Thus, L_n^+ has value $c(n)$.

For L_n^- , any move by Left results in the sum $L_i^- + L_j$ with $i + j = n - 1$, or L_{n-1}^- . By induction, all of these positions have value $c(n - 1)$. Any Right move is to a sum $L_i + L_j$ with $i + j = n - 1$, or to L_{n-1} . Again, all of these positions have value $c(n - 1)$. Thus, L_n^- has value $c(n)$. \square

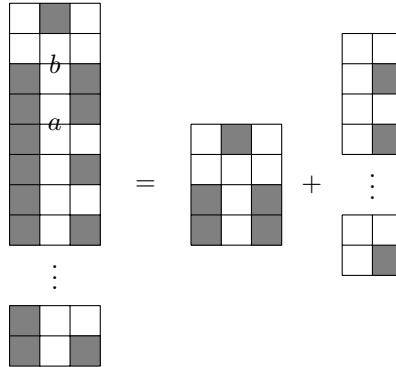
Lemma 2. *For all positive integers n , the position L_n^U has value $2*$ if n is even and 2 if n is odd.*

Proof. We will show that L_n^U is equal to the disjunctive sum of the hook with two squares added and L_n (see Figure 8). We will do so by showing that Left's move placing at a in L_n^U is strictly dominated by the move at b in Figure 8.

When Left places a domino at a in Figure 8, the result is a sum of the hook with a single square attached, an isolated square, and L_{n-1}^+ . This has value $\{2 \mid 1\} + *$ when n is even and $\{2 \mid 1\}$ when n is odd.

On the other hand, when Left places a domino at b in Figure 8, the resulting sum is two vertical dominoes plus L_n^+ , which has value 2 when n is even and $2*$ when n is odd. This is strictly better for Left, thus she will never play at a and the sum claimed above holds.

Since the value of the hook with two squares added is $2*$, the value of L_n^U is as claimed. \square

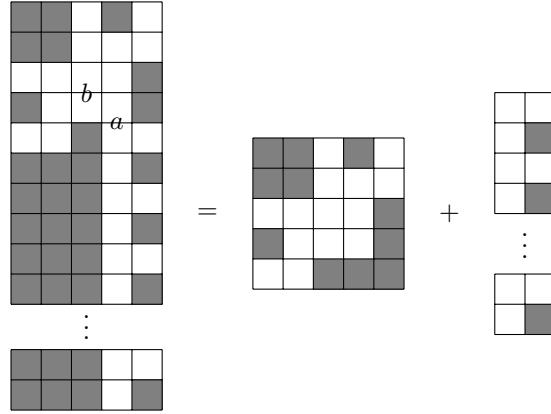
Figure 8: Decomposition of L_n^U .

We will now determine the value of DCL_n in a similar manner.

Proposition 1. *For all non-negative integers n , the position DCL_n has value $\pm(2*)$ if n is even and ± 2 if n is odd. Thus, the temperature of the position DCL_n is 2.*

Proof. When $n = 0$, then DCL_n is the Drummond–Cole position, which is known to have value $\pm(2*)$.

For $n \geq 1$, we will show that DCL_n is equal to the disjunctive sum of L_n and the Drummond–Cole position (see Figure 9). We will do so by showing that Left's move placing at a in DCL_n is strictly dominated by the move in position b in Figure 9.

Figure 9: Decomposition of DCL_n .

When Left places a domino at a in Figure 9, the result is a sum of the Drummond–Cole position with the corner square removed (which has value $\pm\{2 \mid 1\}$), an isolated

square, and L_{n-1}^+ . This has value $\pm\{2 \mid 1\} + *$ when n is even and $\pm\{2 \mid 1\}$ when n is odd.

On the other hand, when Left places a domino at b in Figure 9, the resulting sum is a hook plus L_n^\cup , which has value $2*$ when n is even and 2 when n is odd. This is strictly better for Left, thus she will never play at a and the sum claimed above holds.

Since the value of the Drummond–Cole position is $\pm(2*)$, the result holds. \square

4. Search Results

We have run our exhaustive search program for grids of size 5×5 and 5×6 . While it is possible to run it for larger grids, the number of positions grows exponentially with the size of the grid and the search space becomes too large to be feasible. The main limitation is the memory of the machine required to store the transposition table of already computed positions. It is possible to run the search without the use of a transposition table; however, the search is then too slow, taking multiple seconds for each evaluated position. Each position is stored as a 64-bit number, so even storing the keys for a 6×6 transposition table would require $2^{36} \times 8$ bytes (~ 550 GB) and the estimated memory requirement for the exhaustive 6×6 grid search is about 4 TB.

In the tables below, we are omitting positions that are a rotation or reflection of already included positions as they have the same temperature. For the 5×5 and 5×6 searches, we are including only positions with temperatures higher than $7/4$ in the tables.

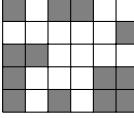
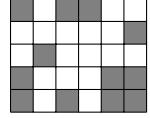
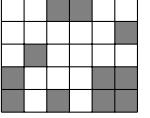
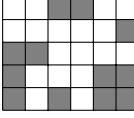
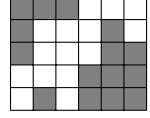
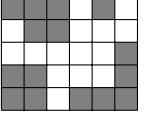
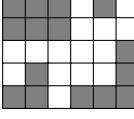
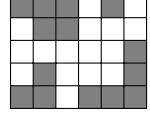
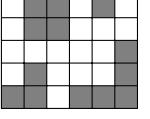
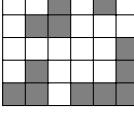
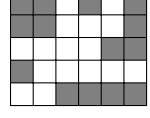
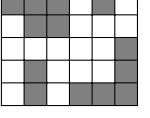
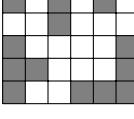
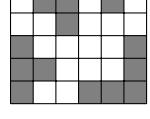
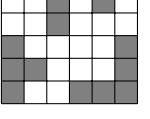
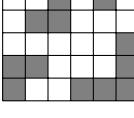
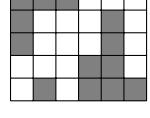
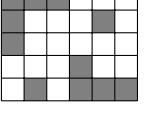
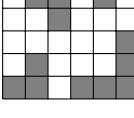
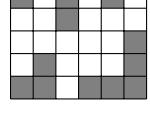
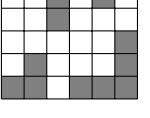
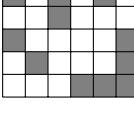
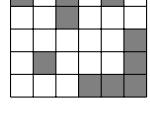
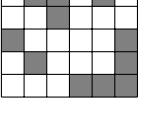
Notice that all positions listed contain the hook. Further, all temperature 2 positions contain the Drummond–Cole position. Although we do not prove so, it is likely that most, if not all, of these positions are a disjunctive sum of the Drummond–Cole position and the position P containing the remaining squares, where P has as its value an infinitesimal or a number, similar to the infinite family DCL_n in Section 3.

Raw JSON result files are available for download at the `cgt-tools` [7] releases page.

4.1. 5×5

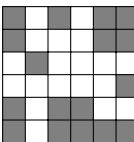
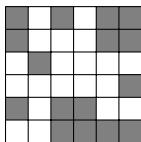
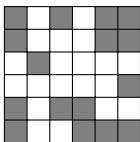
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4.2. 5×6

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	15/8		15/8		15/8
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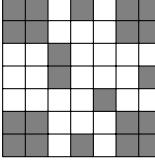
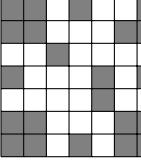
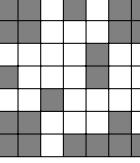
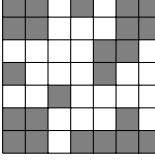
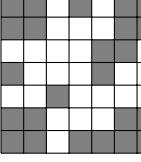
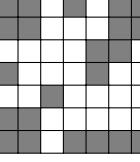
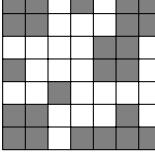
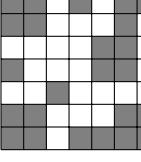
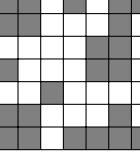
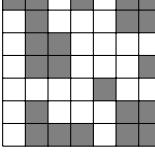
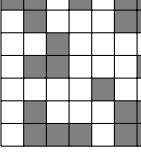
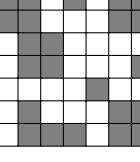
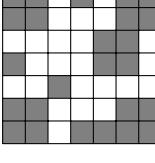
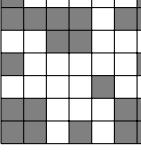
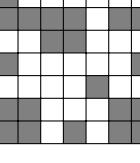
4.3. 6×6

Due to resource limitations, the 6×6 search results are not exhaustive. The search space was limited to positions with no more than 20 empty tiles. The table includes unique, up to rotation and reflection, positions with temperature 2.

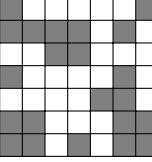
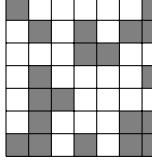
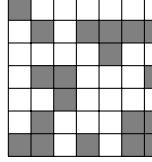
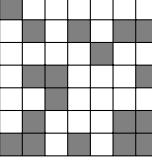
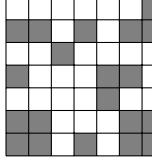
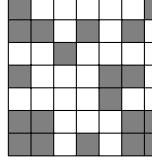
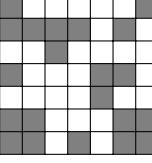
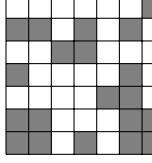
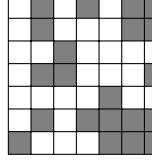
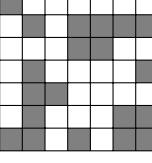
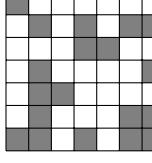
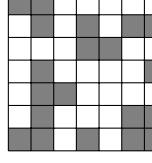
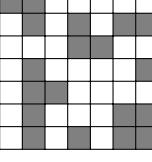
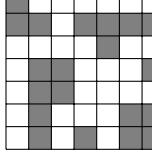
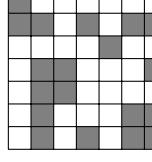
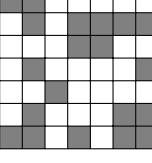
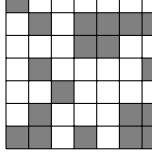
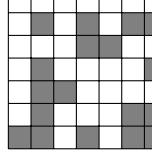
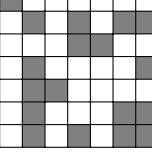
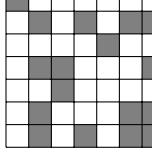
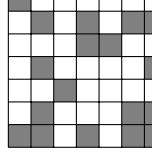
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4.4. 7×7

The 7×7 positions listed below were found using a simple genetic algorithm that started with a seed of manually discovered 7×7 grids with temperature 2 and mutated it while saving positions that maintained a temperature of 2.

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	2		2		2
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Position	Temp.	Position	Temp.	Position	Temp.
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Position	Temp.	Position	Temp.	Position	Temp.
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Position	Temp.	Position	Temp.	Position	Temp.
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Position	Temp.	Position	Temp.	Position	Temp.
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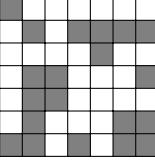
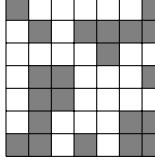
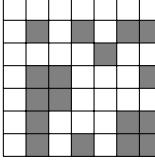
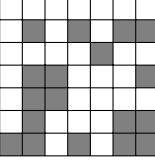
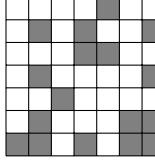
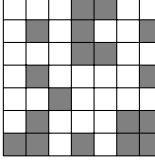
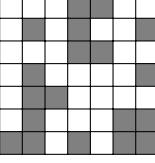
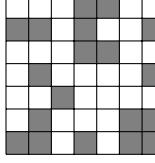
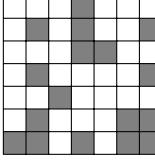
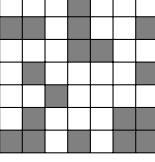
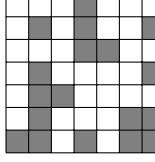
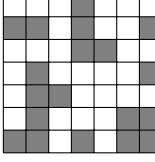
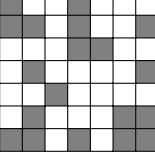
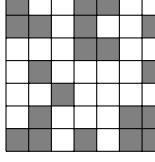
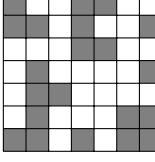
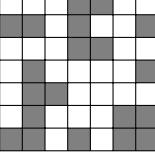
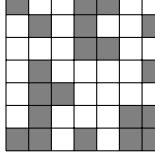
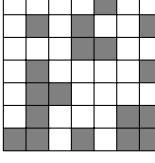
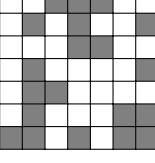
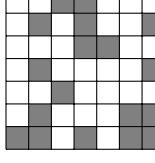
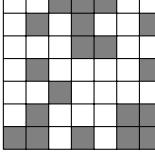
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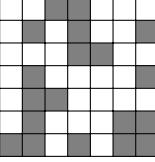
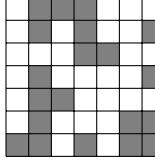
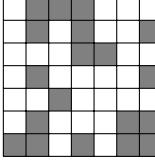
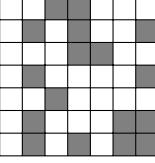
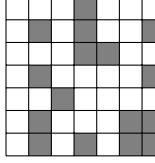
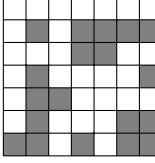
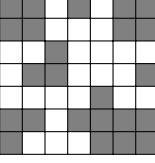
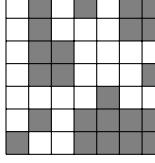
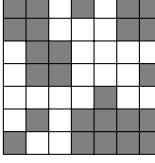
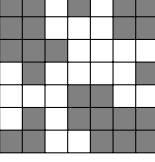
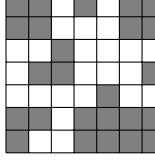
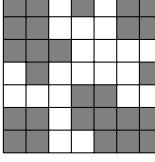
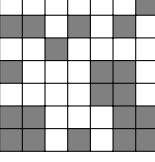
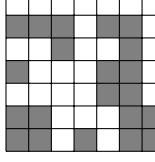
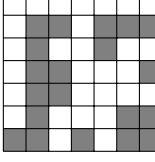
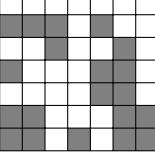
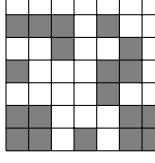
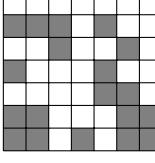
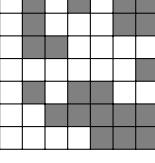
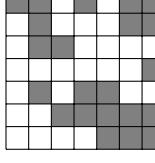
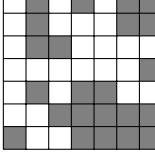
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Position	Temp.	Position	Temp.	Position	Temp.
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Position	Temp.	Position	Temp.	Position	Temp.
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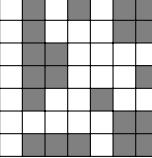
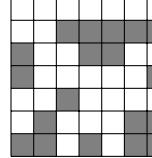
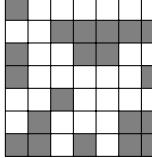
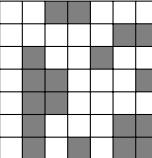
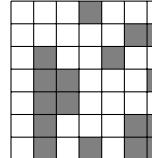
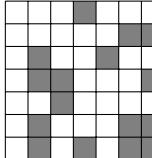
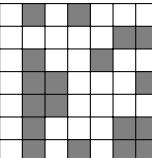
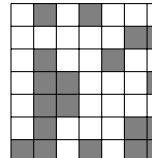
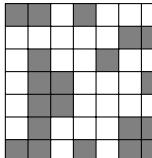
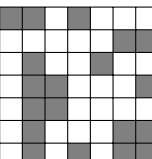
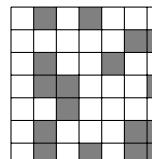
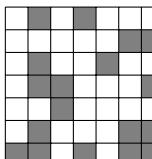
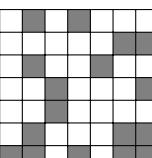
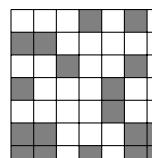
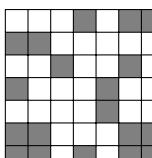
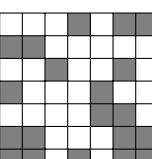
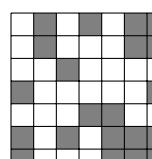
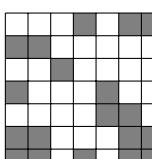
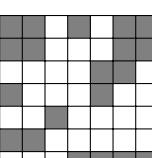
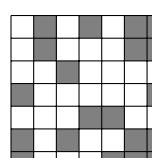
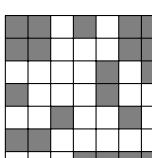
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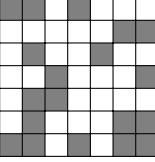
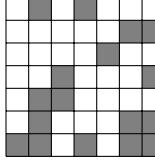
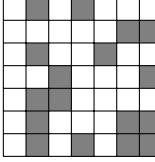
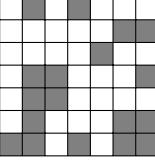
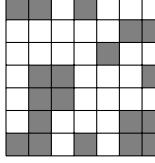
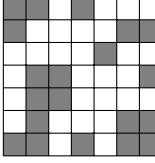
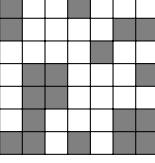
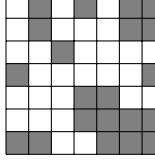
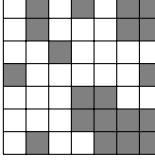
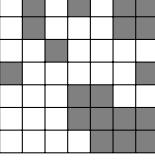
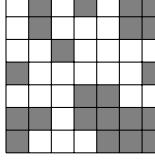
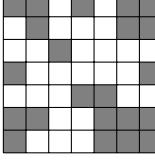
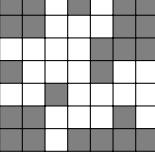
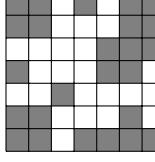
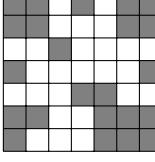
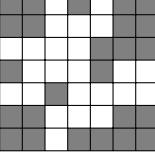
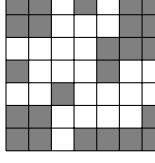
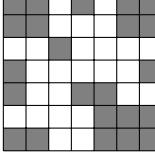
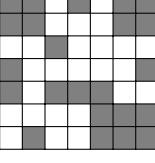
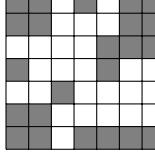
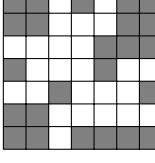
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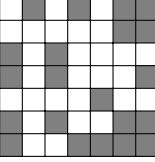
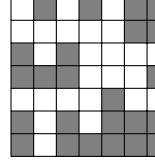
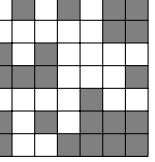
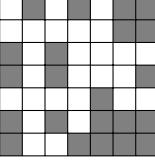
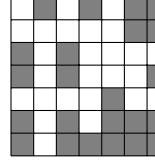
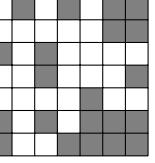
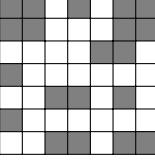
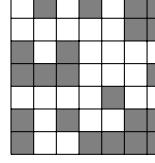
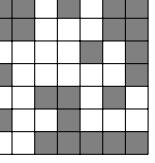
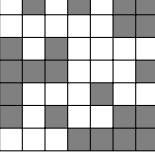
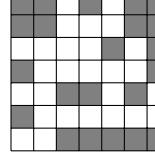
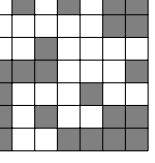
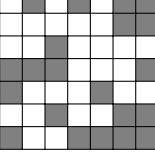
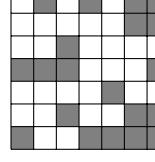
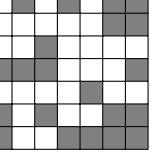
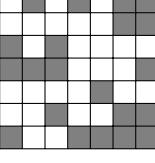
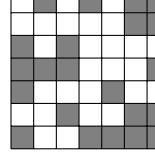
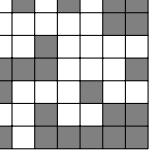
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5. Conclusion

Our search results support the conjecture that every high temperature position contains the hook – this is also the case for all 6×6 positions checked, even those we did not list here. The results further make it seem as if the Drummond–Cole position is contained in every temperature 2 position. A question of interest of course is

whether this continues to hold for those positions not yet checked, especially larger size positions.

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