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CONSECUTIVE PRIMES WHICH ARE WIDELY DIGITALLY DELICATE AND BRIER NUMBERS

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Abstract

Making use of covering systems and a theorem of D. Shiu, the first and second authors showed that for every positive integer k, there exist k consecutive widely digitally delicate primes. They also noted that for every positive integer k, there exist k consecutive primes which are Brier numbers. We show that for every positive integer k, there exist k consecutive primes that are both widely digitally delicate and Brier numbers.

1. Introduction

In 1958, R. M. Robinson [17] made tables of primes that were of the form $k \cdot 2^n + 1$ for odd integers $1 \le k < 100$ and $0 \le n \le 512$. The table found primes for all of these k except 47. These tables led to a result of W. Sierpiński [19] where he proved there are infinitely many odd positive integers k such that $k \cdot 2^n + 1$ is composite for every positive integer n. Such odd k are called Sierpiński numbers. Sierpiński's proof shows that in fact, for every nonnegative integer m, the number

$$k = 15511380746462593381 + 36893488147419103230\, m$$

is a Sierpiński number. Here, we have added the condition that k is odd in the definition of a Sierpiński number, as is common, though the original paper of Sierpiński

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did not have such a condition. This added condition arose in part due to the desire to find the smallest Sierpiński number as we explain below.

Since Sierpiński's work, there have been a number of attempts to find small explicit examples of Sierpiński numbers. J. Selfridge (see [9, Section B21]) found the smallest known Sierpiński number, $k = 78557$. Observe that if we were to remove the condition that k is odd from the definition of a Sierpiński number, then it is likely the number 65536 would be a smaller Sierpiński number. The point here is that, for a positive integer n, the number $65536 \cdot 2^n + 1 = 2^{n+16} + 1$ is a prime only if $n + 16$ is a power of 2. Thus, the number 65536 satisfies $65536 \cdot 2^n + 1$ is composite for all positive integers n if and only if $2^{16} + 1$ is the largest Fermat prime, which is a widely held belief based on heuristic arguments that seems very difficult to prove.

Following the work of Sierpiński [19], Selfridge used what is known as a covering system to prove 78557 is a Sierpiński number; covering systems will be discussed in detail later. While 78857 is the smallest known Sierpiński number, it has not been proven to be the smallest one. Eliminating k as a possible Sierpiński number can be done by finding a prime of the form $k \cdot 2^n + 1$ for some positive integer n, and an effort has been made to find such primes for all odd positive integers $k < 78557$. In March 2002, there were 17 values of k left to check that are smaller than 78557 and possible Sierpiński numbers. At that time, L. Heim and D. Norris started the Seventeen or Bust project which aimed to prove 78557 is the smallest Sierpiński number (see [15]). The project resulted in showing 11 of the 17 possible Sierpiński numbers less than 78557 are in fact not Sierpiński numbers. Then PrimeGrid took over and showed one more of the remaining possibilities less than 78557 is not a Sierpiński number in October 2016 leaving 5 more possible Sierpiński numbers less than 78557. PrimeGrid has continued to run and, as of February 2021, the remaining possible Sierpiński numbers less than 78557 are 21181, 22699, 24737, 55459, and 67607. If k is one of these five remaining numbers, then PrimeGrid has thus far verified that $k \cdot 2^n + 1$ is composite for all $n \leq 31875742$.

Around the same time that Sierpiński was working with his new class of numbers, Hans Riesel [16] showed there are infinitely many odd integers k such that $k \cdot 2^n - 1$ is composite for every positive integer n. Riesel showed that for every nonnegative integer m, the odd number $k = 509203 + 11184810\,m$ has this property. The odd positive integers k with this property are called Riesel numbers.

The number 509203 is conjectured to be the smallest Riesel number (see [9, Section B21]). The Reisel Sieve Project, analogous to Seventeen or Bust, started in August 2003 to prove the odd positive integers less than 509203 are not Riesel. The project, along with others like PrimeGrid, have reduced the number of possible Riesel numbers $k < 509203$ down to 42. As of January 2021, PrimeGrid has taken over and shown that these remaining 42 values of k are such that $k \cdot 2^n - 1$ is composite for all $n \leq 14447000$ (see [15]).

Obtaining Sierpiński and Riesel numbers is typically done by solving a system of linear congruences, where the solution is given in the form of an arithmetic progression. To find such congruences, one makes use of a covering system.

Definition 1 (Covering System)**.** For nonnegative integers a_i and positive integers b_i, the set

$$\{a_1 \pmod{b_1}, \quad a_2 \pmod{b_2}, \quad \ldots, \quad a_m \pmod{b_m}\}$$

is called a *covering system* (or simply a *covering*) if for every integer n, we have $n \equiv a_i \pmod{b_i}$ for at least one $i \in \{1, 2, \ldots, m\}$.

By choosing a_i and b_i as in the definition and choosing p_i prime so that $p_i | (2^{b_i} - 1)$ and the p_i's are distinct, solutions k to the m congruences

$$k \cdot 2^{a_1} + 1 \equiv 0 \pmod{p_1}, \quad \ldots, \quad k \cdot 2^{a_m} + 1 \equiv 0 \pmod{p_m}$$

are Sierpiński, at least for $k > \max_{1 \leq i \leq m}\{p_i\}$. Similarly, a covering system can be used to obtain Riesel numbers.

In this paper, we will use a covering system argument to obtain Brier numbers, that is, odd positive integers k that are both Sierpiński numbers and Riesel numbers. The existence of such numbers was first established by Brier (see [3]). Brier showed that

$$29364695660123543278115025405114452910889$$

is both a Sierpiński and Riesel number.

As was done to obtain Sierpiński and Riesel numbers, Brier found covering systems to determine an arithmetic progression of odd positive integers k for which $k \cdot 2^n + 1$ and $k \cdot 2^n - 1$ are both composite for every positive integer n [1998 unpublished]. In August 2009, A. Wesolowski (see [20, A076335]) showed that the smallest Brier number must be larger than 10^9. The smallest known Brier number is 3316923598096294713661 found by C. Clavier (see [20, A076335]) in December 2013. Clavier showed that any number in the arithmetic progression

$$3316923598096294713661 + 3770214739596601257962594704110\, m,$$

where m is a nonnegative integer, is a Brier number.

One result, which this paper will model its main result after, is the following found in [3].

Theorem 1. *For every positive integer k, there exist k consecutive primes all of which are Brier numbers.*

The theorem is a consequence of the arithmetic progression above. Since

$$3316923598096294713661 \quad \text{and} \quad 3770214739596601257962594704110$$

are relatively prime, a result of D. Shiu [18] implies Theorem 1. An extension of Shiu's theorem due to J. Maynard [12] will play an equivalent role in our main result in this paper. Shiu showed that in any arithmetic progression containing infinitely many primes, there are arbitrarily long sequences of consecutive primes [18]. For an arithmetic progression $Am + B$, where A, B and m are nonnegative integers, to contain infinitely many primes, we want $\gcd(A, B) = 1$ and $A > 0$. We see then that Theorem 1 follows from Clavier's arithmetic progression of Brier numbers and Shiu's theorem. The work of Shiu has been strengthened in the work of W. D. Banks, T. Freiberg and C. L. Turnage-Butterbaugh [1] and J. Maynard [12] (also, see T. Freiberg [7]). For example, Maynard extends Shiu's work in Theorem 3.3 of [12] by showing for every positive integer k, in any arithmetic progression $Am + B$, where $A > 0$, $B \geq 0$ and $m \geq 0$ are integers with A and B fixed and $\gcd(A, B) = 1$, a positive proportion of positive integers ℓ are such that $p_\ell, p_{\ell+1}, \ldots, p_{\ell+k-1}$ are all in the arithmetic progression $Am + B$, where p_j denotes the j^{th} prime.

The main result of this paper not only refers to Brier numbers but also refers to widely digitally delicate primes. First, we define a digitally delicate prime number.

Definition 2 (Digitally Delicate)**.** Let b be an integer greater than or equal to 2. A prime number is called *digitally delicate in base* b (or simply *digitally delicate* for $b = 10$) if changing any base b digit of the prime to another base b digit results in a composite number.

In 1979, P. Erdős [2] proved that there are infinitely many digitally delicate prime numbers. The first of these is 294001. Hence,

$$d94001, \quad 2d4001, \quad 29d001, \quad 294d01, \quad 2940d1, \quad \text{and} \quad 29400d$$

are composite or equal to 294001 for every $d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The proof of Erdős was done using a partial covering system and a sieve argument. In 2011, Tao [21] was able to refine the sieve part of the argument to show that a positive lower asymptotic density of primes are digitally delicate. Konyagin [11] proved a related result for composite numbers using a similar argument showing that, in particular, a positive lower asymptotic density of composite numbers which are coprime to 10 remain composite if any base 10 digit of the number is changed to another base 10 digit. These authors also establish analogous results for arbitrary bases.

In 2021, the first author and J. Southwick [6] considered a more restrictive class of digitally delicate primes in base 10 and showed a similar result to Tao's for this more restrictive class. These numbers also are required to be composite when changing any one of the infinitely many leading zeroes in base 10.

Definition 3 (Widely Digitally Delicate)**.** Let b be an integer greater than or equal to 2. A prime number is called *widely digitally delicate in base* b (or simply *widely* *digitally delicate* for $b = 10$) if changing any base b digit of the prime, including any of the infinitely many leading 0's, to another base b digit results in a composite number.

In the same paper [6], the first author and J. Southwick elaborate that the result is possible in some other bases, not just base 10. To see that the collection of widely digitally delicate primes is more refined than digitally delicate primes, consider the digitally delicate prime listed previously, 294001. The prime is not widely digitally delicate since 10294001 is prime. Furthermore, there are no widely digitally delicate primes in the first 10^9 positive integers [6]. Their result was mentioned in Quanta Magazine [13] along with the next result by the first and third author [3] (also, see [14]). We note that an explicit example of a widely digitally delicate prime has been given by J. Grantham [8]. The example has 4032 digits.

Theorem 2. *For every positive integer k, there exist k consecutive widely digitally delicate primes.*

In this paper, we use the methods from [3] to obtain a result similar to Theorem 2 where the primes are also Brier numbers.

Theorem 3 (Main Theorem)**.** *For every positive integer k, there exist k consecutive primes $p_\ell, p_{\ell+1}, \ldots, p_{\ell+k-1}$, each of which is both a widely digitally delicate prime and a Brier number. Furthermore, the first primes p_ℓ in consecutive lists of k such primes have positive density (depending on k) in the set of prime numbers. In particular, a positive proportion of the primes are both a widely digitally delicate prime and a Brier number.*

According to [6], the density of the digitally delicate primes up to 10^{10} is $0.000071031\ldots$, and Jon Grantham [8] indicates there are no widely digitally delicate primes up to 10^{11}. Given that Theorem 3 concerns itself further with widely digitally delicate primes that are also Brier numbers, it is reasonable to expect that the density alluded to in Theorem 3 is very small even with $k = 1$.

As a corollary to the proof of Theorem 3, we will also establish the following.

Corollary 1. *For every positive integer k, there exist k consecutive primes that are widely digitally delicate in both base 2 and base 10.*

As a simple consequence of an interesting result by D. A. Kaptan [10, Theorem 4] about small gaps between primes in an arithmetic progression, we can further deduce Theorem 4 below. We do not elaborate further on the details of the proof.

Theorem 4. *For every positive integer k, there is a constant $C > 0$ and infinitely many positive integers ℓ such that the k consecutive primes $p_\ell, p_{\ell+1}, \ldots, p_{\ell+k-1}$ are widely digitally delicate primes and Brier numbers which satisfy*

$$p_{\ell+k-1} - p_\ell < C.$$

2. Some Preliminaries and a Proof of Corollary 1

In the introduction, we discussed the proof of Theorem 1 based on the use of Shiu's theorem. This included finding an arithmetic progression $Am + B$ where $\gcd(A, B) = 1$ and every number in the arithmetic progression is a Brier number. The goal for establishing Theorem 3 is to construct an arithmetic progression $Am + B$, with fixed positive relatively prime integers A and B, with A divisible by 10 and having a prime divisor greater than 10, and with a variable nonnegative integer m, such that every integer k of the form $Am + B$ satisfies

- for every nonnegative integer n and $d \in \{-9, \ldots, -1, 1, \ldots, 9\}$, the number $k + d \cdot 10^n$ is divisible by at least one prime p dividing A,
- for every nonnegative integer n, the number $k \cdot 2^n + 1$ is divisible by at least one prime p dividing A, and
- for every nonnegative integer n, the number $k \cdot 2^n - 1$ is divisible by at least one prime p dividing A.

Provided we can also find such a progression that avoids the expressions $k + d \cdot 10^n$, $k \cdot 2^n + 1$ and $k \cdot 2^n - 1$ above from being equal to a prime divisor of A, then Theorem 3 will follow directly from Maynard's extension of Shiu's theorem mentioned earlier (i.e., Theorem 3.3 of [12]). Observe that for $m \geq 2$, we have $k \cdot 2^n + 1$ and $k \cdot 2^n - 1$ are both at least $(2A + B)2^n - 1 \geq 2A$, which is larger than any prime divisor of A. By replacing B then by $2A + B$, we obtain a new arithmetic progression with every element contained in the previous progression, so that the three bulleted items above are still satisfied. Thus, with B so changed, the expressions $k \cdot 2^n + 1$ and $k \cdot 2^n - 1$ cannot equal a prime divisor of A. The next lemma allows us to adjust A and B further so that the expressions $k + d \cdot 10^n$ in the first bullet above cannot equal a prime divisor of A.

Lemma 1. *Let b be an integer greater than or equal to 2. Let A and B be positive relatively prime integers such that A has a prime divisor greater than b and every prime dividing b divides A. Suppose further that for every nonnegative integers m and k and for*

$$d \in \{-(b-1), -(b-2), \ldots, -1\} \cup \{1, 2, \ldots, b-1\},$$

there is a prime p dividing A which also divides $Am + B + d \cdot b^k$. Then, a subprogression $A_0 m + B_0$ can be found, with A_0 and B_0 relatively prime and A dividing A_0, such that for every nonnegative integers m and k and for d as above, there is a prime p dividing A_0 with

$$p \mid \left(A_0 m + B_0 + d \cdot b^k\right) \quad \text{and} \quad A_0 m + B_0 + d \cdot b^k \neq \pm p.$$

Proof. Let A' be the largest positive integer dividing A that is relatively prime to b. Then there is a positive integer u such that A divides $b^u A'$. Furthermore, taking $v = \phi(A')$ and a positive integer $\ell \geq u$, we see that A divides $b^{\ell(v+1)} - b^{\ell}$. We take $\ell \geq \max\{u, 2\}$ sufficiently large so that also $b^{\ell} > A + B$, and set

$$A_0 = b^{\ell(v+2)}A \qquad \text{and} \qquad B_0 = b^{\ell(v+1)} - b^{\ell} + B.$$

Note that $\ell \geq 2$ ensures that $\ell(v+1)-1 \geq 2\ell-1 \geq \ell+1$, which we use momentarily. Since A divides $b^{\ell(v+1)} - b^{\ell}$, we see that for every nonnegative integer m, there is a nonnegative integer m' such that $A_0m + B_0 = Am' + B$, so the arithmetic progression $A_0m + B_0$ is a subprogression of the arithmetic progression $Am + B$. By the properties of the progression $Am+B$, we deduce that for every nonnegative integers m and k and for $d \in \{-(b-1), -(b-2), \ldots, -1\} \cup \{1, 2, \ldots, b-1\}$, there is a prime p dividing A, and hence A_0, with $p \mid \big(A_0m + B_0 + d \cdot b^k\big)$. Furthermore, if $k \leq \ell(v+1) - 1$, we see that

$$\begin{aligned} A_0m + B_0 + d \cdot b^k &= b^{\ell(v+2)}Am + b^{\ell(v+1)} - b^{\ell} + B + d \cdot b^k \\ &\geq b^{\ell(v+1)} - b^{\ell} + B - (b-1)b^{\ell(v+1)-1} \\ &= b^{\ell(v+1)-1} - b^{\ell} + B \geq b^{\ell+1} - b^{\ell} + B \\ &> b^{\ell} > A \end{aligned}$$

for every nonnegative integer m. Also, if $k \geq \ell(v+1)$, then $k \geq 2\ell$ so that

$$A_0m + B_0 + d \cdot b^k \equiv -b^{\ell} + B \pmod{b^{2\ell}}.$$

Since $b^{\ell} > A + B$, we see that

$$-b^{2\ell} + A < b^{\ell} - b^{2\ell} < -b^{\ell} < -b^{\ell} + B < -A.$$

In this case, $A_0m + B_0 + d \cdot b^k$ is in a residue class modulo $b^{2\ell}$ represented by an integer in $(A, b^{2\ell} - A)$. In both cases, that is $k \leq \ell(v+1) - 1$ and $k \geq \ell(v+1)$, we deduce that since $p \leq A$, we have $A_0m + B_0 + d \cdot b^k \neq \pm p$. □

As noted before Lemma 1, we will find an arithmetic progression $Am + B$, with $10 \mid A$ and A divisible by a prime greater than 10, satisfying the bullets above. The comments before Lemma 1 together with Lemma 1 with $b = 10$ allow us to find a subprogression of $Am + B$ such that every prime k in the subprogression is both widely digitally delicate and a Brier number. Maynard's extension of Shiu's theorem will then imply Theorem 3.

We are now ready to prove Corollary 1.

Proof of Corollary 1 (assuming the existence of $Am + B$ as above). To show that a prime k is widely digitally delicate in base 2, it suffices to show that $k \pm 2^n$ is composite for all nonnegative integers n. Let $k = Am + B$, with A and B as above and

m a nonnegative integer. Fix a nonnegative integer n. Let

$$T = n \prod_{p|A} (p-1).$$

Then by the second bullet above, there is a prime p dividing A such that

$$(Am + B) \cdot 2^{T-n} + 1 \equiv 0 \pmod{p}.$$

Note that this congruence implies p is odd. Since $2^T \equiv 1 \pmod{p}$, we deduce

$$(Am + B) \cdot 2^{-n} + 1 \equiv 0 \pmod{p}.$$

Multiplying both sides of the congruence by 2^n gives

$$k + 2^n \equiv 0 \pmod{p}.$$

Thus, for each nonnegative integer n, the number $k + 2^n$ is divisible by a prime dividing A. Similarly, for each nonnegative integer n, we can see that the third bullet above implies that the number $k - 2^n$ is divisible by a prime dividing A. By applying Lemma 1 first with $b = 10$ and then with $b = 2$, we see that there is a subprogression of $Am + B$ containing infinitely many primes such that every prime in the subprogression is widely digitally delicate in both base 10 and base 2. Shiu's theorem applies as before to complete the proof. □

3. The Coverings

As explained in the previous section, we are interested in finding an arithmetic progression $Am + B$, with $\gcd(A, B) = 1$ and with A divisible by 10 and some prime greater than 10, satisfying the bulleted items at the beginning of the previous section. By replacing A with a power of A, we may suppose that $B < A$ and do so. We will refer to properties (i), (ii) and (iii) analogous to the bullets of the previous section as follows.

(i) If $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$, then each number in the set

$$\mathcal{A}_d = \mathcal{A}_d(A, B) = \left\{ Am + B + d \cdot 10^n : m \in \mathbb{Z}^+ \cup \{0\},\ n \in \mathbb{Z}^+ \cup \{0\} \right\}$$

is composite.

(ii) Each number in the set

$$\mathcal{B}_S = \mathcal{B}_S(A, B) = \{(Am + B) \cdot 2^n + 1 : m \in \mathbb{Z}^+ \cup \{0\},\ n \in \mathbb{Z}^+ \cup \{0\}\}$$

is composite.

(iii) Each number in the set

$$\mathcal{B}_R = \mathcal{B}_R(A, B) = \{(Am + B) \cdot 2^n - 1 : m \in \mathbb{Z}^+ \cup \{0\},\ n \in \mathbb{Z}^+ \cup \{0\}\}$$

is composite.

In the above, a negative integer is composite if its absolute value is composite.

The statement (i) is exactly the condition we want for each prime in the progression $Am + B$ to be widely digitally delicate. Similarly, (ii) and (iii), imply that each prime in the progression $Am + B$ is a Sierpiński number and Riesel number, respectively.

Note that we want relatively prime A and B satisfying (i), (ii), and (iii). However, we go about this indirectly by finding relatively prime A_1 and B_1 so that each number in $\mathcal{A}_d(A_1, B_1)$ is composite, by finding relatively prime A_2 and B_2 so that each number in $\mathcal{B}_S(A_2, B_2)$ is composite, and by finding relatively prime A_3 and B_3 so that each number in $\mathcal{B}_R(A_3, B_3)$ is composite. Thus, for example, every positive integer that is B_1 modulo A_1 is such that if we add $d \cdot 10^n$ to the integer, where $n \in \mathbb{Z}^+ \cup \{0\}$ and $d \in \{-9, -8, \ldots, -1\} \cup \{1, 2, \ldots, 9\}$, the resulting number is composite.

Let

$$\mathcal{P}(A) = \{p : p \text{ is prime and } p|A\}.$$

We will take each of A_1, A_2, and A_3 to be a product of distinct primes. In other words, each A_j is squarefree. The A_j's and B_j's will have the properties

$$\begin{gathered} \mathcal{P}(A_1) \cap \mathcal{P}(A_2) = \emptyset, \quad \mathcal{P}(A_1) \cap \mathcal{P}(A_3) = \emptyset, \quad \mathcal{P}(A_2) \cap \mathcal{P}(A_3) = \{2, 5\}, \\ B_2 \equiv B_3 \pmod{10}. \end{gathered} \tag{1}$$

By applying the Chinese Remainder Theorem, we can then establish (i), (ii), and (iii) for some relatively prime A and B by taking $A = A_1 A_2 A_3 / 10$ and $B \in [0, A-1]$ so that

$$\begin{gathered} B \equiv B_1 \pmod{A_1}, \quad B \equiv B_2 \pmod{A_2/10}, \quad B \equiv B_3 \pmod{A_3/10}, \\ B \equiv B_2 \equiv B_3 \pmod{10}. \end{gathered} \tag{2}$$

The first and third authors [3] constructed A_1 squarefree and B_1 so that (i) holds with $\mathcal{A}_d(A, B)$ replaced by $\mathcal{A}_d(A_1, B_1)$. Thus, this paper will focus on constructing the pair (A_2, B_2) as well as the pair (A_3, B_3) such that (1) holds and (ii) and (iii) hold with $\mathcal{B}_S(A, B)$ and $\mathcal{B}_R(A, B)$ replaced by $\mathcal{B}_S(A_2, B_2)$ and $\mathcal{B}_R(A_3, B_3)$, respectively.

Since the construction of the pairs (A_2, B_2) and (A_3, B_3) is similar, we will discuss the construction of the pair (A_2, B_2) and then explain how this translates to constructing the pair (A_3, B_3). To show that $(A_2 m + B_2) \cdot 2^n + 1$ is composite for all nonnegative integers n, we will show that for each nonnegative integer n, there is a prime, $p \in \mathcal{P}(A_2)$ such that p divides $(A_2m + B_2) \cdot 2^n + 1$. We will choose A_2 and B_2 large enough so that each number of the form $(A_2m + B_2) \cdot 2^n + 1$, with m and n in $\mathbb{Z}^+ \cup \{0\}$, is greater than each prime in $\mathcal{P}(A_2)$. Thus, every number of the form $(A_2m + B_2) \cdot 2^n + 1$ will be composite.

For a prime $p \in \mathcal{P}(A_2)$, observe that $(A_2m + B_2) \cdot 2^n + 1$ is divisible by p if and only if $B_2 \cdot 2^n + 1$ is divisible by p. Initially, we do not know the values of A_2 and B_2; we want to construct them. The idea then is to find a finite set $\mathcal{P}_2$ of primes so that for some positive integer B_2 and all nonnegative integers n, the number $B_2 \cdot 2^n + 1$ is divisible by one of the primes in $\mathcal{P}_2$. Then A_2 will be determined by taking A_2 to be the product of the primes in $\mathcal{P}_2$ so that $\mathcal{P}(A_2) = \mathcal{P}_2$. We want to construct B_2 as above in such a way that $\gcd(A_2, B_2) = 1$. The focus now is on finding the set $\mathcal{P}_2$ and how to construct B_2 from this set.

For an odd prime p and an arbitrary integer a, we can determine $B_2 \equiv -2^{-a} \pmod{p}$ so that

$$B_2 \cdot 2^n + 1 \equiv 0 \pmod{p},$$

when $n \equiv a \pmod{b}$ with $b = \mathrm{ord}_p(2)$. The idea now is to determine primes p (our set $\mathcal{P}_2$) so that the orders of 2 modulo these primes and appropriate choices for a as above provide us with a list of congruence classes $n \equiv a \pmod{b}$ that form a covering system for the integers. In this way, for every nonnegative integer n, there will be a prime p such that p divides $B_2 \cdot 2^n + 1$, where p depends on a congruence class satisfied by n.

We now use an example to illustrate how a congruence class in the covering system gives a congruence class for B_2 to satisfy. We take $p = 5$. The order of 2 modulo 5 is 4, so the modulus for the congruence on n will be 4. Suppose we want the congruence $n \equiv 0 \pmod{4}$ in the covering system for n. If we let $B_2 \equiv -2^{-0} \equiv 4 \pmod{5}$, then for some integer t, we have

$$B_2 \cdot 2^n + 1 = B_2 \cdot 2^{4t} + 1 \equiv 4 \cdot 1 + 1 \equiv 0 \pmod{5}.$$

Thus, whenever $n \equiv 0 \pmod{4}$, the number $B_2 \cdot 2^n + 1$ is divisible by 5. Then $n \equiv 0 \pmod{4}$ is part of the covering system we want to help establish (ii), where we want

$$A_2 \equiv 0 \pmod{5} \quad \text{and} \quad B_2 \equiv 4 \pmod{5}.$$

In order for (1) to hold, observe that we need $5 \notin \mathcal{P}(A_1)$. The value of A_1 is given in [3], and throughout this paper, we avoid using primes in $\mathcal{P}(A_1)$. It is the case that $5 \notin \mathcal{P}(A_1)$, so using 5 as above is permissible. We note that the primes 2 and 5 were not in $\mathcal{P}(A_1)$ because, in [3], the authors were interested in choosing moduli for the coverings based on the order of 10 modulo the primes dividing A_1. This led them to avoiding the primes 2 and 5 for which no such order exists. Similarly in this paper, we will want 2 to have an order modulo each of the primes dividing A_2 or A_3, and thus we will seemingly want to avoid the prime 2. However, we have already indicated that we want $A_j m + B_j$ to be odd for $j \in \{2, 3\}$, so we are taking 2 to be in both $\mathcal{P}(A_2)$ and $\mathcal{P}(A_3)$ with the added conditions that B_2 and B_3 are 1 modulo 2.

Now, suppose we want to show that $(A_2m + B_2) \cdot 2^n + 1$ is composite as in (ii) whenever $n \equiv 2 \pmod 4$. Since 5 is the only prime p with 2 of order 4 modulo p, the idea is to work modulo 8 instead and show that $(A_2m+B_2)\cdot 2^n+1$ is composite when $n \equiv 2 \pmod 8$ and when $n \equiv 6 \pmod 8$. Each of these will require a number of congruence classes, particularly since we want to avoid primes in $\mathcal{P}(A_1)$ and the only prime p with 2 of order 8 modulo p is $p = 17 \in \mathcal{P}(A_1)$. For the discussion now, we consider the case of showing $(A_2m + B_2) \cdot 2^n + 1$ is composite when $n \equiv 2 \pmod 8$.

By the above arguments, we want to find primes so that if B_2 satisfies certain congruence classes, then whenever $n \equiv 2 \pmod 8$, one of these primes divides $B_2 \cdot 2^n + 1$. The primes we found are given in the second column in Table 1. Momentarily, we will describe how we determined the primes more clearly, but if $n \equiv a \pmod b$ and p are the columns in a row of Table 1, then the order of 2 modulo p is b. To see that every integer $n \equiv 2 \pmod 8$ satisfies a congruence class in the first column, observe that if $n \equiv 128 \pmod{144}$, then n satisfies one of the last 3 congruence classes in the first column of Table 1. With the prior two congruence classes modulo 144, every $n \equiv 42 \pmod{48}$ will satisfy one of the last 5 congruence classes in the first column. If $n \equiv 74 \pmod{96}$, then it satisfies one of the congruence classes in rows 4-6 in the first column. Thus, if $n \equiv 26 \pmod{48}$, then it satisfies one of the congruence classes in rows 3-6 in the first column. Now, if $n \equiv 10 \pmod{16}$, then it satisfies one of the congruence classes in rows 2-10 in the first column. Since integers which are 2 modulo 16 satisfy the first congruence class in the first column, we see that every integer $n \equiv 2 \pmod 8$ satisfies at least one congruence class in the first column of Table 1.

Congruence class	prime
$n \equiv 2 \pmod{16}$	257
$n \equiv 10 \pmod{48}$	673
$n \equiv 26 \pmod{96}$	22253377
$n \equiv 74 \pmod{288}$	1153
$n \equiv 170 \pmod{288}$	6337
$n \equiv 266 \pmod{288}$	38941695937
$n \equiv 42 \pmod{144}$	577
$n \equiv 90 \pmod{144}$	487824887233
$n \equiv 138 \pmod{432}$	4261383649
$n \equiv 282 \pmod{432}$	209924353
$n \equiv 426 \pmod{432}$	24929060818265360451708193

Table 1: Congruence classes used to satisfy $n \equiv 2 \pmod 8$

So we will choose A_2 so that it is divisible by each prime in the second column of Table 1. Corresponding to each congruence $n \equiv a \pmod{b}$ and prime p in a row, we take $B_2 \equiv -2^{-a} \pmod{p}$ as noted earlier. Then whenever $n \equiv 2 \pmod{8}$ and $m \in \mathbb{Z}$, we have $(A_2 m + B_2) \cdot 2^n + 1$ is divisible by a prime in the second column of Table 1.

Next, we describe more clearly where our choice of primes came from. The idea in the example above is to find a system $\mathcal{C}$ of congruence classes such that every $n \equiv 2 \pmod{8}$ satisfies a congruence $n \equiv a \pmod{b}$ in $\mathcal{C}$. Each such congruence class will correspond to a prime $p \notin \mathcal{P}(A_1)$ for which 2 has order b modulo p. To find the primes p for which 2 has order b modulo p, where b is a modulus we want to use for $\mathcal{C}$, we look at the prime divisors of $\Phi_b(2)$, where $\Phi_b(x)$ is the b^{th} cyclotomic polynomial. As noted in [3] and [5] (with 2 replaced by 10), each prime divisor p of $\Phi_b(2)$ either will be the largest prime divisor of b or will satisfy that 2 has order b modulo p. Thus, looking at prime divisors of $\Phi_b(2)$, which are not in $\mathcal{P}(A_1)$, determines whether we can use the modulus b for $\mathcal{C}$ and how many times we can use b as a modulus. Thus, in general, we found what moduli were possible for our systems of congruence classes by looking at the number of distinct prime divisors of $\Phi_b(2)$, not in $\mathcal{P}(A_1)$, for different choices of b, and then we determined our covering systems based on this information.

Observe in Table 1 that we could have interchanged the primes with 2 of a given order b. For example, for the congruences $n \equiv 74 \pmod{288}$, $n \equiv 170 \pmod{288}$, and $n \equiv 266 \pmod{288}$, we could have selected the primes in the right-most column as 38941695937, 1153, and 6337, respectively.

Tables 3 and 4 describe the results of looking at prime factors of $\Phi_b(2)$ to determine the moduli b we can use for our covering systems and the number of times we can use each modulus (that is, the number of distinct prime factors of $\Phi_b(2)$ that do not divide b and are not in $\mathcal{P}(A_1)$). Table 3 indicates the number of distinct prime factors of $\Phi_b(2)$ that do not divide b and are not in $\mathcal{P}(A_1)$ but are in $\mathcal{P}(A_2) \cup \mathcal{P}(A_3)$. Table 4 shows the remaining number of distinct prime factors of $\Phi_b(2)$ which we know exist and do not divide b and are not in $\mathcal{P}(A_1)$ for each modulus. If a modulus b appears in Table 3 but not in Table 4, then all the distinct prime factors of $\Phi_b(2)$ that do not divide b and are not in $\mathcal{P}(A_1)$ have been used in $\mathcal{P}(A_2) \cup \mathcal{P}(A_3)$.

The above describes how we obtained a covering system for determining A_2 and B_2 so that (ii) holds. We obtain a covering system for determining A_3 and B_3 similarly so that (iii) holds. In this case, each congruence $n \equiv a \pmod{b}$ corresponds to an odd prime p dividing A_3, with b equal to the order of 2 modulo p, and we want

$$B_3 \equiv 2^{-a} \pmod{p} \tag{3}$$

so that

$$B_3 \cdot 2^n - 1 \equiv 0 \pmod{p}$$

for $n \equiv a \pmod{b}$.

For $p = 5$ above, we took $B_2 \equiv 4 \pmod 5$. Also, B_2 is odd, so we have $B_2 \equiv 9 \pmod{10}$. For (1) to hold, we therefore want $B_3 \equiv 9 \pmod{10}$. In particular, $B_3 \equiv 4 \pmod 5$. From (3), with $p = 5$, we want $a = 2$. As the order of 2 modulo 5 is 4, the congruence we want associated with $p = 5$ and (iii) is $n \equiv 2 \pmod 4$.

Each prime counted in Table 3 in the last column, besides $p = 5$ with $b = 4$, can be used to provide a congruence class for the covering system corresponding to either (ii) or (iii) and not both, due to the restriction made in (1) that $\mathcal{P}(A_2) \cap \mathcal{P}(A_3) = \{2, 5\}$. The complete covering systems used for determining A_j and B_j, for $j \in \{2, 3\}$, can be found in the appendix. In the next section, we explain how we verified that the congruence classes we tabulated for the covering systems in fact form covering systems.

4. Verifying the Covering Systems

Consider the collection of congruence classes

$$\mathcal{C}_0 = \{0 \pmod 3,\ 1 \pmod 3,\ 2 \pmod 9,\ 5 \pmod 9,\ 8 \pmod 9\}.$$

We can verify $\mathcal{C}_0$ is a covering system as follows. Every nonnegative integer is either 0, 1 or 2 $\pmod 3$. The congruence classes 0 (mod 3) and 1 (mod 3) are in $\mathcal{C}_0$, so the case when an integer is 2 (mod 3) is left to satisfy. Every integer that is 2 (mod 3) is either 2 (mod 9), 5 (mod 9), or 8 (mod 9). These congruence classes modulo 9 are in $\mathcal{C}_0$. Thus, $\mathcal{C}_0$ is a covering system.

Due to the complexity of the coverings in the previous section, this method for verifying in general a

$$\mathcal{C} = \{a_1 \pmod{b_1}, \quad a_2 \pmod{b_2}, \quad \dots, \quad a_m \pmod{b_m}\}$$

is a covering is quite time consuming. An alternate way of verifying $\mathcal{C}$ is a covering is to check every integer in $[0, \ell - 1]$, where $\ell = \operatorname{lcm}(b_i)$. To see this, suppose that every integer in $[0, \ell - 1]$ is in at least one congruence class in $\mathcal{C}$. We want to show that all integers k are in at least one congruence class in $\mathcal{C}$. We can rewrite k as

$$k = q \cdot \ell + r$$

where q and r are integers with $0 \le r \le \ell - 1$. Since $r \in [0, \ell - 1]$, we have that r is in at least one congruence class, $a \pmod b$, in $\mathcal{C}$. As a consequence of b dividing ℓ, we deduce

$$k \equiv r \equiv a \pmod b.$$

Thus, k is in the congruence class $a \pmod b$ in $\mathcal{C}$, as we wanted.

In the example above with $\mathcal{C}_0$, this process is emulated by checking that every $k \in [0, 8]$ satisfies a congruence class in $\mathcal{C}_0$. Table 2 confirms that $\mathcal{C}_0$ is a covering by listing a congruence class each k satisfies in $\mathcal{C}_0$ in the second column.

k	Congruence Classes in $\mathcal{C}_0$
0	$0 \pmod 3$
1	$1 \pmod 3$
2	$2 \pmod 9$
3	$0 \pmod 3$
4	$1 \pmod 3$
5	$5 \pmod 9$
6	$0 \pmod 3$
7	$1 \pmod 3$
8	$8 \pmod 9$

Table 2: Verifying the Covering $\mathcal{C}_0$

While this method works and is easily implemented by a computer, if $\ell = \operatorname{lcm}(b_i)$ is too large, this process takes a substantial amount of time. In the coverings used in this paper for Sierpiński and Riesel numbers, the least common multiples are 236107872000 and 922078080000, respectively. Also, the number of congruence classes in the coverings are 447 and 459, respectively. Thus, an alternative verification method, as seen in the paper [3], was used.

Let $\mathcal{C}$ be a set of congruence classes $a_i \pmod{b_i}$. Let w be a positive integer and u be an integer in $[0, w-1]$. Let $\mathcal{C}_u$ be the congruence classes $a_i \pmod{b_i}$ in $\mathcal{C}$ such that

$$a_i \equiv u \pmod{\gcd(b_i, w)}.$$

If $|\mathcal{C}_u| = 0$, then $\mathcal{C}$ is not a covering. Now consider the case that $|\mathcal{C}_u| > 0$. Let ℓ' be the lcm of the moduli in $\mathcal{C}_u$, and set $d = \gcd(w, \ell')$. In [3], the authors show the following.

Lemma 2. *With the above notation, if for each* $u \in [0, w-1]$*, every*

$$k = wt + u, \qquad \text{with } t \in [0, (\ell'/d) - 1] \cap \mathbb{Z},$$

satisfies a congruence class in $\mathcal{C}_u$*, then* $\mathcal{C}$ *is a covering system.*

Let $\mathcal{C}$ be one of the systems of congruence classes created in the previous section, that is for constructing arithmetic progressions for either Sierpiński or Riesel numbers. We take $w = 4 \cdot 3 \cdot 5 \cdot q$ where q is the largest prime dividing the least common multiple of the moduli in $\mathcal{C}$ as done in [3]. Applying Lemma 2 with this choice of w allowed us to easily verify our congruence classes form a covering system.

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Appendix

To aid in verifying the computations in this paper, all the data in this appendix can be found in [4] as lists suitable for computations. This appendix begins with Table 3, which lists the number of distinct primes used, $L = L(b)$, that divide $\Phi_b(2)$, do not divide b, and are not in $\mathcal{P}(A_1)$. Note the * indicates the prime 5 is used once in each covering. The b listed correspond to moduli used in our coverings. Table 4 gives a lower bound on the number of distinct primes, $M = M(b)$, that divide $\Phi_b(2)$, do not divide b, are not in $\mathcal{P}(A_1)$, and are not used in either covering. The * in this table represents an $M(b)$ that is a lower bound; that is, we did not completely factor $\Phi_b(2)$.

Tables 5 and 6 list the congruence classes $n \equiv a \pmod{b}$ in the first column that form the covering systems we obtained for $k\cdot 2^n+1$ and $k\cdot 2^n-1$ respectively. Recall that, with the order b of 2 fixed, the choice of a prime p associated with a given congruence class does not matter. We simply put a number in the second column to refer to a prime for which the order of 2 modulo that prime is b, with different numbers referring to different primes. Thus, for example, $\Phi_{64}(2)$ has exactly two prime factors, 641 and 6700417, neither of which are in $\mathcal{P}(A_1)$. These two primes are in the count for $L(64)$ in Table 3. We associate some ordering of these two primes. The second column of Table 5 indicates that one of these primes is associated with the congruence $n \equiv 22 \pmod{64}$ (indicated by "1" in the second column) and the other is associated with the congruence $n \equiv 54 \pmod{64}$ (indicated by "2" in the second column). Note that we do not attempt to clarify which of the two primes is associated with which congruence as it does not matter.

b	L
4	1*
13	1
16	1
25	2
26	1
27	1
32	1
33	1
35	1
39	1
40	1
48	1
50	1
55	1
64	2
65	1
66	1
70	1
75	2
77	1
78	1
80	1
81	3
88	1
91	2
96	1
99	2
100	2
104	2
105	3
108	1
110	2
112	2
117	4
120	1
121	1
125	2
130	2
132	2
135	2
140	2
143	3
144	2
150	1
154	2
156	1
160	2
162	1
165	1
169	3
175	3
176	3
182	3
195	1
198	2
200	4
208	1
216	2
220	2
224	2
225	3
231	1
234	1
240	2
242	2
250	2
260	3
264	2
270	3
273	3
275	2
280	1
286	3
288	3
297	2
300	3
308	3
312	1
324	4
325	2
330	2
336	3
338	2
350	2
351	5
352	2
360	2
363	4
364	2
375	1
378	4
385	3
390	2
396	4
416	5
420	2
429	3
432	3
440	4
448	2
450	2
455	2
462	1
468	5
480	2
484	4
486	3
495	2
507	1
520	5
528	2
540	4
546	2
550	1
560	4
572	6
585	4
594	4
616	3
624	3
625	4
637	7
648	7
650	5
660	5
672	2
676	5
693	3
700	3
702	2
704	4
715	3
720	2
726	3
728	2
750	3
756	2
770	3
780	5
784	5
792	2
810	6
825	4
832	3
840	3
845	3
858	2
864	2
880	4
896	2
910	4
924	2
936	4
960	4
968	4
975	4
980	5
1014	3
1040	4
1053	3
1056	4
1078	3
1080	3
1089	4
1092	5
1100	7
1120	4
1125	3
1134	3
1155	2
1170	5
1183	3
1188	6
1232	2
1248	3
1250	2
1260	3
1274	3
1300	7
1320	4
1344	4
1350	3
1352	4
1365	4
1375	3
1386	1
1400	2
1404	7
1408	3
1430	2
1440	4
1452	2
1456	4
1485	3
1512	1
1540	4
1560	3
1568	2
1584	2
1620	2
1625	5
1638	3
1664	2
1680	4
1690	2
1716	8
1728	4
1755	6
1792	4
1848	5
1872	4
1875	5
1890	3
1960	4
1980	5
2016	3
2028	4
2080	3
2100	3
2106	3
2112	4
2156	5
2160	5
2178	2
2184	6
2200	2
2250	6
2268	9
2275	4
2310	2
2340	3
2366	2
2464	4
2496	4
2520	4
2548	5
2574	4
2600	4
2640	2
2704	4
2730	3
2750	2
2772	8
2808	4
2816	4
2912	4
2925	5
3024	2
3080	3
3120	3
3136	4
3234	2
3276	3
3300	3
3360	5
3432	4
3510	9
3640	4
3696	3
3744	3
3780	2
3822	3
3960	2
4095	4
4160	4
4224	2
4290	4
4312	2
4320	2
4368	3
4550	2
4732	4
5040	3
5096	2
5148	3
5200	2
5280	4
5544	2
5632	3
5824	4
5850	4
6006	4
6160	2
6240	4
6600	4
6825	3
7040	4
7392	2
7800	3
8190	2
8316	3
8580	2
8775	3
9100	4
9240	4
10080	2
10296	2
11264	2
11700	2
12012	4
13650	3
18018	3

Table 3: Number of primes used in both coverings, $L = L(b)$

b	M
225	1
288	1
300	1
350	2
637	1
960	1
968	1*
1080	1
1120	1
1125	1*
1155	1
1232	1
1250	2
1260	1
1350	2*
1352	1
1430	3*
1452	3*
1485	1*
1568	1
1584	2
1620	6
1625	2*
1664	1*
1690	2*
1792	1*
1875	1*
1960	3*
2016	1*
2028	2*
2112	1*

b	M
2200	3*
2310	2*
2340	4*
2704	1*
2730	1*
2750	4*
3024	1*
3120	1*
3136	1*
3234	1*
3276	1*
3744	1*
3780	2*
3960	2*
4312	1
4320	4*
4368	1*
4732	2*
5096	1*
5200	3*
6240	2*
6825	1*
7392	2*
7800	3*
8316	1*
8580	3*
10296	1*
11264	1*
11700	4*
12012	1*

Table 4: Number of primes, $M = M(b)$, not used in both coverings

congruence	p
$n \equiv 0 \pmod{4}$	1
$n \equiv 2 \pmod{16}$	1
$n \equiv 6 \pmod{32}$	1
$n \equiv 22 \pmod{64}$	1
$n \equiv 54 \pmod{64}$	2
$n \equiv 10 \pmod{48}$	1
$n \equiv 26 \pmod{96}$	1
$n \equiv 74 \pmod{288}$	1
$n \equiv 170 \pmod{288}$	2
$n \equiv 266 \pmod{288}$	3
$n \equiv 42 \pmod{144}$	1
$n \equiv 90 \pmod{144}$	2
$n \equiv 138 \pmod{432}$	1
$n \equiv 282 \pmod{432}$	2
$n \equiv 426 \pmod{432}$	3
$n \equiv 14 \pmod{112}$	1
$n \equiv 30 \pmod{112}$	2
$n \equiv 46 \pmod{224}$	1
$n \equiv 158 \pmod{224}$	2
$n \equiv 62 \pmod{336}$	1
$n \equiv 174 \pmod{336}$	2
$n \equiv 286 \pmod{336}$	3
$n \equiv 78 \pmod{448}$	1
$n \equiv 190 \pmod{448}$	2
$n \equiv 302 \pmod{896}$	1
$n \equiv 750 \pmod{896}$	2
$n \equiv 414 \pmod{1792}$	1
$n \equiv 862 \pmod{1792}$	2
$n \equiv 1310 \pmod{1792}$	3
$n \equiv 1758 \pmod{1792}$	4
$n \equiv 94 \pmod{672}$	1
$n \equiv 206 \pmod{672}$	2
$n \equiv 318 \pmod{1344}$	1
$n \equiv 990 \pmod{1344}$	2
$n \equiv 430 \pmod{1344}$	3
$n \equiv 1102 \pmod{1344}$	4
$n \equiv 542 \pmod{2016}$	1
$n \equiv 1214 \pmod{2016}$	2
$n \equiv 1886 \pmod{2016}$	3
$n \equiv 654 \pmod{3360}$	1
$n \equiv 1326 \pmod{3360}$	2
$n \equiv 1998 \pmod{3360}$	3
$n \equiv 2670 \pmod{3360}$	4
$n \equiv 3342 \pmod{3360}$	5
$n \equiv 110 \pmod{784}$	1
$n \equiv 222 \pmod{784}$	2
$n \equiv 334 \pmod{784}$	3
$n \equiv 446 \pmod{784}$	4
$n \equiv 558 \pmod{784}$	5
$n \equiv 670 \pmod{1568}$	1
$n \equiv 1454 \pmod{1568}$	2

congruence	p
$n \equiv 782 \pmod{3136}$	1
$n \equiv 1566 \pmod{3136}$	2
$n \equiv 2350 \pmod{3136}$	3
$n \equiv 3134 \pmod{3136}$	4
$n \equiv 0 \pmod{13}$	1
$n \equiv 1 \pmod{26}$	1
$n \equiv 2 \pmod{39}$	1
$n \equiv 15 \pmod{117}$	1
$n \equiv 54 \pmod{117}$	2
$n \equiv 93 \pmod{117}$	3
$n \equiv 28 \pmod{117}$	4
$n \equiv 67 \pmod{351}$	1
$n \equiv 184 \pmod{351}$	2
$n \equiv 301 \pmod{351}$	3
$n \equiv 106 \pmod{351}$	4
$n \equiv 223 \pmod{351}$	5
$n \equiv 340 \pmod{1053}$	1
$n \equiv 691 \pmod{1053}$	2
$n \equiv 1042 \pmod{1053}$	3
$n \equiv 3 \pmod{156}$	1
$n \equiv 55 \pmod{468}$	1
$n \equiv 211 \pmod{468}$	2
$n \equiv 367 \pmod{468}$	3
$n \equiv 107 \pmod{468}$	4
$n \equiv 263 \pmod{468}$	5
$n \equiv 419 \pmod{1404}$	1
$n \equiv 887 \pmod{1404}$	2
$n \equiv 1355 \pmod{1404}$	3
$n \equiv 29 \pmod{260}$	1
$n \equiv 81 \pmod{260}$	2
$n \equiv 133 \pmod{260}$	3
$n \equiv 185 \pmod{780}$	1
$n \equiv 445 \pmod{780}$	2
$n \equiv 705 \pmod{780}$	3
$n \equiv 237 \pmod{780}$	4
$n \equiv 497 \pmod{780}$	5
$n \equiv 757 \pmod{2340}$	1
$n \equiv 1537 \pmod{2340}$	2
$n \equiv 2317 \pmod{2340}$	3
$n \equiv 17 \pmod{104}$	1
$n \equiv 43 \pmod{104}$	2
$n \equiv 69 \pmod{312}$	1
$n \equiv 173 \pmod{936}$	1
$n \equiv 485 \pmod{936}$	2
$n \equiv 797 \pmod{936}$	3
$n \equiv 277 \pmod{936}$	4
$n \equiv 589 \pmod{1872}$	1
$n \equiv 1525 \pmod{1872}$	2
$n \equiv 901 \pmod{1872}$	3
$n \equiv 1837 \pmod{1872}$	4
$n \equiv 95 \pmod{520}$	1

congruence	p
$n \equiv 199 \pmod{520}$	2
$n \equiv 303 \pmod{520}$	3
$n \equiv 407 \pmod{520}$	4
$n \equiv 511 \pmod{520}$	5
$n \equiv 5 \pmod{208}$	1
$n \equiv 31 \pmod{416}$	1
$n \equiv 239 \pmod{416}$	2
$n \equiv 57 \pmod{416}$	3
$n \equiv 265 \pmod{416}$	4
$n \equiv 83 \pmod{416}$	5
$n \equiv 291 \pmod{2080}$	1
$n \equiv 707 \pmod{2080}$	2
$n \equiv 1123 \pmod{2080}$	3
$n \equiv 1539 \pmod{4160}$	1
$n \equiv 3619 \pmod{4160}$	2
$n \equiv 1955 \pmod{4160}$	3
$n \equiv 4035 \pmod{4160}$	4
$n \equiv 109 \pmod{624}$	1
$n \equiv 317 \pmod{624}$	2
$n \equiv 525 \pmod{624}$	3
$n \equiv 135 \pmod{832}$	1
$n \equiv 343 \pmod{832}$	2
$n \equiv 551 \pmod{832}$	3
$n \equiv 759 \pmod{1664}$	1
$n \equiv 1591 \pmod{1664}$	2
$n \equiv 161 \pmod{1040}$	1
$n \equiv 369 \pmod{1040}$	2
$n \equiv 577 \pmod{1040}$	3
$n \equiv 785 \pmod{1040}$	4
$n \equiv 993 \pmod{3120}$	1
$n \equiv 2033 \pmod{3120}$	2
$n \equiv 3073 \pmod{3120}$	3
$n \equiv 187 \pmod{1248}$	1
$n \equiv 395 \pmod{1248}$	2
$n \equiv 603 \pmod{1248}$	3
$n \equiv 811 \pmod{2496}$	1
$n \equiv 2059 \pmod{2496}$	2
$n \equiv 1019 \pmod{2496}$	3
$n \equiv 2267 \pmod{2496}$	4
$n \equiv 1227 \pmod{3744}$	1
$n \equiv 2475 \pmod{3744}$	2
$n \equiv 3723 \pmod{3744}$	3
$n \equiv 19 \pmod{78}$	1
$n \equiv 45 \pmod{234}$	1
$n \equiv 123 \pmod{702}$	1
$n \equiv 357 \pmod{702}$	2
$n \equiv 591 \pmod{2106}$	1
$n \equiv 1293 \pmod{2106}$	2
$n \equiv 1995 \pmod{2106}$	3
$n \equiv 201 \pmod{1404}$	1
$n \equiv 435 \pmod{1404}$	2

Table 5: Covering information for Sierpiński numbers

congruence	p
$n \equiv 669 \pmod{1404}$	3
$n \equiv 903 \pmod{1404}$	4
$n \equiv 1137 \pmod{2808}$	1
$n \equiv 2541 \pmod{2808}$	2
$n \equiv 1371 \pmod{2808}$	3
$n \equiv 2775 \pmod{2808}$	4
$n \equiv 71 \pmod{858}$	1
$n \equiv 149 \pmod{858}$	2
$n \equiv 227 \pmod{1716}$	1
$n \equiv 1085 \pmod{1716}$	2
$n \equiv 305 \pmod{1716}$	3
$n \equiv 1163 \pmod{1716}$	4
$n \equiv 383 \pmod{1716}$	5
$n \equiv 1241 \pmod{1716}$	6
$n \equiv 461 \pmod{1716}$	7
$n \equiv 1319 \pmod{1716}$	8
$n \equiv 539 \pmod{2574}$	1
$n \equiv 1397 \pmod{2574}$	2
$n \equiv 2255 \pmod{2574}$	3
$n \equiv 617 \pmod{2574}$	4
$n \equiv 1475 \pmod{5148}$	1
$n \equiv 4049 \pmod{5148}$	2
$n \equiv 2333 \pmod{5148}$	3
$n \equiv 4907 \pmod{10296}$	1
$n \equiv 10055 \pmod{10296}$	2
$n \equiv 695 \pmod{3432}$	1
$n \equiv 1553 \pmod{3432}$	2
$n \equiv 2411 \pmod{3432}$	3
$n \equiv 3269 \pmod{3432}$	4
$n \equiv 773 \pmod{4290}$	1
$n \equiv 1631 \pmod{4290}$	2
$n \equiv 2489 \pmod{4290}$	3
$n \equiv 3347 \pmod{4290}$	4
$n \equiv 4205 \pmod{8580}$	1
$n \equiv 8495 \pmod{8580}$	2
$n \equiv 851 \pmod{6006}$	1
$n \equiv 1709 \pmod{6006}$	2
$n \equiv 2567 \pmod{6006}$	3
$n \equiv 3425 \pmod{6006}$	4
$n \equiv 4283 \pmod{12012}$	1
$n \equiv 10289 \pmod{12012}$	2
$n \equiv 5141 \pmod{12012}$	3
$n \equiv 11147 \pmod{12012}$	4
$n \equiv 5999 \pmod{18018}$	1
$n \equiv 12005 \pmod{18018}$	2
$n \equiv 18011 \pmod{18018}$	3
$n \equiv 7 \pmod{130}$	1
$n \equiv 33 \pmod{130}$	2
$n \equiv 59 \pmod{650}$	1
$n \equiv 189 \pmod{650}$	2
$n \equiv 319 \pmod{650}$	3

congruence	p
$n \equiv 449 \pmod{650}$	4
$n \equiv 579 \pmod{650}$	5
$n \equiv 85 \pmod{1170}$	1
$n \equiv 215 \pmod{1170}$	2
$n \equiv 345 \pmod{1170}$	3
$n \equiv 475 \pmod{1170}$	4
$n \equiv 605 \pmod{1170}$	5
$n \equiv 735 \pmod{3510}$	1
$n \equiv 1905 \pmod{3510}$	2
$n \equiv 3075 \pmod{3510}$	3
$n \equiv 865 \pmod{3510}$	4
$n \equiv 2035 \pmod{3510}$	5
$n \equiv 3205 \pmod{3510}$	6
$n \equiv 995 \pmod{3510}$	7
$n \equiv 2165 \pmod{3510}$	8
$n \equiv 3335 \pmod{3510}$	9
$n \equiv 1125 \pmod{5850}$	1
$n \equiv 2295 \pmod{5850}$	2
$n \equiv 3465 \pmod{5850}$	3
$n \equiv 4635 \pmod{5850}$	4
$n \equiv 5805 \pmod{11700}$	1
$n \equiv 11655 \pmod{11700}$	2
$n \equiv 111 \pmod{1300}$	1
$n \equiv 241 \pmod{1300}$	2
$n \equiv 371 \pmod{1300}$	3
$n \equiv 501 \pmod{1300}$	4
$n \equiv 631 \pmod{1300}$	5
$n \equiv 761 \pmod{1300}$	6
$n \equiv 891 \pmod{1300}$	7
$n \equiv 1021 \pmod{2600}$	1
$n \equiv 2321 \pmod{2600}$	2
$n \equiv 1151 \pmod{2600}$	3
$n \equiv 2451 \pmod{2600}$	4
$n \equiv 1281 \pmod{5200}$	1
$n \equiv 3881 \pmod{5200}$	2
$n \equiv 2581 \pmod{7800}$	1
$n \equiv 5181 \pmod{7800}$	2
$n \equiv 7781 \pmod{7800}$	3
$n \equiv 8 \pmod{65}$	1
$n \equiv 21 \pmod{195}$	1
$n \equiv 86 \pmod{585}$	1
$n \equiv 281 \pmod{585}$	2
$n \equiv 476 \pmod{585}$	3
$n \equiv 151 \pmod{585}$	4
$n \equiv 346 \pmod{1755}$	1
$n \equiv 931 \pmod{1755}$	2
$n \equiv 1516 \pmod{1755}$	3
$n \equiv 541 \pmod{1755}$	4
$n \equiv 1126 \pmod{1755}$	5
$n \equiv 1711 \pmod{1755}$	6
$n \equiv 34 \pmod{325}$	1

congruence	p
$n \equiv 99 \pmod{325}$	2
$n \equiv 164 \pmod{975}$	1
$n \equiv 489 \pmod{975}$	2
$n \equiv 814 \pmod{975}$	3
$n \equiv 229 \pmod{975}$	4
$n \equiv 554 \pmod{2925}$	1
$n \equiv 1529 \pmod{2925}$	2
$n \equiv 2504 \pmod{2925}$	3
$n \equiv 879 \pmod{2925}$	4
$n \equiv 1854 \pmod{2925}$	5
$n \equiv 2829 \pmod{8775}$	1
$n \equiv 5754 \pmod{8775}$	2
$n \equiv 8679 \pmod{8775}$	3
$n \equiv 294 \pmod{1625}$	1
$n \equiv 619 \pmod{1625}$	2
$n \equiv 944 \pmod{1625}$	3
$n \equiv 1269 \pmod{1625}$	4
$n \equiv 1594 \pmod{1625}$	5
$n \equiv 47 \pmod{390}$	1
$n \equiv 177 \pmod{390}$	2
$n \equiv 307 \pmod{1560}$	1
$n \equiv 697 \pmod{1560}$	2
$n \equiv 1087 \pmod{1560}$	3
$n \equiv 1477 \pmod{6240}$	1
$n \equiv 3037 \pmod{6240}$	2
$n \equiv 4597 \pmod{6240}$	3
$n \equiv 6157 \pmod{6240}$	4
$n \equiv 60 \pmod{455}$	1
$n \equiv 125 \pmod{455}$	2
$n \equiv 190 \pmod{1365}$	1
$n \equiv 645 \pmod{1365}$	2
$n \equiv 1100 \pmod{1365}$	3
$n \equiv 255 \pmod{1365}$	4
$n \equiv 710 \pmod{4095}$	1
$n \equiv 2075 \pmod{4095}$	2
$n \equiv 3440 \pmod{4095}$	3
$n \equiv 1165 \pmod{4095}$	4
$n \equiv 6625 \pmod{8190}$	1
$n \equiv 3895 \pmod{8190}$	2
$n \equiv 320 \pmod{2275}$	1
$n \equiv 775 \pmod{2275}$	2
$n \equiv 1230 \pmod{2275}$	3
$n \equiv 1685 \pmod{2275}$	4
$n \equiv 2140 \pmod{6825}$	1
$n \equiv 4415 \pmod{6825}$	2
$n \equiv 6690 \pmod{6825}$	3
$n \equiv 385 \pmod{3640}$	1
$n \equiv 1295 \pmod{3640}$	2
$n \equiv 2205 \pmod{3640}$	3
$n \equiv 3115 \pmod{3640}$	4
$n \equiv 905 \pmod{4550}$	1

Table 5: Covering information for Sierpiński numbers (continued)

congruence	p
$n \equiv 1815 \pmod{4550}$	2
$n \equiv 2725 \pmod{9100}$	1
$n \equiv 7275 \pmod{9100}$	2
$n \equiv 3635 \pmod{9100}$	3
$n \equiv 8185 \pmod{9100}$	4
$n \equiv 4545 \pmod{13650}$	1
$n \equiv 9095 \pmod{13650}$	2
$n \equiv 13645 \pmod{13650}$	3
$n \equiv 9 \pmod{91}$	1
$n \equiv 22 \pmod{91}$	2
$n \equiv 35 \pmod{273}$	1
$n \equiv 126 \pmod{273}$	2
$n \equiv 217 \pmod{273}$	3
$n \equiv 139 \pmod{364}$	1
$n \equiv 321 \pmod{364}$	2
$n \equiv 61 \pmod{1092}$	1
$n \equiv 243 \pmod{1092}$	2
$n \equiv 425 \pmod{1092}$	3
$n \equiv 607 \pmod{1092}$	4
$n \equiv 789 \pmod{1092}$	5
$n \equiv 971 \pmod{3276}$	1
$n \equiv 2063 \pmod{3276}$	2
$n \equiv 3155 \pmod{3276}$	3
$n \equiv 74 \pmod{637}$	1
$n \equiv 165 \pmod{637}$	2
$n \equiv 256 \pmod{637}$	3
$n \equiv 347 \pmod{637}$	4
$n \equiv 438 \pmod{637}$	5
$n \equiv 529 \pmod{637}$	6
$n \equiv 620 \pmod{637}$	7
$n \equiv 87 \pmod{728}$	1
$n \equiv 269 \pmod{728}$	2
$n \equiv 451 \pmod{2184}$	1
$n \equiv 1179 \pmod{2184}$	2
$n \equiv 1907 \pmod{2184}$	3
$n \equiv 633 \pmod{2184}$	4
$n \equiv 1361 \pmod{2184}$	5
$n \equiv 2089 \pmod{2184}$	6
$n \equiv 23 \pmod{182}$	1
$n \equiv 49 \pmod{182}$	2
$n \equiv 75 \pmod{182}$	3
$n \equiv 101 \pmod{546}$	1
$n \equiv 283 \pmod{546}$	2
$n \equiv 465 \pmod{1638}$	1
$n \equiv 1011 \pmod{1638}$	2
$n \equiv 1557 \pmod{1638}$	3
$n \equiv 127 \pmod{910}$	1
$n \equiv 309 \pmod{910}$	2
$n \equiv 491 \pmod{910}$	3
$n \equiv 673 \pmod{910}$	4
$n \equiv 855 \pmod{2730}$	1

congruence	p
$n \equiv 1765 \pmod{2730}$	2
$n \equiv 2675 \pmod{2730}$	3
$n \equiv 153 \pmod{1274}$	1
$n \equiv 335 \pmod{1274}$	2
$n \equiv 517 \pmod{1274}$	3
$n \equiv 699 \pmod{2548}$	1
$n \equiv 1973 \pmod{2548}$	2
$n \equiv 881 \pmod{2548}$	3
$n \equiv 2155 \pmod{2548}$	4
$n \equiv 1063 \pmod{2548}$	5
$n \equiv 2337 \pmod{5096}$	1
$n \equiv 4885 \pmod{5096}$	2
$n \equiv 1245 \pmod{3822}$	1
$n \equiv 2519 \pmod{3822}$	2
$n \equiv 3793 \pmod{3822}$	3
$n \equiv 179 \pmod{1456}$	1
$n \equiv 361 \pmod{1456}$	2
$n \equiv 543 \pmod{1456}$	3
$n \equiv 725 \pmod{1456}$	4
$n \equiv 907 \pmod{2912}$	1
$n \equiv 2363 \pmod{2912}$	2
$n \equiv 1089 \pmod{2912}$	3
$n \equiv 2545 \pmod{2912}$	4
$n \equiv 1271 \pmod{4368}$	1
$n \equiv 2727 \pmod{4368}$	2
$n \equiv 4183 \pmod{4368}$	3
$n \equiv 1453 \pmod{5824}$	1
$n \equiv 2909 \pmod{5824}$	2
$n \equiv 4365 \pmod{5824}$	3
$n \equiv 5821 \pmod{5824}$	4
$n \equiv 11 \pmod{143}$	1
$n \equiv 24 \pmod{143}$	2
$n \equiv 37 \pmod{143}$	3
$n \equiv 193 \pmod{286}$	1
$n \equiv 63 \pmod{286}$	2
$n \equiv 219 \pmod{286}$	3
$n \equiv 89 \pmod{429}$	1
$n \equiv 232 \pmod{429}$	2
$n \equiv 375 \pmod{429}$	3
$n \equiv 245 \pmod{572}$	1
$n \equiv 531 \pmod{572}$	2
$n \equiv 115 \pmod{572}$	3
$n \equiv 401 \pmod{572}$	4
$n \equiv 271 \pmod{572}$	5
$n \equiv 557 \pmod{572}$	6
$n \equiv 141 \pmod{715}$	1
$n \equiv 284 \pmod{715}$	2
$n \equiv 427 \pmod{715}$	3
$n \equiv 1285 \pmod{1430}$	1
$n \equiv 713 \pmod{1430}$	2
$n \equiv 12 \pmod{169}$	1

congruence	p
$n \equiv 25 \pmod{169}$	2
$n \equiv 38 \pmod{169}$	3
$n \equiv 51 \pmod{338}$	1
$n \equiv 233 \pmod{338}$	2
$n \equiv 77 \pmod{507}$	1
$n \equiv 753 \pmod{2028}$	1
$n \equiv 1767 \pmod{2028}$	2
$n \equiv 415 \pmod{2028}$	3
$n \equiv 1429 \pmod{2028}$	4
$n \equiv 259 \pmod{676}$	1
$n \equiv 597 \pmod{676}$	2
$n \equiv 103 \pmod{676}$	3
$n \equiv 441 \pmod{676}$	4
$n \equiv 285 \pmod{676}$	5
$n \equiv 623 \pmod{2704}$	1
$n \equiv 1299 \pmod{2704}$	2
$n \equiv 1975 \pmod{2704}$	3
$n \equiv 2651 \pmod{2704}$	4
$n \equiv 129 \pmod{845}$	1
$n \equiv 298 \pmod{845}$	2
$n \equiv 467 \pmod{845}$	3
$n \equiv 1481 \pmod{1690}$	1
$n \equiv 805 \pmod{1690}$	2
$n \equiv 311 \pmod{1014}$	1
$n \equiv 649 \pmod{1014}$	2
$n \equiv 987 \pmod{1014}$	3
$n \equiv 155 \pmod{1183}$	1
$n \equiv 324 \pmod{1183}$	2
$n \equiv 493 \pmod{1183}$	3
$n \equiv 1845 \pmod{2366}$	1
$n \equiv 831 \pmod{2366}$	2
$n \equiv 2183 \pmod{4732}$	1
$n \equiv 4549 \pmod{4732}$	2
$n \equiv 1169 \pmod{4732}$	3
$n \equiv 3535 \pmod{4732}$	4
$n \equiv 337 \pmod{1352}$	1
$n \equiv 675 \pmod{1352}$	2
$n \equiv 1013 \pmod{1352}$	3
$n \equiv 1351 \pmod{1352}$	4

Table 5: Covering information for Sierpiński numbers (continued)

congruence	p
$n \equiv 2 \pmod{4}$	1
$n \equiv 4 \pmod{40}$	1
$n \equiv 24 \pmod{80}$	1
$n \equiv 64 \pmod{160}$	1
$n \equiv 144 \pmod{160}$	2
$n \equiv 8 \pmod{120}$	1
$n \equiv 28 \pmod{240}$	1
$n \equiv 148 \pmod{240}$	2
$n \equiv 48 \pmod{360}$	1
$n \equiv 168 \pmod{360}$	2
$n \equiv 288 \pmod{1080}$	1
$n \equiv 648 \pmod{1080}$	2
$n \equiv 1008 \pmod{1080}$	3
$n \equiv 68 \pmod{480}$	1
$n \equiv 188 \pmod{480}$	2
$n \equiv 308 \pmod{960}$	1
$n \equiv 788 \pmod{960}$	2
$n \equiv 428 \pmod{960}$	3
$n \equiv 908 \pmod{960}$	4
$n \equiv 88 \pmod{720}$	1
$n \equiv 208 \pmod{720}$	2
$n \equiv 328 \pmod{1440}$	1
$n \equiv 1048 \pmod{1440}$	2
$n \equiv 448 \pmod{1440}$	3
$n \equiv 1168 \pmod{1440}$	4
$n \equiv 568 \pmod{2160}$	1
$n \equiv 1288 \pmod{2160}$	2
$n \equiv 2008 \pmod{2160}$	3
$n \equiv 688 \pmod{2160}$	4
$n \equiv 1408 \pmod{2160}$	5
$n \equiv 2128 \pmod{4320}$	1
$n \equiv 4288 \pmod{4320}$	2
$n \equiv 108 \pmod{840}$	1
$n \equiv 228 \pmod{840}$	2
$n \equiv 348 \pmod{840}$	3
$n \equiv 468 \pmod{1680}$	1
$n \equiv 1308 \pmod{1680}$	2
$n \equiv 588 \pmod{1680}$	3
$n \equiv 1428 \pmod{1680}$	4
$n \equiv 708 \pmod{2520}$	1
$n \equiv 1548 \pmod{2520}$	2
$n \equiv 2388 \pmod{2520}$	3
$n \equiv 828 \pmod{2520}$	4
$n \equiv 1668 \pmod{5040}$	1
$n \equiv 4188 \pmod{5040}$	2
$n \equiv 2508 \pmod{5040}$	3
$n \equiv 5028 \pmod{10080}$	1
$n \equiv 10068 \pmod{10080}$	2
$n \equiv 12 \pmod{100}$	1
$n \equiv 32 \pmod{100}$	2
$n \equiv 52 \pmod{200}$	1
$n \equiv 152 \pmod{200}$	2

congruence	p
$n \equiv 72 \pmod{200}$	3
$n \equiv 172 \pmod{200}$	4
$n \equiv 92 \pmod{300}$	1
$n \equiv 192 \pmod{300}$	2
$n \equiv 292 \pmod{300}$	3
$n \equiv 16 \pmod{140}$	1
$n \equiv 36 \pmod{140}$	2
$n \equiv 56 \pmod{280}$	1
$n \equiv 196 \pmod{1120}$	1
$n \equiv 476 \pmod{1120}$	2
$n \equiv 756 \pmod{1120}$	3
$n \equiv 1036 \pmod{1120}$	4
$n \equiv 76 \pmod{420}$	1
$n \equiv 216 \pmod{420}$	2
$n \equiv 356 \pmod{1260}$	1
$n \equiv 776 \pmod{1260}$	2
$n \equiv 1196 \pmod{1260}$	3
$n \equiv 96 \pmod{560}$	1
$n \equiv 236 \pmod{560}$	2
$n \equiv 376 \pmod{560}$	3
$n \equiv 516 \pmod{560}$	4
$n \equiv 116 \pmod{700}$	1
$n \equiv 256 \pmod{700}$	2
$n \equiv 396 \pmod{700}$	3
$n \equiv 536 \pmod{1400}$	1
$n \equiv 1236 \pmod{1400}$	2
$n \equiv 676 \pmod{2100}$	1
$n \equiv 1376 \pmod{2100}$	2
$n \equiv 2076 \pmod{2100}$	3
$n \equiv 136 \pmod{980}$	1
$n \equiv 276 \pmod{980}$	2
$n \equiv 416 \pmod{980}$	3
$n \equiv 556 \pmod{980}$	4
$n \equiv 696 \pmod{980}$	5
$n \equiv 836 \pmod{1960}$	1
$n \equiv 1816 \pmod{1960}$	2
$n \equiv 976 \pmod{1960}$	3
$n \equiv 1956 \pmod{1960}$	4
$n \equiv 0 \pmod{25}$	1
$n \equiv 5 \pmod{25}$	2
$n \equiv 10 \pmod{50}$	1
$n \equiv 35 \pmod{150}$	1
$n \equiv 85 \pmod{450}$	1
$n \equiv 235 \pmod{450}$	2
$n \equiv 385 \pmod{1350}$	1
$n \equiv 835 \pmod{1350}$	2
$n \equiv 1285 \pmod{1350}$	3
$n \equiv 135 \pmod{750}$	1
$n \equiv 285 \pmod{750}$	2
$n \equiv 435 \pmod{750}$	3
$n \equiv 585 \pmod{2250}$	1
$n \equiv 1335 \pmod{2250}$	2

congruence	p
$n \equiv 2085 \pmod{2250}$	3
$n \equiv 735 \pmod{2250}$	4
$n \equiv 1485 \pmod{2250}$	5
$n \equiv 2235 \pmod{2250}$	6
$n \equiv 15 \pmod{75}$	1
$n \equiv 40 \pmod{75}$	2
$n \equiv 65 \pmod{225}$	1
$n \equiv 140 \pmod{225}$	2
$n \equiv 215 \pmod{225}$	3
$n \equiv 20 \pmod{125}$	1
$n \equiv 45 \pmod{125}$	2
$n \equiv 70 \pmod{250}$	1
$n \equiv 195 \pmod{250}$	2
$n \equiv 95 \pmod{375}$	1
$n \equiv 220 \pmod{1125}$	1
$n \equiv 595 \pmod{1125}$	2
$n \equiv 970 \pmod{1125}$	3
$n \equiv 345 \pmod{1875}$	1
$n \equiv 720 \pmod{1875}$	2
$n \equiv 1095 \pmod{1875}$	3
$n \equiv 1470 \pmod{1875}$	4
$n \equiv 1845 \pmod{1875}$	5
$n \equiv 120 \pmod{625}$	1
$n \equiv 245 \pmod{625}$	2
$n \equiv 370 \pmod{625}$	3
$n \equiv 495 \pmod{625}$	4
$n \equiv 620 \pmod{1250}$	1
$n \equiv 1245 \pmod{1250}$	2
$n \equiv 7 \pmod{35}$	1
$n \equiv 49 \pmod{70}$	1
$n \equiv 21 \pmod{105}$	1
$n \equiv 56 \pmod{105}$	2
$n \equiv 91 \pmod{105}$	3
$n \equiv 28 \pmod{175}$	1
$n \equiv 63 \pmod{175}$	2
$n \equiv 98 \pmod{175}$	3
$n \equiv 133 \pmod{350}$	1
$n \equiv 343 \pmod{350}$	2
$n \equiv 0 \pmod{27}$	1
$n \equiv 3 \pmod{81}$	1
$n \equiv 30 \pmod{81}$	2
$n \equiv 57 \pmod{81}$	3
$n \equiv 33 \pmod{108}$	1
$n \equiv 87 \pmod{756}$	1
$n \equiv 195 \pmod{756}$	2
$n \equiv 303 \pmod{2268}$	1
$n \equiv 1059 \pmod{2268}$	2
$n \equiv 1815 \pmod{2268}$	3
$n \equiv 411 \pmod{2268}$	4
$n \equiv 1167 \pmod{2268}$	5
$n \equiv 1923 \pmod{2268}$	6
$n \equiv 519 \pmod{2268}$	7

Table 6: Covering information for Riesel numbers

congruence	p
$n \equiv 1275 \pmod{2268}$	8
$n \equiv 2031 \pmod{2268}$	9
$n \equiv 627 \pmod{1512}$	1
$n \equiv 1383 \pmod{3024}$	1
$n \equiv 2895 \pmod{3024}$	2
$n \equiv 9 \pmod{135}$	1
$n \equiv 36 \pmod{135}$	2
$n \equiv 63 \pmod{540}$	1
$n \equiv 333 \pmod{540}$	2
$n \equiv 117 \pmod{540}$	3
$n \equiv 387 \pmod{540}$	4
$n \equiv 39 \pmod{162}$	1
$n \equiv 93 \pmod{486}$	1
$n \equiv 255 \pmod{486}$	2
$n \equiv 417 \pmod{486}$	3
$n \equiv 147 \pmod{810}$	1
$n \equiv 309 \pmod{810}$	2
$n \equiv 471 \pmod{810}$	3
$n \equiv 633 \pmod{1620}$	1
$n \equiv 1443 \pmod{1620}$	2
$n \equiv 15 \pmod{216}$	1
$n \equiv 69 \pmod{216}$	2
$n \equiv 123 \pmod{648}$	1
$n \equiv 339 \pmod{648}$	2
$n \equiv 555 \pmod{648}$	3
$n \equiv 177 \pmod{864}$	1
$n \equiv 393 \pmod{864}$	2
$n \equiv 609 \pmod{1728}$	1
$n \equiv 1473 \pmod{1728}$	2
$n \equiv 825 \pmod{1728}$	3
$n \equiv 1689 \pmod{1728}$	4
$n \equiv 99 \pmod{270}$	1
$n \equiv 153 \pmod{270}$	2
$n \equiv 207 \pmod{270}$	3
$n \equiv 261 \pmod{810}$	1
$n \equiv 531 \pmod{810}$	2
$n \equiv 801 \pmod{810}$	3
$n \equiv 21 \pmod{324}$	1
$n \equiv 75 \pmod{324}$	2
$n \equiv 129 \pmod{324}$	3
$n \equiv 183 \pmod{324}$	4
$n \equiv 237 \pmod{648}$	1
$n \equiv 561 \pmod{648}$	2
$n \equiv 291 \pmod{648}$	3
$n \equiv 615 \pmod{648}$	4
$n \equiv 51 \pmod{378}$	1
$n \equiv 159 \pmod{378}$	2
$n \equiv 213 \pmod{378}$	3
$n \equiv 267 \pmod{378}$	4
$n \equiv 321 \pmod{1134}$	1
$n \equiv 699 \pmod{1134}$	2
$n \equiv 1077 \pmod{1134}$	3

congruence	p
$n \equiv 753 \pmod{1890}$	1
$n \equiv 1131 \pmod{1890}$	2
$n \equiv 1509 \pmod{1890}$	3
$n \equiv 1887 \pmod{3780}$	1
$n \equiv 3777 \pmod{3780}$	2
$n \equiv 11 \pmod{33}$	1
$n \equiv 22 \pmod{99}$	1
$n \equiv 55 \pmod{99}$	2
$n \equiv 88 \pmod{297}$	1
$n \equiv 187 \pmod{297}$	2
$n \equiv 583 \pmod{594}$	1
$n \equiv 1 \pmod{55}$	1
$n \equiv 67 \pmod{165}$	1
$n \equiv 122 \pmod{495}$	1
$n \equiv 287 \pmod{495}$	2
$n \equiv 452 \pmod{1485}$	1
$n \equiv 947 \pmod{1485}$	2
$n \equiv 1442 \pmod{1485}$	3
$n \equiv 23 \pmod{275}$	1
$n \equiv 78 \pmod{275}$	2
$n \equiv 133 \pmod{825}$	1
$n \equiv 683 \pmod{825}$	2
$n \equiv 188 \pmod{825}$	3
$n \equiv 463 \pmod{825}$	4
$n \equiv 243 \pmod{1375}$	1
$n \equiv 518 \pmod{1375}$	2
$n \equiv 793 \pmod{1375}$	3
$n \equiv 2443 \pmod{2750}$	1
$n \equiv 1343 \pmod{2750}$	2
$n \equiv 89 \pmod{330}$	1
$n \equiv 199 \pmod{330}$	2
$n \equiv 13 \pmod{66}$	1
$n \equiv 35 \pmod{264}$	1
$n \equiv 101 \pmod{264}$	2
$n \equiv 167 \pmod{792}$	1
$n \equiv 431 \pmod{792}$	2
$n \equiv 695 \pmod{1584}$	1
$n \equiv 1487 \pmod{1584}$	2
$n \equiv 233 \pmod{1320}$	1
$n \equiv 497 \pmod{1320}$	2
$n \equiv 761 \pmod{1320}$	3
$n \equiv 1289 \pmod{1320}$	4
$n \equiv 3 \pmod{77}$	1
$n \equiv 25 \pmod{231}$	1
$n \equiv 179 \pmod{693}$	1
$n \equiv 410 \pmod{693}$	2
$n \equiv 641 \pmod{693}$	3
$n \equiv 113 \pmod{308}$	1
$n \equiv 267 \pmod{308}$	2
$n \equiv 47 \pmod{308}$	3
$n \equiv 201 \pmod{1540}$	1
$n \equiv 509 \pmod{1540}$	2

congruence	p
$n \equiv 817 \pmod{1540}$	3
$n \equiv 1433 \pmod{1540}$	4
$n \equiv 58 \pmod{385}$	1
$n \equiv 212 \pmod{385}$	2
$n \equiv 289 \pmod{385}$	3
$n \equiv 751 \pmod{1155}$	1
$n \equiv 1136 \pmod{1155}$	2
$n \equiv 223 \pmod{462}$	1
$n \equiv 377 \pmod{1386}$	1
$n \equiv 839 \pmod{2772}$	1
$n \equiv 2225 \pmod{2772}$	2
$n \equiv 1301 \pmod{2772}$	3
$n \equiv 2687 \pmod{2772}$	4
$n \equiv 15 \pmod{88}$	1
$n \equiv 37 \pmod{440}$	1
$n \equiv 213 \pmod{440}$	2
$n \equiv 301 \pmod{440}$	3
$n \equiv 389 \pmod{440}$	4
$n \equiv 59 \pmod{528}$	1
$n \equiv 235 \pmod{528}$	2
$n \equiv 323 \pmod{2112}$	1
$n \equiv 851 \pmod{2112}$	2
$n \equiv 1379 \pmod{2112}$	3
$n \equiv 1907 \pmod{2112}$	4
$n \equiv 499 \pmod{2640}$	1
$n \equiv 1027 \pmod{2640}$	2
$n \equiv 2083 \pmod{5280}$	1
$n \equiv 4723 \pmod{5280}$	2
$n \equiv 2611 \pmod{5280}$	3
$n \equiv 5251 \pmod{5280}$	4
$n \equiv 81 \pmod{616}$	1
$n \equiv 169 \pmod{616}$	2
$n \equiv 257 \pmod{616}$	3
$n \equiv 345 \pmod{1232}$	1
$n \equiv 961 \pmod{1232}$	2
$n \equiv 433 \pmod{2464}$	1
$n \equiv 1049 \pmod{2464}$	2
$n \equiv 1665 \pmod{2464}$	3
$n \equiv 2281 \pmod{2464}$	4
$n \equiv 521 \pmod{3080}$	1
$n \equiv 1137 \pmod{3080}$	2
$n \equiv 1753 \pmod{3080}$	3
$n \equiv 2369 \pmod{6160}$	1
$n \equiv 5449 \pmod{6160}$	2
$n \equiv 27 \pmod{110}$	1
$n \equiv 49 \pmod{110}$	2
$n \equiv 71 \pmod{220}$	1
$n \equiv 181 \pmod{220}$	2
$n \equiv 93 \pmod{550}$	1
$n \equiv 203 \pmod{1100}$	1
$n \equiv 753 \pmod{1100}$	2
$n \equiv 313 \pmod{1100}$	3

Table 6: Covering information for Riesel numbers (continued)

congruence	p
$n \equiv 863 \pmod{1100}$	4
$n \equiv 423 \pmod{1100}$	5
$n \equiv 973 \pmod{1100}$	6
$n \equiv 533 \pmod{1100}$	7
$n \equiv 1083 \pmod{2200}$	1
$n \equiv 2183 \pmod{2200}$	2
$n \equiv 6 \pmod{121}$	1
$n \equiv 17 \pmod{242}$	1
$n \equiv 149 \pmod{242}$	2
$n \equiv 160 \pmod{363}$	1
$n \equiv 281 \pmod{363}$	2
$n \equiv 50 \pmod{363}$	3
$n \equiv 292 \pmod{363}$	4
$n \equiv 61 \pmod{484}$	1
$n \equiv 303 \pmod{484}$	2
$n \equiv 193 \pmod{484}$	3
$n \equiv 435 \pmod{484}$	4
$n \equiv 83 \pmod{726}$	1
$n \equiv 325 \pmod{726}$	2
$n \equiv 215 \pmod{726}$	3
$n \equiv 457 \pmod{1452}$	1
$n \equiv 1183 \pmod{1452}$	2
$n \equiv 105 \pmod{968}$	1
$n \equiv 347 \pmod{968}$	2
$n \equiv 589 \pmod{968}$	3
$n \equiv 831 \pmod{968}$	4
$n \equiv 116 \pmod{1089}$	1
$n \equiv 358 \pmod{1089}$	2
$n \equiv 479 \pmod{1089}$	3
$n \equiv 721 \pmod{1089}$	4
$n \equiv 1931 \pmod{2178}$	1
$n \equiv 2173 \pmod{2178}$	2
$n \equiv 7 \pmod{132}$	1
$n \equiv 29 \pmod{132}$	2
$n \equiv 73 \pmod{660}$	1
$n \equiv 337 \pmod{660}$	2
$n \equiv 469 \pmod{660}$	3
$n \equiv 601 \pmod{660}$	4
$n \equiv 227 \pmod{660}$	5
$n \equiv 359 \pmod{1980}$	1
$n \equiv 1019 \pmod{1980}$	2
$n \equiv 1679 \pmod{1980}$	3
$n \equiv 491 \pmod{1980}$	4
$n \equiv 1151 \pmod{1980}$	5
$n \equiv 1811 \pmod{3960}$	1
$n \equiv 3791 \pmod{3960}$	2
$n \equiv 623 \pmod{3300}$	1
$n \equiv 1283 \pmod{3300}$	2
$n \equiv 1943 \pmod{3300}$	3
$n \equiv 2603 \pmod{6600}$	1
$n \equiv 5903 \pmod{6600}$	2
$n \equiv 3263 \pmod{6600}$	3

congruence	p
$n \equiv 6563 \pmod{6600}$	4
$n \equiv 19 \pmod{154}$	1
$n \equiv 41 \pmod{154}$	2
$n \equiv 239 \pmod{770}$	1
$n \equiv 393 \pmod{770}$	2
$n \equiv 547 \pmod{770}$	3
$n \equiv 701 \pmod{2310}$	1
$n \equiv 1471 \pmod{2310}$	2
$n \equiv 107 \pmod{924}$	1
$n \equiv 415 \pmod{924}$	2
$n \equiv 569 \pmod{2772}$	1
$n \equiv 1493 \pmod{2772}$	2
$n \equiv 2417 \pmod{2772}$	3
$n \equiv 877 \pmod{2772}$	4
$n \equiv 1801 \pmod{5544}$	1
$n \equiv 4573 \pmod{5544}$	2
$n \equiv 2725 \pmod{8316}$	1
$n \equiv 5497 \pmod{8316}$	2
$n \equiv 8269 \pmod{8316}$	3
$n \equiv 129 \pmod{1078}$	1
$n \equiv 283 \pmod{1078}$	2
$n \equiv 437 \pmod{1078}$	3
$n \equiv 591 \pmod{2156}$	1
$n \equiv 1669 \pmod{2156}$	2
$n \equiv 745 \pmod{2156}$	3
$n \equiv 1823 \pmod{2156}$	4
$n \equiv 899 \pmod{2156}$	5
$n \equiv 1977 \pmod{4312}$	1
$n \equiv 4133 \pmod{4312}$	2
$n \equiv 2131 \pmod{3234}$	1
$n \equiv 3209 \pmod{3234}$	2
$n \equiv 151 \pmod{1848}$	1
$n \equiv 305 \pmod{1848}$	2
$n \equiv 613 \pmod{1848}$	3
$n \equiv 767 \pmod{1848}$	4
$n \equiv 1075 \pmod{1848}$	5
$n \equiv 1229 \pmod{3696}$	1
$n \equiv 3077 \pmod{3696}$	2
$n \equiv 1537 \pmod{3696}$	3
$n \equiv 3385 \pmod{7392}$	1
$n \equiv 7081 \pmod{7392}$	2
$n \equiv 1691 \pmod{9240}$	1
$n \equiv 3539 \pmod{9240}$	2
$n \equiv 5387 \pmod{9240}$	3
$n \equiv 9083 \pmod{9240}$	4
$n \equiv 9 \pmod{176}$	1
$n \equiv 31 \pmod{176}$	2
$n \equiv 53 \pmod{176}$	3
$n \equiv 75 \pmod{352}$	1
$n \equiv 251 \pmod{352}$	2
$n \equiv 97 \pmod{704}$	1
$n \equiv 273 \pmod{704}$	2

congruence	p
$n \equiv 449 \pmod{704}$	3
$n \equiv 625 \pmod{704}$	4
$n \equiv 119 \pmod{880}$	1
$n \equiv 471 \pmod{880}$	2
$n \equiv 647 \pmod{880}$	3
$n \equiv 823 \pmod{880}$	4
$n \equiv 317 \pmod{1056}$	1
$n \equiv 493 \pmod{1056}$	2
$n \equiv 845 \pmod{1056}$	3
$n \equiv 1021 \pmod{1056}$	4
$n \equiv 163 \pmod{1408}$	1
$n \equiv 339 \pmod{1408}$	2
$n \equiv 515 \pmod{1408}$	3
$n \equiv 691 \pmod{2816}$	1
$n \equiv 2099 \pmod{2816}$	2
$n \equiv 867 \pmod{2816}$	3
$n \equiv 2275 \pmod{2816}$	4
$n \equiv 1043 \pmod{4224}$	1
$n \equiv 3859 \pmod{4224}$	2
$n \equiv 1219 \pmod{5632}$	1
$n \equiv 2627 \pmod{5632}$	2
$n \equiv 4035 \pmod{5632}$	3
$n \equiv 5443 \pmod{11264}$	1
$n \equiv 11075 \pmod{11264}$	2
$n \equiv 2803 \pmod{7040}$	1
$n \equiv 4211 \pmod{7040}$	2
$n \equiv 5619 \pmod{7040}$	3
$n \equiv 7027 \pmod{7040}$	4
$n \equiv 43 \pmod{198}$	1
$n \equiv 65 \pmod{198}$	2
$n \equiv 109 \pmod{396}$	1
$n \equiv 307 \pmod{396}$	2
$n \equiv 131 \pmod{396}$	3
$n \equiv 329 \pmod{396}$	4
$n \equiv 175 \pmod{594}$	1
$n \equiv 373 \pmod{594}$	2
$n \equiv 571 \pmod{594}$	3
$n \equiv 197 \pmod{1188}$	1
$n \equiv 395 \pmod{1188}$	2
$n \equiv 593 \pmod{1188}$	3
$n \equiv 791 \pmod{1188}$	4
$n \equiv 989 \pmod{1188}$	5
$n \equiv 1187 \pmod{1188}$	6

Table 6: Covering information for Riesel numbers (continued)