INVERSION SEQUENCES AVOIDING A SET OF LENGTH-3 PATTERNS

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Abstract
An *inversion sequence* of size n is a sequence of integers $e = e_0 \cdots e_n$ such that $0 \leq e_i \leq i$, for all $i = 0, 1, \ldots, n$. For a set of patterns B, let $\mathbf{I}_n(B)$ be the set of inversion sequences of length n that avoid all the patterns from B. We say that two sets of patterns B and C are I-Wilf-equivalent if $|\mathbf{I}_n(B)| = |\mathbf{I}_n(C)|$, for all $n \geq 0$. Let w_d be the number of distinct I-Wilf-equivalence classes of subsets of exactly d length-3 patterns. This paper aims to prove that $w_5 = 219$, $w_6 = 167$, $w_7 = 105$, $w_8 = 61$, $w_9 = 35$, $w_{10} = 21$, $w_{11} = 10$, $w_{12} = 4$, and $w_{13} = 1$.

1. Introduction

For a set K, we let K^n denote the set of words of size n over the alphabet K, that is, all the n-tuples $w_1 w_2 \cdots w_n$ with $w_i \in K$. We say that a word $x = x_1 \cdots x_n \in K^n$ is *order-isomorphic* to a word $y = y_1 \cdots y_n \in K^n$ if for every pair of indices i, j, we have $x_i(<,=,>)x_j$ if and only if $y_i(<,=,>)y_j$, respectively. We say that a word $w = w_1 \cdots w_n$ *contains* a word $\pi = \pi_1 \cdots \pi_m$ if w contains a subsequence of size m which is order-isomorphic to π. Otherwise, we say that w *avoids* π. In this context, π is called a *length-m pattern.* Let S_n be the symmetric group of all permutations of $[n] = \{1, 2, \ldots, n\}$. For a set B of patterns, we define $S_n(B)$ to be the set of all permutations of $[n]$ that avoid each pattern in B. We say that two sets of patterns P and Q are *P-Wilf-equivalent* if $|S_n(P)| = |S_n(Q)|$ for all $n \geq 0$. Let s_d^k be the number of distinct P-Wilf-equivalence classes of sets of d length-k patterns. Finding s_d^3 has been considered in [13]. As a next step, finding s_d^4 has received a lot of attention, where it showed

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$$\begin{array}{lllll} s^4_0 = 1, & s^4_1 = 3, & s^4_2 = 38, & s^4_3 = 242, & s^4_4 = 1100, \\ s^4_5 = 3441, & s^4_6 = 8438, & s^4_7 = 15392, & s^4_8 = 19002, & s^4_9 = 16293, \\ s^4_{10} = 10624, & s^4_{11} = 5857, & s^4_{12} = 3044, & s^4_{13} = 1546, & s^4_{14} = 786, \\ s^4_{15} = 393, & s^4_{16} = 198, & s^4_{17} = 105, & s^4_{18} = 55, & s^4_{19} = 28, \\ s^4_{20} = 14, & s^4_{21} = 8, & s^4_{22} = 4, & s^4_{23} = 2, & s^4_{24} = 1. \end{array}$$

See [9] and references therein.

We are interested in extending these results to the case of inversion sequences. An *inversion sequence* [6, 11] of size n is a sequence of integers $e = e_0 \cdots e_n$ such that $0 \leq e_i \leq i$, for all $i = 0, 1, \ldots, n$. We denote the set of all inversion sequences of size n by $\mathbf{I}_n$. We say that a word $w = w_1 \cdots w_n$ is *reduced*, if each number from the set $\{0, 1, \ldots, \max_i w_i\}$ appears at least once in w. Note that if π and π' are order-isomorphic, then a word w contains π if and only if w contains π'. So, when dealing with pattern avoidance, we may without loss of generality restrict our attention to reduced patterns. Throughout this paper, we use the term *pattern* as one of the following thirteen length-3 patterns: 000, 001, 010, 011, 012, 021, 100, 101, 102, 110, 120, 201, and 210.

For set B of patterns, we define $\mathbf{I}_n(B)$ to be the set of all inversion sequences of size n that avoid all the patterns in B. To avoid notational clutter, we often omit nested braces and write, for example, $\mathbf{I}_n(\pi, \pi')$ instead of $\mathbf{I}_n(\{\pi, \pi'\})$. We say that two sets of patterns P and Q are *I-Wilf-equivalent*, denoted $P \overset{\mathbf{I}}{\sim} Q$, if $|\mathbf{I}_n(P)| = |\mathbf{I}_n(Q)|$, for all $n \geq 0$. Let w_d be the number of distinct I-Wilf-equivalence classes of sets of d length-3 patterns.

The systematic study of pattern-avoidance for inversion sequences started around 2015 [6,11], which shows that $w_1 = 11$. The results of [1,5,12,14] determined all the I-Wilf-equivalence classes of pairs of length-3 patterns, which implies that $w_2 = 48$. In [2] it is shown that $137 \leq w_3 \leq 139$, and in [4] that $212 \leq w_4 \leq 215$. The present paper aims to prove the following result.

Theorem 1. *We have that* $w_5 = 219$, $w_6 = 167$, $w_7 = 105$, $w_8 = 61$, $w_9 = 35$, $w_{10} = 21$, $w_{11} = 10$, $w_{12} = 4$, *and* $w_{13} = 1$.

2. Background and Our Strategy

This paper continues the work from the previous papers [2, 4], further exploring the values of w_d. More precisely, we determine the values of w_d, for $d = 5, 6, \ldots, 13$. Since the number of sets of d length-3 patterns is $\binom{13}{d}$, it seems to be impossible to reach by constructing explicit bijections and direct enumeration between sets of inversion sequences. As in [2, 4], the way out is to use the procedure in [8] to do the work for us. To reach the results of Theorem 1, we follow the following procedure, which is given by the following steps.

`Step 1` (Reduction). Let B be any set of patterns. We say that B is *reducible* if there exists $C \subsetneqq B$ such that $\mathbf{I}_n(B) = \mathbf{I}_n(C)$, for all $n \geq 0$. Otherwise, we say that B cannot be reduced. If B is reducible to set C, then we write $B \overset{r}{\sim} C$. Clearly, $B \overset{r}{\sim} C$ implies $B \overset{\mathbf{I}}{\sim} C$. Reference [4] gives the following reductions.

Theorem 2. *We have the following:*

- $\{001\} \overset{r}{\sim} \{001, 101\}$, $\{001\} \overset{r}{\sim} \{001, 102\}$, $\{001\} \overset{r}{\sim} \{001, 201\}$;
- $\{011\} \overset{r}{\sim} \{011, 101\}$, $\{011\} \overset{r}{\sim} \{011, 110\}$;
- $\{012\} \overset{r}{\sim} \{012, 102\}$, $\{012\} \overset{r}{\sim} \{012, 120\}$;
- $\{021\} \overset{r}{\sim} \{021, 201\}$, $\{021\} \overset{r}{\sim} \{021, 210\}$;
- $\{000, 001\} \overset{r}{\sim} \{000, 001, 100\}$;
- $\{000, 011\} \overset{r}{\sim} \{000, 011, \tau\}$, *for any* $\tau = 100, 201, 210$;
- $\{000, 012\} \overset{r}{\sim} \{000, 012, \tau\}$, *for any* $\tau = 201, 210$;
- $\{000, 021\} \overset{r}{\sim} \{000, 021, 100\}$;
- $\{001, 010\} \overset{r}{\sim} \{001, 010, \tau\}$, *for any* $\tau = 021, 100, 110, 120, 210$;
- $\{001, 011\} \overset{r}{\sim} \{001, 011, 021\}$;
- $\{001, 012\} \overset{r}{\sim} \{001, 012, 021\}$;
- $\{001, 110\} \overset{r}{\sim} \{001, 110, 210\}$;
- $\{001, 120\} \overset{r}{\sim} \{001, 120, 210\}$;
- $\{010, 011\} \overset{r}{\sim} \{010, 011, 100\}$;
- $\{010, 012\} \overset{r}{\sim} \{010, 012, \tau\}$, *for any* $\tau = 101, 201$;
- $\{010, 021\} \overset{r}{\sim} \{010, 021, \tau\}$, *for any* $\tau = 100, 101, 102, 110, 120$;
- $\{001, 011, 012\} \overset{r}{\sim} \{001, 011, 012, 210\}$;
- $\{001, 011, 100\} \overset{r}{\sim} \{001, 011, 100, 210\}$;
- $\{001, 012, 100\} \overset{r}{\sim} \{001, 012, 100, 210\}$.

Note that when we have a reducible subset of d length-3 patterns, we can consider the references [2, 4, 12, 14] or the case of s length-3 patterns with $1 \leq s \leq d-1$. Note that these references considered the I-Wilf-equivalences and enumerations of $|\mathbf{I}_n(B)|$ whenever B is any pair, triple, or quadruple of length-3 patterns. In the tables below, we mark the set of patterns B by $\bullet$ whenever B cannot be reduced.

`Step 2` (Generating trees). Let B be any set of patterns. Following [8], we define the generating tree (see [15]) $\mathcal{T}(B)$ for the class $\cup_{n=0}^{\infty}\mathbf{I}_n(B)$ as a plain tree as follows. Set that the root of the tree $\mathcal{T}(B)$ is 0 and it stays at level 0. We construct the remainder of the nodes of the tree $\mathcal{T}(B)$ as follows: the children of $e = e_0e_1\cdots e_{n-1} \in \mathbf{I}_{n-1}(B)$ are obtained from the set $\{e_0e_1\cdots e_{n-1}e_n \mid e_n = 0, 1, \ldots, n\}$ by obeying the pattern-avoiding restrictions of the patterns in B. In this case, we write $e \rightsquigarrow v^{(1)}, \ldots, v^{(s)}$, where $v^{(1)}, \ldots, v^{(s)}$ are the children of $e \in \mathcal{T}(B)$ and e is called a *father*.

We define an equivalence relation on nodes of $\mathcal{T}(B)$. Let $\mathcal{T}(B; e)$ be the subtree consisting of the inversion sequence e as the root and its descendants in $\mathcal{T}(B)$. We say that e is *equivalent* to e' if and only if $\mathcal{T}(B; e) \cong \mathcal{T}(B; e')$ (in the sense of plain trees). Let $\mathcal{T}'(B)$ be the same tree $\mathcal{T}(B)$ where we replace each node e by the first node $e' \in \mathcal{T}(B)$ from top to bottom and from left to right in $\mathcal{T}(B)$ such that $\mathcal{T}(B; e) \cong \mathcal{T}(B; e')$. From now, we identify $\mathcal{T}'(B)$ with $\mathcal{T}(B)$.

The main results of this paper are applications of the following procedure, *Main-Procedure*, (see [8]):

- Let ℓ be any positive integer.
- We initialize the tree $\mathcal{T}(B)$ by the root 0, where we define $Q_0 = \{0\}$ to be the set of nodes at level 0 and $GT_\ell(B) = \emptyset$ to be the set of the succession rules up to level 0.
- For all $i = 1, 2, \ldots, \ell$,
  - For any $w \in Q_{i-1}$, we denote the set of all children of w in $\mathcal{T}(B)$ by N_w. Define $M_i = \cup_{w \in Q_{i-1}} N_w$. If $M_i = \emptyset$, then we stop the procedure.
  - We initialize the set Q_i (set of new equivalence classes at ith step) to be empty set. For each child w in M_i,
    * We find $v \in \cup_{j=0}^{i-1} Q_j$, if possible, such that $w \sim v$, where we use [8, Lemma 2.1] to check that $w \sim v$ holds or not;
    * Otherwise, we add the equivalence class w to Q_i.
  - Add the rule $w \rightsquigarrow v_1v_2\cdots v_s$ to the set $GT_\ell(B) = \emptyset$, where v_j is the label of the jth child of w, from left to right, in $\mathcal{T}(B)$.
- If we stop the procedure, then we have the finite set of labels $\cup_{j=0}^{i-1} Q_j$ and finite set of succession rules $GT_\ell(B) = \emptyset$ that specifies the tree $\mathcal{T}(B)$ with the root 0.

- Otherwise, we have a set of succession rules $GT_\ell(B) = \emptyset$ that specifies the tree $\mathcal{T}(B)$ with its root 0 up to level ℓ. We could guess, if possible, all of the set of succession rules of $\mathcal{T}(B)$ based on $GT_\ell(B) = \emptyset$, then use [8, Lemma 2.1] to prove this claim. In case we fail to guess the whole set of the succession rules, then either we increase ℓ or we say that our procedure does not lead us to determine all the succession rules of $\mathcal{T}(B)$.

Let B be any set of length-3 patterns. We apply Main-Procedure up to level ℓ (say $\ell = 8$). The result is a set $GT_\ell(B)$ of rules that describe all the rules that produce all the nodes of $\mathcal{T}(B)$ from its root up to level ℓ.

Example 1. Let $B = \{010, 011, 012, 100, 201\}$. Then by Main-Procedure with $\ell = 8$, we obtain the following rules:

$0 \rightsquigarrow 00, 01$

$00 \rightsquigarrow 000, 01, 002,$

$01 \rightsquigarrow$

$000 \rightsquigarrow 0000, 01, 002, 0003,$

$002 \rightsquigarrow 01,$

$0000 \rightsquigarrow 00000, 01, 002, 0003, 00004,$

$0003 \rightsquigarrow 01, 002,$

$00000 \rightsquigarrow 000000, 01, 002, 0003, 00004, 000005,$

$00004 \rightsquigarrow 01, 002, 0003,$

$000000 \rightsquigarrow 0000000, 01, 002, 0003, 00004, 000005, 0000006,$

$000005 \rightsquigarrow 01, 002, 0003, 00004,$

$0000000 \rightsquigarrow 00000000, 01, 002, 0003, 00004, 000005, 0000006, 00000007,$

$0000006 \rightsquigarrow 01, 002, 0003, 00004, 000005,$

$00000000 \rightsquigarrow 000000000, 01, 002, 0003, 00004, 000005, 0000006, 00000007, 000000008,$

$00000007 \rightsquigarrow 01, 002, 0003, 00004, 000005, 0000006,$

To shorten the notation, we define a^m to be the word $aa \cdots a$ with m letters, for any letter a and a nonnegative integer m. Also, we define $\{s_i\}_{i=1}^m$ to be the sequence $s_1, s_2, \cdots, s_m$ with m terms, for any nonnegative integer m. Now, we try to guess the set of the rules of $\mathcal{T}(B)$ based on the set of the rules $GT_\ell(B)$. If we fail to do that, either we try again after we increase ℓ, or we say that we *failed to guess* $\mathcal{T}(B)$.

Example 2. Let $B = \{010, 011, 012, 100, 201\}$. Based on Example 1, we can guess that the set of the rules of $\mathcal{T}(B)$ are given by

$$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m, \qquad b_m \rightsquigarrow b_1, \ldots, b_{m-1},$$

where $a_m = 0^m$ and $b_m = a_m m$. To prove this, we append a new letter to the right of each father. In the case of the father a_m, we have the child a_{m+1} and the children a_mj with $1 \leq j \leq m$. Note that $a_mj\pi' \in \mathbf{I}_B$ if any only if $a_jj\pi' \in \mathbf{I}_B$, so $\mathcal{T}(B;a_mj) \cong \mathcal{T}(B;a_jj)$, for all $j = 1,2,\ldots,m$. Hence, we can state the rule $a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$. In the case of the father b_m, we have the children b_mj with $1 \leq j \leq m-1$. Note that $b_mj\pi' \in \mathbf{I}_B$ if and only if $b_j\pi' \in \mathbf{I}_B$, which leads to the fact that $\mathcal{T}(B;b_mj) \cong \mathcal{T}(B,;b_j)$, for all $j = 1,2,\ldots,m-1$. Hence, we can state the rule $b_m \rightsquigarrow b_1, \ldots, b_{m-1}$. This completes the proof.

`Step 3` (Generating functions). After we guessed and proved (if possible) the rules of the generating tree $\mathcal{T}(B)$, we translate these rules into a system of equations and we solve for

$$F_B(x) = \sum_{n\geq 0} |\mathbf{I}_n(B)| x^{n+1}.$$

Note that the rule $e \rightsquigarrow v^{(1)}, \ldots, v^{(s)}$ can be translated to

$$A_e(x) = x + x\sum_{j=1}^{s} A_{v^{(s)}}(x),$$

where $A_w(x) = \sum_{n\geq 0}(\#\text{the nodes at level } n \text{ in } \mathcal{T}(B;w))x^{n+1}$ is the generating function for the number of nodes at level $n \geq 0$ in the subtree of $\mathcal{T}(B;w)$, where its root stays at level 0. As continuation of Example 2, we present the details for finding $F_{\{010,011,012,100,201\}}(x)$ as follows. First, we define $A_m(x)$ (respectively, $B_m(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B;a_m)$ (respectively, $\mathcal{T}(B;b_m)$), where its root stays at level 0. Then, the rules can be translated as

$$A_m(x) = x + xA_{m+1}(x) + x\sum_{j=1}^{m} B_j(x), \qquad B_m(x) = x + x\sum_{j=1}^{m-1} B_j(x),$$

for all $m \geq 1$. Now, to solve such a system of recurrence relations, we use the kernel method (for example, see [7] and references therein). More precisely, we define $A(v) = \sum_{m\geq 1} A_m(x)v^{m-1}$ and $B(v) = \sum_{m\geq 1} B_m(x)v^{m-1}$. Then our recurrences can be written as

$$A(v) = \frac{x}{1-v} + \frac{x}{v}(A(x) - A(0)) + \frac{x}{1-v}B(v), \quad B(v) = \frac{x}{1-v} + \frac{xv}{1-v}B(v).$$

By taking $v = x$, we obtain

$$A(0) = \frac{x}{1-x}(1 + B(x)) = \frac{x}{1-x}\left(1 + \frac{x}{1-x-x^2}\right) = \frac{x(1+x)}{1-x-x^2}.$$

Hence, $F_{\{010,011,012,100,201\}}(x) = \frac{x(1+x)}{1-x-x^2}$. For other examples, see the end of the next sections, and as well as we refer the reader to [2, 4, 8].

`Full procedure`. Now, we are ready to present our main procedure.

- Fix d such that $5 \leq d \leq 13$.
- We denote the set of all sets of d length-3 patterns by $\mathcal{P}_d$. Create all the sets of d patterns in $\mathcal{P}_d$. Clearly, we have $|\mathcal{P}_d| = \binom{13}{d}$.
- For any set of d length-3 patterns B, find the number of inversion sequences in $\mathbf{I}_n(B)$, for $n = 0, 1, \ldots, 9$. We present the output as a table of $\binom{13}{d}$ lines such that each line has the form $I_n(B) = \{|I_n(B)|\}_{n=0}^{9}$. By sorting this table with respect to the sequences $\{|I_n(B)|\}_{n=0}^{9}$ and separating different sequences by a space line, we complete the first step. Here, we call this table by d-sorted table and present it as a text file called InvSeqDataSorted[d].txt; see [10].
- For any set of patterns B in the file InvSeqDataSorted[d].txt, we apply Main-Procedure for finding $GT_\ell(B)$ (with small ℓ). The output of all $GT_\ell(B)$ with $B \in \mathcal{P}_d$ is stored in a pdf file called InvSeqAvoid[d]pat3pdf.pdf; see [10]. Note that if Main-Procedure shows that $T(B)$ has a finite number of labels, then we found all the succession rules of $T(B)$ and we wrote "DONE" at the end of the output of each class.
- By considering the rules $GT_\ell(B)$ in InvSeqAvoid[d]pat3pdf.pdf, we guess and prove (if possible) all the rules of the generating tree $\mathcal{T}(B)$. In the case we failed to guess/prove, we wrote nothing in the tables; see [10].
- By translating the rules of the generating tree $\mathcal{T}(B)$ to recurrence relations, and solving (if possible) for the generating function

  $$F_B(x) = \sum_{n \geq 0} |\mathbf{I}_n(B)| x^{n+1},$$

  the generating function for the number of nodes at level n in $\mathcal{T}(B)$, where the root stays at level 0. Again, if we fail to find $F_B(x)$, we write nothing in the tables; see [10].
- Now, we create a table, called d-Table, which presents all distinct I-Wilf-equivalences of sets of d length-3 patterns, which contains w_d lines and four columns, where the first column presents a number of the I-Wilf-equivalence, the second column presents the set of length-3 patterns, the third column presents the rules of $\mathcal{T}(B)$ (if we succeeded to find them), and the fourth column presents the generating function $F_B(x)$ (if we succeeded to find it) or a reference for $F_B(x)$.

The aims of this paper are to find all the d-Tables with $d = 5, 6, \ldots, 13$. Since we have very long tables and all the computations for finding $F_B(x)$ (if possible) very similar to the techniques and methods in [2,4], we present the proof details only for the few cases and mostly those cases that our algorithm does not lead to an explicit formula for the generating function $F_B(x)$.

3. 5-Table

In this section, we show that $w_5 = 219$. Moreover, by our procedure, we present the generating function for many of the 219 I-Wilf-equivalences. Actually, we see that there are only 162 sets among $\binom{13}{5} = 1287$ sets of 5 length-3 patterns that cannot be reduced to smaller sets of length-3 patterns. In Table 1, we present all the I-Wilf-equivalences of sets of 5 length-3 patterns. As we mentioned, due to the similarity with the work in [2,4], we only present the case, where we failed to apply the KMY algorithm, that is, we present bijections between some classes of inversion sequences with respect to left-right-maxima structure; see the theorems at end of this section.

Theorem 3. *Let* $B = \{010, 101, 102, 201, 210\}$ *and* $C = \{011, 101, 110, 201, 210\}$. *Then*

$$F_B(x) = F_C(x) = \frac{x}{(1-x)\left(2 - \frac{1-\sqrt{1-4x}}{2x}\right)}.$$

Proof. First, we focus on finding an explicit formula for the generating function $F_C(x)$. By the procedure, we succeed in finding the rules of the generating tree $\mathcal{T}(C)$:

$$\begin{aligned}
a_m &\rightsquigarrow a_{m+1}, a_m, b_{m,2}, \dots, b_{m,m},\\
b_{m,j} &\rightsquigarrow a_{m+2-j}, (c_{m+2-j})^{j-1}, b_{m,j}, \dots, b_{m,m},\\
c_m &\rightsquigarrow (c_m)^2, d_{m,5}, \dots, d_{m,m+2},\\
d_{m,j} &\rightsquigarrow (c_{m+4-j})^{j-3}, d_{m,j}, \dots, d_{m,m+2},
\end{aligned}$$

where $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 12$, and $d_{m,j} = a_m 12j$.

Define $A_m(x)$ (respectively, $B_{m,j}(x)$, $C_m(x)$, and $D_{m,j}(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(C; a_m)$ (respectively, $\mathcal{T}(C; b_{m,j})$, $\mathcal{T}(C; c_m)$, $\mathcal{T}(C; d_{m,j})$), where its root stays at level 0. Thus, the above rules lead to

$$A_m(x) = x + xA_{m+1}(x) + xA_m(x) + x\sum_{j=2}^{m} B_{m,j}(x), \quad m \geq 1;$$

$$B_{m,j}(x) = x + xA_{m+2-j}(x) + (j-1)xC_{m+2-j}(x) + x\sum_{i=j}^{m} B_{m,i}(x), \quad 2 \leq j \leq m;$$

$$C_m(x) = x + 2xC_m(x) + x\sum_{i=5}^{m+2} D_{m,i}(x), \quad m \geq 2;$$

$$D_{m,j}(x) = x + (j-3)xC_{m+4-j}(x) + x\sum_{i=j}^{m+2} D_{m,i}(x), \quad 5 \leq j \leq m+2.$$

Now, we define the generating functions

$$A(v)=\sum_{m\geq 1}A_m(x)v^{m-1},\ C(v)=\sum_{m\geq 2}A_m(x)v^{m-2},$$

$$B(v,u)=\sum_{m\geq 2}\sum_{j=2}^{m}B_{m,j}(x)u^{m-j}v^{m-2},\ D(v,u)=\sum_{m\geq 3}\sum_{j=5}^{m+2}D_{m,j}(x)u^{m+2-j}v^{m-3}.$$

Hence, the last recurrence relations can be written as

$$A(v)=\frac{x}{1-v}+\frac{x}{v}(A(v)-A(0))+xA(v)+xvB(v,1), \tag{1}$$

$$\begin{aligned}B(v,u)=&\frac{x}{(1-v)(1-vu)}+\frac{x}{vu(1-v)}(A(vu)-A(0))\\&+\frac{x}{(1-v)^2}C(vu)+\frac{x}{1-u}(B(v,u)-uB(vu,1)),\end{aligned} \tag{2}$$

$$C(v)=\frac{x}{1-v}+2xC(v)+xvD(v,1), \tag{3}$$

$$D(v,u)=\frac{x}{(1-v)(1-vu)}+\frac{x(2-v)}{(1-v)^2}C(vu)+\frac{x}{1-u}(D(v,u)-uD(vu,1)). \tag{4}$$

From (3) and (4), we have

$$\begin{aligned}D(v,u)=&\frac{x}{(1-v)(1-vu)}+\frac{x(2-v)}{(1-v)^2}\left(\frac{x}{(1-vu)(1-2x)}+\frac{xvu}{1-2x}D(xu,1)\right)\\&+\frac{x}{1-u}(D(v,u)-uD(vu,1)).\end{aligned} \tag{5}$$

By taking $u=1-x$, and then replacing v by $v/(1-x)$, we obtain

$$D(v,1)=\frac{x}{(1-x)(1-2x-v)(1-v)}.$$

Thus, by (3), we have

$$C(v)=\frac{x(1-x-v)}{(1-x)(1-2x-v)(1-v)}.$$

Now, we assume that $A(0)=\frac{x}{(1-x)(2-\frac{1-\sqrt{1-4x}}{2x})}$. Then, by finding $A(v)$ from (1), (2) with $u=1-x$, gives

$$B(v,1)=\frac{x(x(1-v)(1-2x-v)-\sqrt{1-4x}(v^2x-2v^2-2vx+2v-x)}{2\sqrt{1-4x}(1-x)(1-v)(v^2-v+x)(1-2x-v)}.$$

By using expressions of $A(0)$ and $B(v,1)$, we obtain an explicit formula for $B(v,u)$. Then, by (1), we obtain

$$\begin{aligned}A(v)=&\frac{x^2(1-v)}{2\sqrt{1-4x}(1-x)(v^2-v+x)}\\&+\frac{x\sqrt{1-4x}(v^3x-2v^3-5v^2x+4v^2+3vx-2x^2-2v+x)}{2\sqrt{1-4x}(1-x)(1-v)(v^2-v+x)(1-2x-v)}.\end{aligned}$$

Note that the expressions of $A(v)$, $B(v,u)$, $C(v)$, and $D(v,u)$ satisfy (1)-(4). Moreover, by expression $A(v)$, we get that the expression of $FC(x)$ equals $A(0)$ as we assumed, which completes the first part of the proof.

Now, we focus on finding an explicit formula for the generating function $F_B(x)$. By the procedure, we succeed in finding the rules of the generating tree $\mathcal{T}(B)$:

$$\begin{aligned} a_m &\rightsquigarrow a_{m+1}, a_m, b_{m,2}, \dots, b_{m,m}, \\ b_{m,j} &\rightsquigarrow (0021)^{j-1}, b_{m+1,j}, b_{m,j}, \dots, b_{m,m}, \\ 0021 &\rightsquigarrow 0021, \end{aligned}$$

where $a_m = 0^m$ and $b_{m,j} = a_m j$. Define $A_m(x)$ (respectively, $B_{m,j}(x)$, $C(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B; a_m)$ (respectively, $\mathcal{T}(B; b_{m,j})$, $\mathcal{T}(B; 0021)$), where its root stays at level 0. Thus, the above rules lead to

$$\begin{aligned} A_m(x) &= x + xA_{m+1}(x) + xA_m(x) + x\sum_{j=2}^{m} B_{m,j}(x), \quad m \geq 1, \\ B_{m,j}(x) &= x + (j-1)xC(x) + xB_{m+1,j}(x) + x\sum_{i=j}^{m} B_{m,i}(x), \quad 2 \leq j \leq m, \\ C(x) &= x + xC(x). \end{aligned}$$

We define

$$A(v) = \sum_{m\geq 1} A_m(x)v^{m-1} \text{ and } B(v,u) = \sum_{m\geq 2}\sum_{j=2}^{m} B_{m,j}(x)u^{m-j}v^{m-2}.$$

Hence, the last recurrence relations can be written as

$$A(v) = \frac{x}{1-v} + \frac{x}{v}(A(v) - A(0)) + xA(v) + xvB(v,1), \tag{6}$$

$$\begin{aligned} B(v,u) =& \frac{x(1-v+vx)}{(1-x)(1-v)^2(1-vu)} + \frac{x}{vu(1-v)}(B(v,u) - B(v,0)) \\ &+ \frac{x}{1-u}(B(v,u) - uB(vu,1)), \end{aligned} \tag{7}$$

In order to solve this system, we assume that

$$A(0) = \frac{x}{(1-x)(2 - \frac{1-\sqrt{1-4x}}{2x})}, \tag{8}$$

$$B(0,0) = \frac{1}{2} - \frac{1-5x}{2(1-x)\sqrt{1-4x}}, \tag{9}$$

$$B(v,0) = \frac{B(0,0) - vA(0)}{(1-v)^2}. \tag{10}$$

Note that the solution (if it exists) of (6)-(10) gives a solution for (6)-(7). From (7), we have

$$\begin{aligned} B(v,u/v) =& \frac{x(1-v+vx)}{(1-x)(1-v)^2(1-u)} + \frac{x}{u(1-v)}\left(B(v,u/v) - \frac{B(0,0)-vA(0)}{(1-v)^2}\right) \\ &+ \frac{x}{v-u}(vB(v,u/v) - uB(u,1)). \end{aligned}$$

So by taking $v = \frac{u(x-u)}{ux-u+x}$, we obtain

$$\begin{aligned} B(v,1) =& \frac{vx(v-x)A(0)}{(v^2-v+x)^2} + \frac{x(vx-v+x)B(0,0)}{(v^2-v+x)^2} \\ &+ \frac{v(v^2x-vx^2-v^2+v-x)}{(v^2-v+x)^2(1-x)(1-v)}. \end{aligned}$$

Hence,

$$B(x/(1-x),1) = \frac{(1-x)^2\sqrt{1-4x}-(1-x)(1-3x)}{x(1-2x)(1-4x)-x(1-2x)\sqrt{1-4x}}.$$

By (6) with $v = x/(1-x)$, we obtain that $A(0) = \frac{x}{1-2x} + \frac{x^2}{(1-x)^2}B(x/(1-x),1)$, which equals the same expression as is assumed in (8). It is not hard to get explicit formulas for $A(v)$ and $B(v,u)$ from (6)-(7), which presents a solution that satisfies (6)-(10) (here we omit the long explicit formulas of $A(v)$ and $B(v,u)$, to save space). □

Table 1: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 5$.

Beginning of Table 1			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012●	$0 \rightsquigarrow (00)^2$	$x+2x^2$
2	000,001,010,012,021 000,001,010,012,100 000,001,010,012,101 000,001,010,012,102 000,001,010,012,110 000,001,010,012,120 000,001,010,012,201 000,001,010,012,210 000,001,011,012,021 000,001,011,012,100 000,001,011,012,101 000,001,011,012,102 000,001,011,012,110 000,001,011,012,120 000,001,011,012,201 000,001,011,012,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x(1+x)^2$
3	000,001,010,011,021 000,001,010,011,100 000,001,010,011,101 000,001,010,011,102 000,001,010,011,110 000,001,010,011,120 000,001,010,011,201 000,001,010,011,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021 001,010,011,012,100 001,010,011,012,101 001,010,011,012,102 001,010,011,012,110 001,010,011,012,120 001,010,011,012,201 001,010,011,012,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	$x^2 + \frac{x}{1-x}$
4	000,001,012,021,110 000,001,012,100,110 000,001,012,101,110 000,001,012,102,110 000,001,012,110,120 000,001,012,110,201 000,001,012,110,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000,010,011,012,021●	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x+2x^2+2x^3$

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
5	000,001,012,021,100		
	000,001,012,021,101		
	000,001,012,021,102		
	000,001,012,021,120		
	000,001,012,021,201		
	000,001,012,021,210		
	000,001,012,100,101		
	000,001,012,100,102		
	000,001,012,100,120		
	000,001,012,100,201		
	000,001,012,100,210		
	000,001,012,101,102		
	000,001,012,101,120		
	000,001,012,101,201		
	000,001,012,101,210		
	000,001,012,102,120		
	000,001,012,102,201		
	000,001,012,102,210		
	000,001,012,120,201		
	000,001,012,120,210		
	000,001,012,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;\ 011 \rightsquigarrow 00$	
	000,010,011,012,100		
	000,010,011,012,101		
	000,010,011,012,102		
	000,010,011,012,110		
	000,010,011,012,120		
	000,010,011,012,201		
	000,010,011,012,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
6	000,001,011,021,120		
	000,001,011,100,120		
	000,001,011,101,120		
	000,001,011,102,120		
	000,001,011,110,120		
	000,001,011,120,201		
	000,001,011,120,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 012;\ 012 \rightsquigarrow 012$	
	001,011,012,021,100		
	001,011,012,100,101		
	001,011,012,100,102		
	001,011,012,100,110		
	001,011,012,100,120		
	001,011,012,100,201		
	001,011,012,100,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010$	$x^2 + x^3 + \frac{x}{1-x}$
7	000,001,010,021,100		
	000,001,010,021,101		
	000,001,010,021,102		
	000,001,010,021,110		
	000,001,010,021,120		
	000,001,010,021,201		
	000,001,010,021,210		
	000,001,010,100,101		
	000,001,010,100,102		
	000,001,010,100,110		
	000,001,010,100,120		
	000,001,010,100,201		
	000,001,010,100,210		
	000,001,010,101,102		
	000,001,010,101,110		
	000,001,010,101,120		
	000,001,010,101,201		
	000,001,010,101,210		
	000,001,010,102,110		
	000,001,010,102,120		
	000,001,010,102,201		
	000,001,010,102,210		
	000,001,010,110,120		
	000,001,010,110,201		
	000,001,010,110,210		
	000,001,010,120,201		
	000,001,010,120,210		
	000,001,010,201,210		
	000,001,011,021,100		
	000,001,011,021,101		
	000,001,011,021,102		
	000,001,011,021,110		
	000,001,011,021,201		
	000,001,011,021,210		
	000,001,011,100,101		
	000,001,011,100,102		
	000,001,011,100,110		
	000,001,011,100,201		
	000,001,011,100,210		
	000,001,011,101,102		
	000,001,011,101,110		
	000,001,011,101,201		
	000,001,011,101,210		
	000,001,011,102,110		
	000,001,011,102,201		
	000,001,011,102,210		
	000,001,011,110,201		
	000,001,011,110,210		
	000,001,011,201,210	$0 \rightsquigarrow 00, 0$	
	001,010,011,021,100		
	001,010,011,021,101		
	001,010,011,021,102		
	001,010,011,021,110		
	001,010,011,021,120		
	001,010,011,021,201		
	001,010,011,021,210		
	001,010,011,100,101		
	001,010,011,100,102		
	001,010,011,100,110		
	001,010,011,100,120		
	001,010,011,100,201		
	001,010,011,100,210		
	001,010,011,101,102		
	001,010,011,101,110		
	001,010,011,101,120		
	001,010,011,101,201		
	001,010,011,101,210		
	001,010,011,102,110		
	001,010,011,102,120		
	001,010,011,102,201		
	001,010,011,102,210		
	001,010,011,110,120		
	001,010,011,110,201		
	001,010,011,110,210		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,010,011,120,201 001,010,011,120,210 001,010,011,201,210 001,010,012,021,100 001,010,012,021,101 001,010,012,021,102 001,010,012,021,110 001,010,012,021,120 001,010,012,021,201 001,010,012,021,210 001,010,012,100,101 001,010,012,100,102 001,010,012,100,110 001,010,012,100,120 001,010,012,100,201 001,010,012,100,210 001,010,012,101,102 001,010,012,101,110 001,010,012,101,120 001,010,012,101,201 001,010,012,101,210 001,010,012,102,110 001,010,012,102,120 001,010,012,102,201 001,010,012,102,210 001,010,012,110,120 001,010,012,110,201 001,010,012,110,210 001,010,012,120,201 001,010,012,120,210 001,010,012,201,210 001,011,012,021,101 001,011,012,021,102 001,011,012,021,110 001,011,012,021,120 001,011,012,021,201 001,011,012,021,210 001,011,012,101,102 001,011,012,101,110 001,011,012,101,120 001,011,012,101,201 001,011,012,101,210 001,011,012,102,110 001,011,012,102,120 001,011,012,102,201 001,011,012,102,210 001,011,012,110,120 001,011,012,110,201 001,011,012,110,210 001,011,012,120,201 001,011,012,120,210 001,011,012,201,210	$0 \rightsquigarrow (00)^2;\ 00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
8	000,011,012,021,100 000,011,012,021,101 000,011,012,021,102 000,011,012,021,110 000,011,012,021,120 000,011,012,021,201 000,011,012,021,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (001)^2;\ 01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
9	000,011,012,100,101 000,011,012,100,102 000,011,012,100,110 000,011,012,100,120 000,011,012,100,201 000,011,012,100,210 000,011,012,101,102 000,011,012,101,110 000,011,012,101,120 000,011,012,101,201 000,011,012,101,210 000,011,012,102,110 000,011,012,102,120 000,011,012,102,201 000,011,012,102,210 000,011,012,110,120 000,011,012,110,201 000,011,012,110,210 000,011,012,120,201 000,011,012,120,210 000,011,012,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 01;\ 01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
10	000,010,012,021,100 000,010,012,021,101 000,010,012,021,102 000,010,012,021,110 000,010,012,021,120 000,010,012,021,201 000,010,012,021,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01)62;\ 01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
11	000,001,021,110,120•	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2, 012;\ 012 \rightsquigarrow 00, 012$	
	001,011,021,100,120 001,011,100,101,120 001,011,100,102,120 001,011,100,110,120 001,011,100,120,201 001,011,100,120,210 001,012,021,100,110 001,012,100,101,110 001,012,100,102,110 001,012,100,110,120 001,012,100,110,201 001,012,100,110,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010, 00$	$x + x^3 + \frac{2x^2}{1-x}$
12	000,010,012,100,110•	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 01 \rightsquigarrow 011;$ $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 3x^3 + 3x^4$
13	000,010,012,100,101 000,010,012,100,102 000,010,012,100,120 000,010,012,100,201 000,010,012,100,210 000,010,012,101,110 000,010,012,102,110 000,010,012,110,120 000,010,012,110,201		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,012,110,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 011$	$x + 2x^2 + 3x^3 + 3x^4 + x^5$
14	000,010,012,101,102		
	000,010,012,101,120		
	000,010,012,101,201		
	000,010,012,101,210		
	000,010,012,102,120		
	000,010,012,102,201		
	000,010,012,102,210		
	000,010,012,120,201		
	000,010,012,120,210		
	000,010,012,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 0022$; $0022 \rightsquigarrow 01$	$x+2x^2+3x^3+3x^4+2x^5+x^6$
15	000,001,021,100,120		
	000,001,021,101,120		
	000,001,021,102,120		
	000,001,021,120,201		
	000,001,021,120,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 012$; $011 \rightsquigarrow 00$; $012 \rightsquigarrow 00, 012$	$x + x^3 + x^4 + \frac{2x^2}{1-x}$
16	000,001,021,100,110		
	000,001,021,101,110		
	000,001,021,102,110		
	000,001,021,110,201		
	000,001,021,110,210		
	000,001,100,110,120		
	000,001,101,110,120		
	000,001,102,110,120		
	000,001,110,120,201		
	000,001,110,120,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow (00)^2, 01$	
	001,011,021,100,101		
	001,011,021,100,102		
	001,011,021,100,110		
	001,011,021,100,201		
	001,011,021,100,210		
	001,011,100,101,102		
	001,011,100,101,110		
	001,011,100,101,201		
	001,011,100,101,210		
	001,011,100,102,110		
	001,011,100,102,201		
	001,011,100,102,210		
	001,011,100,110,201		
	001,011,100,110,210		
	001,011,100,201,210		
	001,012,021,100,101		
	001,012,021,100,102		
	001,012,021,100,120		
	001,012,021,100,201		
	001,012,021,100,210		
	001,012,100,101,102		
	001,012,100,101,120		
	001,012,100,101,201		
	001,012,100,101,210		
	001,012,100,102,120		
	001,012,100,102,201		
	001,012,100,102,210		
	001,012,100,120,201		
	001,012,100,120,210		
	001,012,100,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 01$	
	001,011,021,101,120		
	001,011,021,102,120		
	001,011,021,110,120		
	001,011,021,120,201		
	001,011,021,120,210		
	001,011,101,102,120		
	001,011,101,110,120		
	001,011,101,120,201		
	001,011,101,120,210		
	001,011,102,110,120		
	001,011,102,120,201		
	001,011,102,120,210		
	001,011,110,120,201		
	001,011,110,120,210		
	001,011,120,201,210		
	001,012,021,101,110		
	001,012,021,102,110		
	001,012,021,110,120		
	001,012,021,110,201		
	001,012,021,110,210		
	001,012,101,102,110		
	001,012,101,110,120		
	001,012,101,110,201		
	001,012,101,110,210		
	001,012,102,110,120		
	001,012,102,110,201		
	001,012,102,110,210		
	001,012,110,120,201		
	001,012,110,120,210		
	001,012,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2$	
17	000,001,021,100,101		
	000,001,021,100,102		
	000,001,021,100,201		
	000,001,021,100,210		
	000,001,021,101,102		
	000,001,021,101,201		
	000,001,021,101,210		
	000,001,021,102,201		
	000,001,021,102,210		
	000,001,021,201,210		
	000,001,100,101,120		
	000,001,100,102,120		
	000,001,100,120,201		
	000,001,100,120,210		
	000,001,101,102,120		
	000,001,101,120,201		
	000,001,101,120,210		
	000,001,102,120,201		
	000,001,102,120,210		
	000,001,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 01$; $011 \rightsquigarrow 00$	$x + 2x^2 + 3x^3 + \frac{4x^4}{1-x}$
18	000,001,100,101,110		
	000,001,100,102,110		
	000,001,100,110,201		
	000,001,100,110,210		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,101,102,110		
	000,001,101,110,201		
	000,001,101,110,210		
	000,001,102,110,201		
	000,001,102,110,210		
	000,001,110,201,210	$a_m \rightsquigarrow (00)^{m+1}, a_{m+1}$; $a_m = 01\cdots m$	
	000,010,011,021,100		
	000,010,011,021,101		
	000,010,011,021,102		
	000,010,011,021,110		
	000,010,011,021,120		
	000,010,011,021,201		
	000,010,011,021,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 01$	
	001,010,021,100,101		
	001,010,021,100,102		
	001,010,021,100,110		
	001,010,021,100,120		
	001,010,021,100,201		
	001,010,021,100,210		
	001,010,021,101,102		
	001,010,021,101,110		
	001,010,021,101,120		
	001,010,021,101,201		
	001,010,021,101,210		
	001,010,021,102,110		
	001,010,021,102,120 •		
	001,010,021,102,201		
	001,010,021,102,210		
	001,010,021,110,120		
	001,010,021,110,201		
	001,010,021,110,210		
	001,010,021,120,201		
	001,010,021,120,210		
	001,010,021,201,210		
	001,010,100,101,102		
	001,010,100,101,110		
	001,010,100,101,120		
	001,010,100,101,201		
	001,010,100,101,210		
	001,010,100,102,110		
	001,010,100,102,120		
	001,010,100,102,201		
	001,010,100,102,210		
	001,010,100,110,120		
	001,010,100,110,201		
	001,010,100,110,210		
	001,010,100,120,201		
	001,010,100,120,210		
	001,010,100,201,210		
	001,010,101,102,110		
	001,010,101,102,120		
	001,010,101,102,201		
	001,010,101,102,210		
	001,010,101,110,120		
	001,010,101,110,201		
	001,010,101,110,210		
	001,010,101,120,201		
	001,010,101,120,210		
	001,010,101,201,210		
	001,010,102,110,120		
	001,010,102,110,201		
	001,010,102,110,210		
	001,010,102,120,201		
	001,010,102,120,210		
	001,010,102,201,210		
	001,010,110,120,201		
	001,010,110,120,210		
	001,010,110,201,210		
	001,010,120,201,210		
	001,011,021,101,102		
	001,011,021,101,110		
	001,011,021,101,201		
	001,011,021,101,210		
	001,011,021,102,110		
	001,011,021,102,201		
	001,011,021,102,210		
	001,011,021,110,201		
	001,011,021,110,210		
	001,011,021,201,210		
	001,011,101,102,110		
	001,011,101,102,201		
	001,011,101,102,210		
	001,011,101,110,201		
	001,011,101,110,210		
	001,011,101,201,210		
	001,011,102,110,201		
	001,011,102,110,210		
	001,011,102,201,210		
	001,011,110,201,210		
	001,012,021,101,102		
	001,012,021,101,120		
	001,012,021,101,201		
	001,012,021,101,210		
	001,012,021,102,120		
	001,012,021,102,201		
	001,012,021,102,210		
	001,012,021,120,201		
	001,012,021,120,210		
	001,012,021,201,210		
	001,012,101,102,120		
	001,012,101,102,201		
	001,012,101,102,210		
	001,012,101,120,201		
	001,012,101,120,210		
	001,012,101,201,210		
	001,012,102,120,201		
	001,012,102,120,210		
	001,012,102,201,210		
	001,012,120,201,210	$0 \rightsquigarrow 00, 0$; $00 \rightsquigarrow 00$	
	010,011,012,021,100		
	010,011,012,021,101		
	010,011,012,021,102		
	010,011,012,021,110		
	010,011,012,021,120		
	010,011,012,021,201		
	010,011,012,021,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $a_m = 0^m$	$\frac{x}{(1-x)^2}$

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
19	000,010,011,102,120●	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
20	000,001,100,101,210 000,001,100,102,210 000,001,100,201,210 000,001,101,102,210 000,001,101,201,210 000,001,102,201,210	$a_m \rightsquigarrow (00)^m, b_m, a_{m+1}$; $b_m \rightsquigarrow (00)^m$, $a_m = 01\cdots m$, $b_m = a_m m$	
	000,010,011,100,102 000,010,011,101,102 000,010,011,102,110 000,010,011,102,201 000,010,011,102,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 002$	$\frac{x(1+x^3)}{(1-x)^2}$
21	010,011,012,100,210● 010,011,012,101,210 010,011,012,102,210 010,011,012,110,210 010,011,012,120,210 010,011,012,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (01)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
22	000,001,100,101,102 000,001,100,101,201 000,001,100,102,201 000,001,101,102,201	$a_m \rightsquigarrow b_0, \ldots, b_m, a_{m+1}$; $b_m \rightsquigarrow b_0, \ldots, b_{m-1}$, $a_m = 01\cdots m$, $b_m = a_m m$	
	000,010,011,100,120 000,010,011,101,120 000,010,011,110,120 000,010,011,120,201 000,010,011,120,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 0$; $01 \rightsquigarrow 01$	
	010,011,012,100,101 010,011,012,100,102 010,011,012,100,110 010,011,012,100,120 010,011,012,100,201 010,011,012,101,102 010,011,012,101,110 010,011,012,101,120 010,011,012,101,201 010,011,012,102,110 010,011,012,102,120 010,011,012,102,201 010,011,012,110,120 010,011,012,110,201 010,011,012,120,201	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow b_1, \ldots, b_{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1+x)}{1-x-x^2}$
23	000,010,011,100,101 000,010,011,100,110 000,010,011,100,201 000,010,011,100,210 000,010,011,101,110 000,010,011,101,201 000,010,011,101,210 000,010,011,110,201 000,010,011,110,210 000,010,011,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (00)^2$; $01 \rightsquigarrow 01$	$\frac{x(1-x-x^2)}{(1-x)(1-2x)}$
24	000,010,100,102,120●		
25	000,010,102,110,120●		
26	000,010,101,102,120●		
27	000,010,100,102,110●		
28	000,010,101,102,110●		
29	000,010,100,101,102●		
30	000,010,102,120,201●		
31	000,010,102,120,210●		
32	000,010,102,110,201●		
33	000,010,102,110,210●		
34	000,010,100,102,210●		
35	000,010,100,102,201●		
36	000,010,101,102,210●		
37	000,010,101,102,201●		
38	000,010,100,101,110●		
39	000,010,100,101,120● 000,010,100,110,120●		Theorem 4
40	000,010,101,110,120●		
41	000,010,102,201,210●		
42	000,010,100,120,201●		
43	000,010,100,120,210●		
44	000,010,100,110,201● 000,010,100,110,210● 000,010,101,110,201●	$a_{m,j} \rightsquigarrow$ $\{a_{2m+i-j,2m+2i-1-j}\}_{i=1}^{j-m}, b_{2m-j}$, $\{a_{m,i}\}_{i=j}^{2m}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$, $a_{m,j} = 0^2\cdots(m-1)^2 j$, $b_m = 0^2\cdots m^2$	
45	000,010,101,110,210●		
46	000,010,110,120,201●		
47	000,010,101,120,201● 000,010,101,120,210● 000,010,110,120,210●		Theorem 5
48	000,010,100,101,201●		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,100,101,210●	$a_{m,j} \rightsquigarrow \{b_{2m+i-j,2m+2i-j}\}_{i=0}^{j-m}, \{a_{m,i}\}_{i=j}^{2m}$; $b_{m,j} \rightsquigarrow \{b_{2m+i-j,2m+2i-j-1}\}_{i=1}^{j-m}$, $\{a_{m+1,i}\}_{i=j+1}^{2m+2}$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_{m,j} = a_{m,j}j$	
49	000,010,120,201,210●		
50	000,010,110,201,210●		
51	000,010,100,201,210●		
52	000,010,101,201,210●		
53	000,012,021,101,110●	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001)^2$; $01 \rightsquigarrow 010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^4$
54	000,012,021,100,101 000,012,021,101,102 000,012,021,101,120 000,012,021,101,201 000,012,021,101,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$, $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000,012,021,100,110 000,012,021,102,110 000,012,021,110,120 000,012,021,110,201 000,012,021,110,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	
	000,012,100,101,110●	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 3x^4$
55	000,012,101,102,110 000,012,101,110,120 000,012,101,110,201 000,012,101,110,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 010$	$x + 2x^2 + 4x^3 + 3x^4 + x^5$
56	000,012,021,100,102 000,012,021,100,120 000,012,021,100,201 000,012,021,100,210 000,012,021,102,120 000,012,021,102,201 000,012,021,102,210 000,012,021,120,201 000,012,021,120,210 000,012,021,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$	
	000,012,100,102,110 000,012,100,110,120 000,012,100,110,201 000,012,100,110,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$; $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 4x^3 + 4x^4$
57	000,012,100,101,102 000,012,100,101,120 000,012,100,101,201 000,012,100,101,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000,012,102,110,120 000,012,102,110,201 000,012,102,110,210 000,012,110,120,201 000,012,110,120,210 000,012,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	$x + 2x^2 + 4x^3 + 4x^4 + x^5$
58	000,012,101,102,120 000,012,101,102,201 000,012,101,102,210 000,012,101,120,201 000,012,101,120,210 000,012,101,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 0022$; $0022 \rightsquigarrow 001$	$x+2x^2+4x^3+4x^4+2x^5+x^6$
59	000,011,021,102,120●	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	
	001,021,100,110,120●	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2-2x^3+x^4)}{(1-x)^2}$
60	000,012,100,102,120 000,012,100,102,201 000,012,100,102,210 000,012,100,120,201 000,012,100,120,210 000,012,100,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$; $002 \rightsquigarrow 0011, 001$	$x + 2x^2 + 4x^3 + 5x^4 + x^5$
61	000,012,102,120,201 000,012,102,120,210 000,012,102,201,210 000,012,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$; $002 \rightsquigarrow 001, 0022$; $0022 \rightsquigarrow 001$	$x+2x^2+4x^3+5x^4+2x^5+x^6$
62	000,011,021,100,102 000,011,021,101,102 000,011,021,102,110 000,011,021,102,201 000,011,021,102,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 01$; $002 \rightsquigarrow 002$	
	000,011,100,102,120 000,011,101,102,120 000,011,102,110,120 000,011,102,120,201 000,011,102,120,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 012$; $012 \rightsquigarrow 012$	
	001,021,100,101,110 001,021,100,102,110 001,021,100,110,201 001,021,100,110,210 001,100,101,110,120 001,100,102,110,120 001,100,110,120,201 001,100,110,120,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 01$	
	001,021,100,101,120 001,021,100,102,120 001,021,100,120,201		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,021,100,120,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 012$; $011 \rightsquigarrow 010, 011$; $012 \rightsquigarrow 00, 012$	
	001,021,101,110,120 001,021,102,110,120 001,021,110,120,201 001,021,110,120,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2-x^3)}{(1-x)^2}$
63	000,011,021,100,120 000,011,021,101,120 000,011,021,110,120 000,011,021,120,201 000,011,021,120,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	000,011,100,101,102 000,011,100,102,110 000,011,100,102,201 000,011,100,102,210 000,011,101,102,110 000,011,101,102,201 000,011,101,102,210 000,011,102,110,201 000,011,102,110,210 000,011,102,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 01$	
	001,021,100,101,102 001,021,100,101,201 001,021,100,101,210 001,021,100,102,201 001,021,100,102,210 001,021,100,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 01$; $011 \rightsquigarrow 010, 011$	
	001,021,101,102,110 001,021,101,110,201 001,021,101,110,210 001,021,102,110,201 001,021,102,110,210 001,021,110,201,210 001,101,102,110,120 001,101,110,120,201 001,101,110,120,210 001,102,110,120,201 001,102,110,120,210 001,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 01$	
	001,021,101,102,120 001,021,101,120,201 001,021,101,120,210 001,021,102,120,201 001,021,102,120,210 001,021,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 00, (011)^2$; $011 \rightsquigarrow 00, 011$	
	001,100,101,102,110 001,100,101,110,201 001,100,101,110,210 001,100,102,110,201 001,100,102,110,210 001,100,110,201,210	$a_m \rightsquigarrow (01)^m, 00, a_{m+1}$; $00 \rightsquigarrow 00$, $a_m = 01 \cdots m$	
	001,100,101,102,120 001,100,101,120,201 001,100,101,120,210 001,100,102,120,201 001,100,102,120,210 001,100,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 01$; $011 \rightsquigarrow 010, 011$	
	011,012,021,100,101 011,012,021,100,102 011,012,021,100,110 011,012,021,100,120 011,012,021,100,201 011,012,021,100,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010$, $a_m = 0^m$	$\frac{x(1+x^2)}{(1-x)^2}$
64	000,011,021,100,101 000,011,021,100,110 000,011,021,100,201 000,011,021,100,210 000,011,021,101,110 000,011,021,101,201 000,011,021,101,210 000,011,021,110,201 000,011,021,110,210 000,011,021,201,210	$0 \rightsquigarrow 00, 0$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	001,021,101,102,201 001,021,101,102,210 001,021,101,201,210 001,021,102,201,210 001,101,102,120,201 001,101,102,120,210 001,101,120,201,210 001,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 00, 011, 01$; $011 \rightsquigarrow 00, 011$	
	001,100,101,102,210 001,100,101,201,210 001,100,102,201,210	$a_m \rightsquigarrow (010)^m, b_m, a_{m+1}$; $b_m \rightsquigarrow (010)^m, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	001,101,102,110,201 001,101,102,110,210 001,101,110,201,210 001,102,110,201,210	$a_m \rightsquigarrow (00)^{m+1}, a_{m+1}$; $00 \rightsquigarrow 00$, $a_m = 0^m$	
	010,012,021,100,101 010,012,021,100,102 010,012,021,100,110 010,012,021,100,120 010,012,021,100,201 010,012,021,100,210 010,012,021,101,102 010,012,021,101,110 010,012,021,101,120 010,012,021,101,201 010,012,021,101,210 010,012,021,102,110 010,012,021,102,120 010,012,021,102,201 010,012,021,102,210		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,012,021,110,120 010,012,021,110,201 010,012,021,110,210 010,012,021,120,201 010,012,021,120,210 010,012,021,201,210 011,012,021,101,102 011,012,021,101,110 011,012,021,101,120 011,012,021,101,201 011,012,021,101,210 011,012,021,102,110 011,012,021,102,120 011,012,021,102,201 011,012,021,102,210 011,012,021,110,120 011,012,021,110,201 011,012,021,110,210 011,012,021,120,201 011,012,021,120,210 011,012,021,201,210	$a_m \rightsquigarrow (01)^{m+1}, a_{m+1}$; $01 \rightsquigarrow 01$, $a_m = 0^m$	
	011,012,100,201,210 •	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^2)}{(1-x)^3}$
65	011,012,100,101,210 011,012,100,102,210 011,012,100,110,210 011,012,100,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow 01, 010$; $b_m \rightsquigarrow c_m, (010)^{m-1}$; $c_m \rightsquigarrow (010)^{m-1}$; $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^4+x^2-x+1)}{(1-x)^3}$
66	000,011,100,101,120 000,011,100,110,120 000,011,100,120,201 000,011,100,120,210 000,011,101,110,120 000,011,101,120,201 000,011,101,120,210 000,011,110,120,201 000,011,110,120,210 000,011,120,201,210	$0ru0, 01$; $01 \rightsquigarrow 0, 012$; $012 \rightsquigarrow 012$	
	001,100,101,102,201	$a_m \rightsquigarrow a_{m+1}, b_m, \{c_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 01\cdots m$, $b_m = a_m m$, $c_m = a_m(m-1)$	
	011,012,100,101,201 011,012,100,102,201 011,012,100,110,201 011,012,100,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x}{(1-x)(1-x-x^2)}$
67	011,012,100,101,102 011,012,100,101,110 011,012,100,101,120 011,012,100,102,110 011,012,100,102,120 011,012,100,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow (01)^2$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m-1}$; $c_m \rightsquigarrow c_1, 01, \{c_i\}_{i=3}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x^2-x^3)}{(1-x-x^2)^2}$
68	010,012,100,110,210 • 011,012,101,201,210 011,012,102,201,210 011,012,110,201,210 011,012,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (0021)^{m-1}, b_1$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^2+x^3)}{(1-x)^3}$
69	010,012,100,101,110 010,012,100,102,110 010,012,100,110,120 010,012,100,110,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=2}^m, b_1$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x((1-x)^2+x^3-2x^4)}{(1-x)^3(1-x-x^2)}$
70	001,101,102,201,210	$a_m \rightsquigarrow a_{m+1}, (b_0)^m, b_m$; $b_m \rightsquigarrow (b_0)^m, b_m$, $a_m = 01\cdots m$, $b_m = a_m m$	
	010,012,100,101,210 010,012,100,102,210 010,012,100,120,210 010,012,100,201,210 011,012,101,102,210 011,012,101,110,210 011,012,101,120,210 011,012,102,110,210 011,012,102,120,210 011,012,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (0021)^{m-1}, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	010,012,101,110,210 010,012,102,110,210 010,012,110,120,210 010,012,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (b_1)^m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-2x+2x^2)}{(1-x)^4}$
71	010,012,100,101,102 010,012,100,101,120 010,012,100,101,201 010,012,100,102,120 010,012,100,102,201		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,012,100,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=2}^{m}, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	
	010,012,101,102,110 010,012,101,110,120 010,012,101,110,201 010,012,102,110,120 010,012,102,110,201 010,012,110,120,201 011,012,101,102,201 011,012,101,110,201 011,012,101,120,201 011,012,102,110,201 011,012,102,120,201 011,012,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, b_1$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^3)}{(1-x-x^2)(1-x)^2}$
72	010,012,101,102,210 010,012,101,120,210 010,012,101,201,210 010,012,102,120,210 010,012,102,201,210 010,012,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (b_1)^{m-1}, b_m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-3x+4x^2-2x^3+x^4)}{(1-x)^5}$
73	000,011,100,101,110 000,011,100,101,201 000,011,100,101,210 000,011,100,110,201 000,011,100,110,210 000,011,100,201,210 000,011,101,110,201 000,011,101,110,210 000,011,101,201,210 000,011,110,201,210	$0 \rightsquigarrow (0)^2$	
	010,011,021,100,101 010,011,021,100,102 010,011,021,100,110 010,011,021,100,120 010,011,021,100,201 010,011,021,100,210 010,011,021,101,102 010,011,021,101,110 010,011,021,101,120 010,011,021,101,201 010,011,021,101,210 010,011,021,102,110 010,011,021,102,120 010,011,021,102,201 010,011,021,102,210 010,011,021,110,120 010,011,021,110,201 010,011,021,110,210 010,011,021,120,201 010,011,021,120,210 010,011,021,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$	
	010,012,101,102,120 010,012,101,102,201 010,012,101,120,201 010,012,102,120,201 011,012,101,102,110 011,012,101,102,120 011,012,101,110,120 011,012,102,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x}{1-2x}$
74	010,011,102,120,201•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j,1}$, $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	
	010,011,102,120,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j,1}$, $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m1$	$\frac{x(2x^3-2x+1)}{(1-x)(1-2x)(1-x-x^2)}$
75	010,011,100,102,120• 010,011,101,102,120 010,011,102,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_{j,i}\}_{i=1}^{j-2}, c_{j,1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_{m,1} \rightsquigarrow \{c_{m-1,i}\}_{i=1}^{m-3}, c_{m-1,1}$; $c_{m,j} \rightsquigarrow \{c_{j,i}\}_{i=1}^{j-2}, c_{j,1}, \{c_{m-j,i}\}_{i=1}^{m-2-j}, c_{m-j,1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = a_m mj$	$\frac{x(3x^3-x^2-2x+1)}{(1-x^2)(1-2x)^2}$
76	010,011,102,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, \{b_{m,i}\}_{i=j}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(x^3+2x^2-3x+1)}{(1-x)(1-2x)^2}$

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
77	010,011,100,102,210 010,011,101,102,210 010,011,102,110,210	$\mathcal{T}(\{010, 011, 102, 210\})$	
78	010,011,100,102,201 010,011,101,102,201 010,011,102,110,201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m,i}\}_{i=j}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$	$\frac{x(1-2x)}{(1-x)(1-2x-x^2)}$
79	010,011,100,101,102 010,011,100,102,110 010,011,101,102,110	$\mathcal{T}(\{010, 011, 102\})$	
80	000,010,021,100,101 000,010,021,100,102 000,010,021,100,110 000,010,021,100,120 000,010,021,100,201 000,010,021,100,210 000,010,021,101,102 000,010,021,101,110 000,010,021,101,120 000,010,021,101,201 000,010,021,101,210 000,010,021,102,110 000,010,021,102,120 000,010,021,102,201 000,010,021,102,210 000,010,021,110,120 000,010,021,110,201 000,010,021,110,210 000,010,021,120,201 000,010,021,120,210 000,010,021,201,210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^{m}$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - x$
81	010,011,120,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(b_{m+2-j,1})^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-2x)}{(1-3x+x^2)(1-x)}$
82	010,011,100,120,201 010,011,100,120,210 010,011,101,120,201 010,011,101,120,210 010,011,110,120,201 010,011,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{b_{m+1+i-j,i}\}_{i=1}^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
83	010,011,100,101,120 010,011,100,110,120 010,011,101,110,120	$\mathcal{T}(\{010, 011, 120\})$	
84	010,011,100,201,210 010,011,101,201,210 010,011,110,201,210	$\mathcal{T}(\{010, 011, 201, 210\})$	$\frac{x(1-3x+x^2)}{(1-3x)(1-x)^2}$
85	010,011,100,101,201 010,011,100,110,201 010,011,101,110,201	$\mathcal{T}(\{010, 011, 201\})$	
	010,011,100,101,210 010,011,100,110,210 010,011,101,110,210	$\mathcal{T}(\{010, 011, 210\})$	[2]
86	010,011,100,101,110	$\mathcal{T}(\{010, 011\})$	[14]
87	012,021,100,101,102 012,021,100,101,120 012,021,100,101,201 012,021,100,101,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$, $a_m = 0^m$	
	012,021,100,102,110 012,021,100,110,120 012,021,100,110,201 012,021,100,110,210 012,021,101,102,110 012,021,101,110,120 012,021,101,110,201 012,021,101,110,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow (010)^2$; $010 \rightsquigarrow 010$, $a_m = 0^m$	$\frac{x(1-x+2x^2)}{(1-x)^3}$
88	012,100,101,110,210•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (010)^m, 011$; $c_m \rightsquigarrow (010)^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x+2x^2+x^4)}{(1-x)^3}$
89	012,100,101,110,201•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=1}^{m}, 011$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x(1-2x+2x^2-x^3-x^4)}{(1-x)^3(1-x-x^2)}$
90	012,100,101,102,110 012,100,101,110,120	$\mathcal{T}(\{012, 100, 101, 110\})$	$\frac{x(2x^6-x^4+(1-x)^3)}{(1-x)^3(1-x-x^2)^2}$
91	011,021,100,102,120•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}$, $a{=}0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x((1-x)^2+x^2-2x^3)}{(1-x)^2(1-2x)}$
92	000,021,101,102,120•	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 010, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,021,102,110,120•	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 002$; $010 \rightsquigarrow 0101$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3 + x^4$
93	000,021,101,102,110•	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 01$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + \frac{x^3}{1-x}$
94	012,100,110,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^2, (0021)^{m-1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+2x^2-x+1)}{(1-x)^3}$
95	012,100,102,110,210 012,100,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (0021)^{m-1}, 010$; $c_m \rightsquigarrow (0021)^{m-1}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^4+x^3+2x^2-x+1)}{(1-x)^3}$
96	012,021,100,102,120 012,021,100,102,201 012,021,100,102,210 012,021,100,120,201 012,021,100,120,210 012,021,100,201,210 012,021,101,102,120 012,021,101,102,201 012,021,101,102,210 012,021,101,120,201 012,021,101,120,210 012,021,101,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012,021,102,110,120 012,021,102,110,201 012,021,102,110,210 012,021,110,120,201 012,021,110,120,210 012,021,110,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 011, 01$; $011 \rightsquigarrow 011$, $a_m = 0^m$	
	012,100,101,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^m, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	012,101,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^{m+1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-2x+3x^2-x^3)}{(1-x)^4}$
97	012,100,102,110,201 012,100,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow 00210, sseqc_i i = 2m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^2(1-2x^2-x^3)+(1-x)^2)}{(1-x)^3(1-x-x^2)}$
98	000,021,100,102,120 000,021,102,120,201 000,021,102,120,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 0101, a_1, 002$; $010 \rightsquigarrow 0101$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3 + 2x^4$
99	012,100,102,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{d_i\}_{i=2}^{m}, c_1$; $c_1 \rightsquigarrow c_1$; $c_2 \rightsquigarrow e_2, c_1$; $c_m \rightsquigarrow e_2, d_2, \{e_i\}_{i=4}^{m}, c_1$; $d_2 \rightsquigarrow e_2$; $d_3 \rightsquigarrow (d_2)^2$; $d_m \rightsquigarrow e_m, \{d_i\}_{i=2}^{m-1}$; $e_3 \rightsquigarrow$; $e_m \rightsquigarrow e_2, d_2, \{e_i\}_{i=4}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$, $d_m = a_m m(m-1)$, $e_m = a_m m0(m-1)$	$\frac{x(1-x+2x^2)}{(1-x)^3} + \frac{x^4(1+x-3x^2-x^3)}{(1-x)^2(1-x-x^2)^2}$
100	012,100,101,102,210 012,100,101,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (010)^{m-1}, b_m$; $c_m \rightsquigarrow (010)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^4-x^3+3x^2-2x+1)}{(1-x)^4}$
101	012,101,102,110,210 012,101,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (010)^m$; $c_m \rightsquigarrow c_m, (010)^{m-1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^2-x+1)(2x^2-2x+1)}{(1-x)^5}$
102	012,100,101,102,201 012,100,101,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=1}^{m}, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	
	012,101,102,110,201 012,101,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^2, \{b_i\}_{i=1}^{m-1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+x^2-x+1)}{(1-x)^2(1-x-x^2)}$

Continuation of Table 1			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
103	011,021,101,102,120 011,021,102,110,120 011,021,102,120,201 011,021,102,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^2, \{c_i\}_{i=2}^m$; $c_m \rightsquigarrow 010, \{c_i\}_{i=2}^m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-3x+4x^2-3x^3)}{(1-x)^3(1-2x)}$
104	000,021,100,102,110 000,021,102,110,201 000,021,102,110,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 012$; $010 \rightsquigarrow 0101$; $012 \rightsquigarrow 0101, b_0, 012$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + \frac{x^3}{1-x}$
105	012,100,101,102,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow c_1, b_1$; $b_m \rightsquigarrow c_m, c_2, \{d_i\}_{i=3}^m, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $d_2 \rightsquigarrow c_1$; $d_m \rightsquigarrow c_{m-1}, c_2, \{d_i\}_{i=3}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$, $d_m = a_m m(m-1)$	$\frac{x((1-x)^2+x^3-x^4-x^5)}{(1-x-x^2)^2(1-x)^2}$
106	011,021,100,101,102 011,021,100,102,110 011,021,100,102,201 011,021,100,102,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^m$, $a_m = 0^m$, $b_m = a_m 1$	$\frac{x(1-x+x^2)}{(1-x)(1-2x)}$
	012,101,102,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m-1}, 010$; $c_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m-1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	
107	011,100,102,120,201•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^j, d_{m+1-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$, $d_m = a_m 12$	$\frac{x(1-x)}{(1-2x)(1-x-x^2)}$
	011,100,102,120,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^j, d_{m+1-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m0$, $d_m = a_m 12$	
108	011,100,101,102,120 011,100,102,110,120	$\mathcal{T}(\{011, 100, 102, 120\})$	$\frac{x(1-x-x^2)}{(1+x)(1-2x)^2}$
109	000,021,100,101,102 000,021,101,102,201 000,021,101,102,210	$\mathcal{T}(\{000, 021, 101, 102\})$	$\frac{1-x-x^2-x^3-2x^4-2x^5}{2x^2} - \frac{(1+x^2)\sqrt{1-2x-3x^2})}{2x^2}$
110	000,021,101,110,120•	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (b_0)^2, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + \frac{4x^3}{(1-x+\sqrt{1-2x-3x^2})^2}$
111	000,101,102,110,120•		
112	012,100,102,201,210 012,100,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, (0021)^{m-1}, b_m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(3x^2-2x+1)}{(1-x)^4}$
	012,102,110,201,210 012,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_1, (011)^m$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$	
113	000,021,100,102,201 000,021,100,102,210 000,021,102,201,210	$\mathcal{T}(\{000, 021, 102\})$	[2]
114	012,100,102,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow c_m, (0021)^{m-1}, b_m$; $c_m \rightsquigarrow (0021)^{m-1}, c_m$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-3x+5x^2-3x^3+x^4)}{(1-x)^5}$
	012,101,102,201,210 012,101,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m, b_m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	
	012,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_m, (011)^m$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$	

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
115	012,100,102,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, b_m$; $c_1 \rightsquigarrow c1$; $c_m \rightsquigarrow 00210, \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	
	012,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_1, \{c_i\}_{i=2}^{m-1}, 011$; $011 \rightsquigarrow 011$; $c_m \rightsquigarrow (011)^2, \{c_i\}_{i=2}^m$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x(1-3x+4x^2-x^3-2x^4)}{(1-x)^4(1-x-x^2)}$
116	011,102,120,201,210•		
117	012,101,102,120,210	$\mathcal{T}(\{012, 101, 210\})$	[2]
118	011,021,100,101,120 011,021,100,110,120 011,021,100,120,201 011,021,100,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^m, 012$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_m = a_m 1$	
	011,021,101,102,110 011,021,101,102,201 011,021,101,102,210 011,021,102,110,201 011,021,102,110,210 011,021,102,201,210 012,101,102,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^m, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$	
	011,101,102,120,201 011,102,110,120,201	$\mathcal{T}(\{011, 102, 120, 201\})$	[2]
	011,101,102,120,210 011,102,110,120,210	$\mathcal{T}(\{011, 102, 120, 210\})$	[2]
	012,021,102,120,201 012,021,102,120,210 012,021,102,201,210 012,021,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow (01)^2$, $a_m = 0^m$	$\frac{x(1-2x+2x^2)}{(1-x)^2(1-2x)}$
119	011,100,102,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (010)^j, \{b_{m,i}\}_{i=j}^m$, $a_m = 0^m$, $b_{m,j} = a_m j$	
	011,101,102,110,120	$\mathcal{T}(\{011, 102, 120\})$, Theorem 6	$\frac{x(1-x)^2}{(1-2x)^2}$
120	011,100,101,102,210 011,100,102,110,210	$\mathcal{T}(\{011, 100, 102, 210\})$	
121	011,100,101,102,201 011,100,102,110,201	$\mathcal{T}(\{011, 100, 102, 201\})$	
122	000,021,100,101,110 000,021,101,110,201 000,021,101,110,210	$\mathcal{T}(\{000, 021, 101, 110\})$	
123	000,021,100,101,120 000,021,100,110,120 000,021,101,120,201 000,021,101,120,210 000,021,110,120,201 000,021,110,120,210	$\mathcal{T}(\{000, 021, 101, 120\}) =$ $\mathcal{T}(\{000, 021, 110, 120\})$, [4]	[4]
124	000,100,102,110,120•		
125	000,100,101,102,120•		
126	011,100,101,102,110	$\mathcal{T}(\{011, 100, 102\})$	
127	000,102,110,120,201•		
128	000,101,102,120,201• 000,101,102,120,210• 000,102,110,120,210•	Theorem 7	
129	000,100,101,102,110•		
130	000,101,102,110,210•		
131	000,101,102,110,201•		
132	000,100,102,120,201•		
134	000,100,102,120,210•		
135	012,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_1, (0021)^{m-1}, b_m$, $a_m = 0^m$, $b_m = a_m$	$\frac{x^5-3x^4+7x^3-7x^2+4x-1}{(1-x)^5}$ $+\frac{1}{1-2x}$
136	011,101,102,201,210 011,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow 010, (0021)^{j-1}, \{b_{m,i}\}_{i=j}^m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-4x+6x^2-3x^3-x^4)}{(1-x)^2(1-2x)^2}$
137	000,021,100,120,201 000,021,100,120,210 000,021,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (a_1)^2, 002$; $002 \rightsquigarrow b_0, 002$, $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \ldots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $\frac{4x^2(1-\sqrt{1-2x-3x^2})}{(1-x+\sqrt{1-2x-3x^2})^2}$
138	000,102,120,201,210•		
139	000,100,102,110,201•		
140	011,021,100,101,110 011,021,100,101,201 011,021,100,101,210 011,021,100,110,201 011,021,100,110,210 011,021,100,201,210	$\mathcal{T}(\{011, 021, 100\})$	
	011,021,101,110,120 011,021,101,120,201 011,021,101,120,210 011,021,110,120,201 011,021,110,120,210 011,021,120,201,210	$\mathcal{T}(\{011, 021, 120\})$	
	011,101,102,110,210	$\mathcal{T}(\{011, 102, 210\})$	[2]

Continuation of Table 1			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
141	011,101,102,110,201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,1} \rightsquigarrow c_1, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (c_1)^2, \{c_i\}_{i=3}^j, \{b_{m,i}\}_{i=j}^m$; $c_1 \rightsquigarrow c_1$; $c_m \rightsquigarrow (c_1)^2, \{c_i\}_{i=3}^m$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	$\frac{x(x^3+(1-x)^2)}{(1-x)^2(1-2x-x^2)}$
142	000,021,100,110,201 000,021,100,110,210 000,021,110,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow a_1, b_0, 01$; $002 \rightsquigarrow b_0, 002$, $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \ldots (m-1)^2 m$, $b_m = a_m m$	$\frac{3x^3-3x^2-4x+2}{2x^2(1-x)} +$ $\frac{(x^2+2x-2)\sqrt{1-2x-3x^2}}{2x^2(1-x)}$
143	000,102,110,201,210•		
144	000,100,102,110,210•		
145	011,100,120,201,210•		
146	000,021,100,101,201 000,021,100,101,210 000,021,101,201,210	$\mathcal{T}(\{000, 021, 101\})$	[3]
147	011,100,101,120,201 011,100,110,120,201 011,100,101,120,210 011,100,110,120,210	$\mathcal{T}(\{011, 100, 120, 201\}) =$ $\mathcal{T}(\{011, 100, 120, 210\})$	[4]
148	000,100,101,102,210•		
149	000,101,102,201,210•		
150	000,100,101,102,201•		
151	011,100,101,110,120		
152	000,100,101,110,120•		
153	000,101,110,120,201•		
154	000,101,110,120,210•		
155	000,100,102,201,210•		
156	000,021,100,201,210	$\mathcal{T}(\{000, 021\})$	[14]
157	011,101,120,201,210 011,110,120,201,210	$\mathcal{T}(\{011, 120, 201, 210\})$	
158	011,100,101,201,210 011,100,110,201,210	$\mathcal{T}(\{011, 100, 201, 210\})$; $a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (b_{m+1-j,1})^j, \{b_{m,i}\}_{i=j}^m$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-2x)}{(1-x)(1-3x)}$
159	011,101,110,120,201 011,101,110,120,210	$\mathcal{T}(\{011, 120, 201\})$ $\mathcal{T}(\{011, 120, 210\})$	[2]
160	010,021,100,101,102 010,021,100,101,110 010,021,100,101,120 010,021,100,101,201 010,021,100,101,210 010,021,100,102,110 010,021,100,102,120 010,021,100,102,201 010,021,100,102,210 010,021,100,110,120 010,021,100,110,201 010,021,100,110,210 010,021,100,120,201 010,021,100,120,210 010,021,100,201,210 010,021,101,102,110 010,021,101,102,120 010,021,101,102,201 010,021,101,102,210 010,021,101,110,120 010,021,101,110,201 010,021,101,110,210 010,021,101,120,201 010,021,101,120,210 010,021,101,201,210 010,021,102,110,120 010,021,102,110,201 010,021,102,110,210 010,021,102,120,201 010,021,102,120,210 010,021,102,201,210 010,021,110,120,201 010,021,110,120,210 010,021,110,201,210 010,021,120,201,210 011,021,101,110,201 011,021,101,110,210 011,021,101,201,210 011,021,110,201,210	$a_m \rightsquigarrow a_{m+1}, \ldots, a_1$, $a_m = 0^m$	$\frac{1-2x-\sqrt{1-4x}}{2x}$
161	000,100,101,110,201• 000,100,101,110,210•		Modification of $\{000, 100, 101, 201\} \overset{\mathbf{I}}{\sim}$ $\{000, 100, 101, 210\}$ in [4]
161	011,100,101,110,201 011,100,101,110,210	$\mathcal{T}(\{011, 100, 201\})$ $\mathcal{T}(\{011, 100, 210\})$	Example 4.9 in [2]
162	000,100,101,120,201 000,100,101,120,210		Theorem 8
163	000,101,110,201,210•		
164	000,101,120,201,210•		
165	000,100,110,120,201• 000,100,110,120,210•		Theorem 8
166	000,110,120,201,210•		
167	010,100,102,110,120•		
168	010,100,101,102,120•		
169	010,101,102,110,120•		
170	010,100,101,102,110•		
171	010,100,102,120,201• 010,102,110,120,201•		Theorem 9
172	010,100,102,120,210•		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,101,102,120,201● 010,101,102,120,210● 010,102,110,120,210●		Theorem 9
173	010,100,102,110,210●		
174	010,100,102,110,201●		
175	010,101,102,110,210●		
176	010,101,102,110,201●		
177	010,100,101,102,210●		
178	010,100,101,102,201●		
179	010,100,101,110,120●		
180	000,100,120,201,210●		
181	010,102,120,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (0021)^j, b_{m+1,j}, a_{m+1-j}$, $\{b_{m+2-j,i}\}_{i=2}^{m+2-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
182	010,102,110,201,210●		
183	010,100,102,201,210●		
184	010,101,102,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, b_{m,2}, \ldots, b_{m,m}$; $b_{m,j} \rightsquigarrow$ $(0021)^{j-1}, b_{m+1,j}, b_{m,j}, \ldots, b_{m,m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
	011,101,110,201,210	$\mathcal{T}(\{011, 201, 210\})$	Theorem 3
185	000,100,101,201,210● 000,100,110,201,210●		Following from Theorem 5.9 in [2]
186	010,100,101,120,201●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow \{c_{m+i-j,i}\}_{i=2}^{j}, b_{m+1,j}, a_{m+1-j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $c_{m,j} \rightsquigarrow a_{m+2-j}, \{c_{m+1+i-j,i}\}_{i=2}^{j-1}$, $\{b_{m+2-j,i}\}_{i=2}^{m+2-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = a_m j(j-1)$	
	010,100,101,120,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow \{c_{m+i-j,i}\}_{i=2}^{j}, b_{m+1,j}, a_{m+1-j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $c_{m,j} \rightsquigarrow a_{m+2-j}, \{c_{m+1+i-j,i}\}_{i=2}^{j-1}$, $\{b_{m+2-j,i}\}_{i=2}^{m+2-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = 0^m j1$	
	010,100,110,120,201● 010,100,110,120,210● 010,101,110,120,201●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow a_{m+2-j}, \{b_{m+1+i-j,i}\}_{i=2}^{j-1}$, $a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	Theorem 11
187	010,101,110,120,210●		
188	010,100,101,110,201●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_{m+i-j,i}\}_{i=2}^{j}, a_{m+2-j}, \{b_{m,i}\}_{i=j}^{m}$; $c_{m,j} \rightsquigarrow \{c_{m++1+i-j,i}\}_{i=2}^{j-1}, \{b_{m,i}\}_{i=j-1}^{m}$; $b_{m,1} = a_m$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = a_m j(j-1)$	
	010,100,101,110,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_{m+i-j,i}\}_{i=2}^{j}, a_{m+2-j}, \{b_{m,i}\}_{i=j}^{m}$; $c_{m,j} \rightsquigarrow \{c_{m++1+i-j,i}\}_{i=2}^{j-1}, \{b_{m,i}\}_{i=j-1}^{m}$; $b_{m,1} = a_m$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = a_m j1$	
189	010,100,110,201,210● 010,101,110,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+2-j})^j, \{b_{m,i}\}_{i=j}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{1}{2} - \frac{1}{2}\sqrt{\frac{1-5x}{1-x}}$
190	010,100,120,201,210● 010,101,120,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+2-j})^{j-1}, a_{m+1-j}, b_{m+1,j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
	010,110,120,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow$ $(b_{m+3-j,2})^{j-1}, a_{m+2-j}, a_{m+1-j}$, $b_{m+1,j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	Theorem 12
191	010,100,101,201,210●		
192	012,021,100,101,110●	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $01 \rightsquigarrow 010, 011$; $011 \rightsquigarrow 011$	$\frac{x(1-x(1-x)^2)}{(1-x)^3}$
193	021,100,101,102,120●	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, b_{m+1}, \{c_i\}_{i=1}^{m}$, $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	
	021,100,102,110,120●		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	021,101,102,110,120•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m}, 012$, $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $010 \rightsquigarrow 010$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-x)^3}$
194	021,100,101,102,110•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, c_m, \{b_i\}_{i=1}^{m}$, $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-x)(1-2x)}$
195	021,100,102,120,201 021,100,102,120,210 021,101,102,120,201 021,101,102,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$; $00 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m12$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-x)^4}$
	021,102,110,120,201 021,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m}, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $010 \rightsquigarrow 010, 0101$; $012 \rightsquigarrow c_1, 012$; $0101 \rightsquigarrow 0101$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	
196	021,100,102,110,201 021,100,102,110,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, c_m, \{d_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $d_m \rightsquigarrow \{d_i\}_{i=1}^{m}, 0120$; $010 \rightsquigarrow 010$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$, $d_m = a_m12$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3(1-x-x^2)}{(1-x)^3(1-2x)}$
197	021,101,102,110,201 021,101,102,110,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $010 \rightsquigarrow 010$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-x)^2(1-2x)}$
198	021,100,101,102,201 021,100,101,102,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m1$	$\frac{(1+x)(1-2x-\sqrt{1-4x})}{2x^2} - x - \frac{1}{1-x}$
	021,100,101,110,120•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, c_{m-1}, \{c_i\}_{i=1}^{m+1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 010$; $010 \rightsquigarrow c_1, 010$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	
199	100,101,102,110,120•		
200	021,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, c_m, \{d_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $d_m \rightsquigarrow \{d_i\}_{i=1}^{m}, c_m, 0101$; $010 \rightsquigarrow 010, 0101$; $0113 \rightsquigarrow c_1, 0113$; $0101 \rightsquigarrow 0101$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$, $d_m = a_m12$	[2]
201	021,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$; $010 \rightsquigarrow (010)^2$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m12$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-2x)(1-x)^3}$
202	021,100,102,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0120$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m12$	$\frac{3x^5-x^4-4x^3+8x^2-5x+1}{2x(1-x^4)} - \frac{(1-x-x^2+x^3)\sqrt{1-4x}}{2x(1-x)^2}$
203	021,101,102,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m+1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m1$	$\frac{1-3x+2x^2-2x^3}{2x(1-x)^2} - \frac{\sqrt{1-4x}}{2x(1-x)}$
204	021,100,101,110,201 021,100,101,110,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_1 \rightsquigarrow 010, c_1, b_1$; $b_m \rightsquigarrow c_{m-1}, c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m11$	$\frac{1-3x+x^2}{2x(1-2x)} + \frac{(x^2+x-1)\sqrt{1-4x}}{2x(1-2x)}$
205	021,100,101,120,201 021,100,101,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{c_i\}_{i=1}^{m+1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
	021,100,110,120,201 021,100,110,120,210 021,101,110,120,201		

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	021,101,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{c_i\}_{i=1}^{m}, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	$\frac{4x^2-7x+2+(3x-2)\sqrt{1-4x}}{2x(1-x)}$
206	100,102,110,120,201●		
207	100,101,102,120,201●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=1}^{j}, b_{m+1,j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_mj$, $c_m = a_mm(m-1)$	
	100,101,102,120,210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=1}^{j}, b_{m+1,j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow c_1, \ldots, c_{m-1}$, $a_m = 0^m$, $b_{m,j} = a_mj$, $c_m = a_mm0$	
	100,102,110,120,210● 101,102,110,120,201●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=1}^{j}, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}, c_1$, $a_m = 0^m$, $b_{m,j} = a_mj$, $c_m = a_mm0$	Theorem 10
208	101,102,110,120,210●		
209	100,101,102,110,210●		
210	100,101,102,110,201●		
211	100,102,120,201,210● 101,102,120,201,210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, b_{m+1,j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_{m,j} = a_mj$	
	102,110,120,201,210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $010 \rightsquigarrow 010, 0101$; $0101 \rightsquigarrow 0101$, $a_m = 0^m$, $b_{m,j} = a_mj$	Theorem 10
212	100,102,110,201,210●		
213	021,100,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_{m+1}, c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0103$; $0103 \rightsquigarrow c_1, 0103$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
214	021,100,120,201,210 021,101,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow b_{m+1}, \{c_i\}_{i=1}^{m}, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
	021,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow b_1, 012$, $a_m = 0^m$, $b_m = a_m1$	
	101,102,110,201,210●	$a_m \rightsquigarrow a_{m+1}\{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, a_{m+2-j}, \{b_{m+1,i}\}_{i=j+1}^{m+1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_{m,j} = a_mj$	$\frac{1-x+2x^2-(1-x)\sqrt{1-4x})}{(1-x)(1-2x+\sqrt{1-4x})}$
215	021,100,101,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m+1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
	021,101,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow a_{m+1}, c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m1$, $c_m a_m 11$	
	100,101,102,201,210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, \{b_{m+1,i}\}_{i=j}^{m+1}$, $a_m = 0^m$, $b_{m,j} = a_mj$	$\frac{x}{\sqrt{1-4x}}$
216	100,101,110,120,201●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow a_{m+1-j}, \{c_{m+i-j,i}\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_{m,j} \rightsquigarrow a_{m+2-j}, \{c_{m+1+i-j,i}\}_{i=2}^{j-1}, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_mj$, $c_{m,j} = a_mj(j-1)$	
	100,101,110,120,210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow a_{m+1-j}, \{c_{m+i-j,i}\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_{m,j} \rightsquigarrow a_{m+2-j}, \{c_{m+1+i-j,i}\}_{i=2}^{j-1}, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_mj$, $c_{m,j} = a_mj0$	

Continuation of Table 1

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
217	100,101,110,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+1-j})^j, a_{m+2-j}, \{b_{m+1,i}\}_{i=j+1}^{m+1}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{2}{1+x+\sqrt{1-6x+5x^2}} - 1$
218	100,101,120,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+1-j})^j, b_{m+1,j}, \{b_{m+1-j,i}\}_{i=1}^{m+1=j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	Theorem 12
	$\overline{1}0\overline{1},\overline{1}1\overline{0},\overline{1}2\overline{0},\overline{2}0\overline{1},\overline{2}1\overline{0}$•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (c_{m+2-j})^j, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow c_{m+1}, a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 10$	
219	100,110,120,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+2-j})^{j+1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	[12]

End of Table 1

A *left to right maxima (LRmax)* in an inversion sequence $e = e_0e_1 \dots e_n$ is an entry e_i such that $e_i \geq e_j$ for all $j < i$. If $e_i = m$ we say m is the *value* of the LRmax. For example, if $e = 011021211$, then the LRmax are e_0, e_1, e_2, e_4, e_6 with values $0, 1, 1, 2, 2$, respectively. So any inversion sequence $e \in \mathbf{I}_n$ can be decomposed uniquely as $m_1\pi_1 \cdots m_k\pi_k$ where $m_1, \dots, m_k$ are all of the LRmax entries in e; thus $0 \leq m_1 \leq \cdots \leq m_k \leq n$ and $m_i > \pi_i$ (entrywise). We call this the WLRmax decomposition of e (W for weakly). Say $\mathbf{m}_1 < \mathbf{m}_2 < \cdots < \mathbf{m}_k$ are the first occurrences of the (distinct) LRmax values of an inversion sequence e. Then e can be decomposed as $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ for some $k \geq 1$ with $\mathbf{m}_i \geq \pi_i$ (entrywise) for $1 \leq i \leq k$. This is the *LRmax decomposition* of e.

Theorem 4. *Let $B_\tau = \{000, 010, 100, \tau, 120\}$. Then $B_{101} \overset{\mathbf{I}}{\sim} B_{110}$.*

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(B_{101})$ if and only if,

- (i) If there is a letter m_i that appears in π_i, then it appears as the leftmost letter of π_i, for all $i = 1, 2, \dots, k$;
- (ii) $\mathbf{m}_1\pi_1 < \mathbf{m}_2\pi_2 < \cdots < \mathbf{m}_k\pi_k$;
- (iii) Any π_i avoids B_{101}.

(2) $e \in \mathbf{I}_n(B_{110})$ if and only if,

- (i) If there is a letter m_i that appears in π_i, then it appears as the rightmost letter of π_i, for all $i = 1, 2, \dots, k$;
- (ii) $\mathbf{m}_1\pi_1 < \mathbf{m}_2\pi_2 < \cdots < \mathbf{m}_k\pi_k$;
- (iii) Any π_i avoids B_{110}.

Now, we are ready to define a recursive bijection $f : \mathbf{I}_n(B_{101}) \mapsto \mathbf{I}_n(B_{110})$. We define, $f(a) = a$, for any letter $0 \leq a \leq n$. For any inversion sequence $e \in \mathbf{I}_n(B_{101})$ with LRmax decomposition $\mathbf{m}_1\pi_1 \cdots \mathbf{m}_k\pi_k$, we define $f(e) = \mathbf{m}_1\beta_1 \cdots \mathbf{m}_k\beta_k$, where

- if $\pi_i = m_i\pi_i'$, then β_i is defined to be $f(\pi_i')m_i$,
- otherwise, we define β_i as $f(\pi_i)$.

So, $e \in \mathbf{I}_n(B_{101})$ if and only if $f(e) \in \mathbf{I}_n(B_{110})$. For example, if $e = 002216654987$, then $f(e) = 002f(1)26f(54)69f(87) = 002126546987$. □

Theorem 5. *Let $A = \{000, 010, 101, 120, 201\}$, $B = \{000, 010, 101, 120, 210\}$, and $C = \{000, 010, 110, 120, 210\}$. Then $A \overset{\mathbf{I}}{\sim} B \overset{\mathbf{I}}{\sim} C$.*

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \ldots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(A)$ if and only if,

   (i) If there is a letter m_i that appears in π_i, then it appears as the leftmost letter of π_i, for all $i = 1, 2, \ldots, k$;

   (ii) $\mathbf{m}_1\pi_1 < \mathbf{m}_2\pi_2 < \cdots < \mathbf{m}_k\pi_k$;

   (iii) Any π_i is a non-increasing sequence such that each letter of π_i appears at most twice.

(2) $e \in \mathbf{I}_n(B)$ if and only if,

   (i) If there is a letter m_i that appears in π_i, then it appears as the leftmost letter of π_i, for all $i = 1, 2, \ldots, k$;

   (ii) $\mathbf{m}_1\pi_1 < \mathbf{m}_2\pi_2 < \cdots < \mathbf{m}_k\pi_k$;

   (iii) Any π_i is a non-decreasing sequence such that each letter of π_i appears at most twice.

(3) $e \in \mathbf{I}_n(C)$ if and only if,

   (i) If there is a letter m_i that appears in π_i, then it appears as the rightmost letter of π_i, for all $i = 1, 2, \ldots, k$;

   (ii) $\mathbf{m}_1\pi_1 < \mathbf{m}_2\pi_2 < \cdots < \mathbf{m}_k\pi_k$;

   (iii) Any π_i is a non-decreasing sequence such that each letter of π_i appears at most twice.

Define $rev(w_1w_2\cdots w_m) = w_m\cdots w_2w_1$. Now, we ready to define bijections $f : \mathbf{I}_n(A) \mapsto \mathbf{I}_n(B)$ and $g : \mathbf{I}_n(B) \mapsto \mathbf{I}_n(C)$. We define, $f(a) = g(a) = a$, for any letter $0 \le a \le n$. For any inversion sequence $e \in \mathbf{I}_n(A)$ with LRmax decomposition $\mathbf{m}_1\pi_1\cdots\mathbf{m}_k\pi_k$, we define $f(e) = \mathbf{m}_1\beta_1\cdots\mathbf{m}_k\beta_k$, where

- if $\pi_i = m_i\pi_i'$, then β_i is defined to be $m_i rev(\pi_i')$,
- otherwise, we define β_i as $rev(\pi_i)$.

For any inversion sequence $e \in \mathbf{I}_n(B)$ with LRmax decomposition $\mathbf{m}_1\pi_1 \cdots \mathbf{m}_k\pi_k$, we define $g(e) = \mathbf{m}_1\gamma_1 \cdots \mathbf{m}_k\gamma_k$, where

- if $\pi_i = m_i\pi_i'$, then γ_i is defined to be $\pi_i' m_i$,
- otherwise, we define γ_i as π_i.

So, $e \in \mathbf{I}_n(A)$ if and only if $f(e) \in \mathbf{I}_n(B)$ and $e \in \mathbf{I}_n(B)$ if and only if $g(e) \in \mathbf{I}_n(C)$. □

Theorem 6. *Let $B = \{011, 102, 120\}$. Then*

$$F_B(x) = \frac{x(1-x)^2}{(1-2x)^2}.$$

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e.

- If $\mathbf{m}_k = 1$, then $e = 0^{a+1}10^b$ with $a + 1 + b = n$ and $a, b \geq 0$.
- If $\mathbf{m}_k \geq 2$, then since e avoids 011 and 102, we have that $\pi_2 = \cdots = \pi_{k-1} = \emptyset$. Since e avoids 011 and 120, we have that $\mathbf{m}_k > \pi_k > \mathbf{m}_{k-1}$. Thus, e can be written as $e = 0^d\mathbf{m}_2\mathbf{m}_3 \cdots \mathbf{m}_k\pi_k$ such that π_k is a permutation (no repeated letters) of $\{\mathbf{m}_{k-1} + 1, \dots, \mathbf{m}_k - 1\}$ avoids that 102, 120.

Now, let $G_k(x)$ be the generating function for the number of inversion sequences $e \in \mathbf{I}_n(B)$ with exactly k LRmax and where the largest letter in e is $\mathbf{m}_k$. Clearly, $G_1(x) = \frac{x}{1-x}$. With simple arguments, one can show that $G_2(x) = \frac{x^2(x^2+x-1)^2}{(1-x)(1-x^2)(1-2x)^2}$, $G_3(x) = \frac{x^3(1-x-x^2)}{(1-x)^2(1-x^2)(1-2x)}$, and $G_k(x) = \frac{x}{1-x}G_{k-1}(x)$ for $k \geq 4$. Hence, the generating function $F_B(x) = G_1(x) + G_2(x) + \frac{(1-x)G_3(x)}{1-2x}$, which completes the proof. □

Theorem 7. *Let $A = \{000, 101, 102, 120, 201\}$, $B = \{000, 101, 102, 120, 210\}$, and $C = \{000, 102, 110, 120, 210\}$. Then $A \overset{\mathbf{I}}{\sim} B \overset{\mathbf{I}}{\sim} C$.*

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(A)$ if and only if,

 (i) either $\pi_i = m_i$ or $\pi_i =$, for all $i = 1, 2, \dots, k-1$;

 (ii) if $\pi_{k-1} = \emptyset$, then $\pi_k \geq m_{k-1}$ such that $m_k\pi_k$ is weakly decreasing and each letter of $m_k\pi_k$ appears at most twice, except m_{k-1} appears at most once;

 (iii) if $\pi_{k-1} = m_{k-1}$, then $\pi_k > m_{k-1}$ such that $m_k\pi_k$ is weakly decreasing and each letter of $m_k\pi_k$ appears at most twice.

(2) $e \in \mathbf{I}_n(B)$ if and only if,

(i) either $\pi_i = m_i$ or $\pi_i =$, for all $i = 1, 2, \ldots, k-1$;

(ii) if $\pi_{k-1} = \emptyset$, then $\pi_k \geq m_{k-1}$ such that either $\pi_k = m_k\beta$ or $\pi_k = \beta$, where β is weakly increasing and each letter appears at most twice, except m_{k-1} appears at most once;

(iii) if $\pi_{k-1} = m_{k-1}$, then $\pi_k > m_{k-1}$ such that either $\pi_k = m_k\beta$ or $\pi_k = \beta$, where β is weakly increasing and each letter appears at most twice.

(3) $e \in \mathbf{I}_n(C)$ if and only if,

(i) either $\pi_i = m_i$ or $\pi_i =$, for all $i = 1, 2, \ldots, k-1$;

(ii) if $\pi_{k-1} = \emptyset$, then $\pi_k \geq m_{k-1}$ such that either $\pi_k = \beta m_k$ or $\pi_k = \beta$, where β is weakly increasing and each letter appears at most twice, except m_{k-1} appears at most once;

(iii) if $\pi_{k-1} = m_{k-1}$, then $\pi_k > m_{k-1}$ such that either $\pi_k = \beta m_k$ or $\pi_k = \beta$, where β is weakly increasing and each letter appears at most twice, except the letter m_k appears at most once.

Based on the characterization of sets $\mathbf{I}_n(A)$, $\mathbf{I}_n(B)$, and $\mathbf{I}_n(C)$, each having k LRmax elements, one can establish bijections between them. □

Theorem 8. *Let* $A_\tau = \{000, 100, 101, 120, \tau\}$ *and* $B_\tau = \{000, 100, 110, 120, \tau\}$. *Then* $A_{201} \overset{\mathbf{I}}{\sim} A_{210}$ *and* $B_{201} \overset{\mathbf{I}}{\sim} B_{210}$.

Proof. Since the proof of both cases are similar, we present only the proof of the case $A_{201} \overset{\mathbf{I}}{\sim} A_{210}$. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \ldots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then $e \in \mathbf{I}_n(A_{201})$ (respectively, $e \in \mathbf{I}_n(A_{210})$) if and only if,

(i) If there is a letter m_i that appears in π_i, then it appears as the leftmost letter of π_i, for all $i = 1, 2, \ldots, k$;

(ii) if the leftmost letter of π is m_i, then $\mathbf{m}_i\pi_i > \mathbf{m}_{i-1}$ (here, $\mathbf{m}_0 = 0$) and $\mathbf{m}_i\pi_i$ is weakly decreasing (respectively, increasing) and each letter of $\mathbf{m}_i\pi_i$ appears at most twice, for all $i = 1, 2, \ldots, k$;

(iii) if the leftmost letter of π is not m_i, then $\mathbf{m}_i\pi_i \geq \mathbf{m}_{i-1}$ (here, $\mathbf{m}_0 = 0$) and $\mathbf{m}_i\pi_i$ is weakly decreasing (respectively, increasing) and each letter of $\mathbf{m}_i\pi_i$ appears at most twice, except the letter m_{i-1} which appears at most once, for all $i = 1, 2, \ldots, k$.

Thus, by mapping the LRmax decomposition of inversion sequences in $\mathbf{I}_n(A_{201})$ to $\mathbf{I}_n(A_{210})$, we complete the proof. □

Theorem 9. *We have the following:*

(1) Let $A_\tau = \{010, \tau, 102, 120, 201\}$*. Then*

$$A_{100} \overset{\mathbf{I}}{\sim} A_{110}.$$

(2) Let $A_\tau = \{010, \tau, 102, 120, 210\}$ *and* $B = \{010, 101, 102, 120, 201\}$*. Then*

$$A_{100} \overset{\mathbf{I}}{\sim} A_{101} \overset{\mathbf{I}}{\sim} A_{110} \overset{\mathbf{I}}{\sim} B.$$

Proof. Since the proof of both cases are similar, we present only the proof of (1). Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then $e \in \mathbf{I}_n(A_\tau)$ with $\tau \in \{100, 110\}$) if and only if,

- (i) If there are letters m_i appear in π_i, then it they appear as leftmost letters of π_i, for all $i = 1, 2, \dots, k-1$;
- (ii) If π_i is not empty then each letter in π_i is m_i, for all $i = 1, 2, \dots, k-1$;
- (iii) When $\tau = 100$ and π_k is not empty, we see that π_k can be written as $\pi_k^{(1)} m_k \cdots \pi_k^{(s)} m_k \pi^{(s+1)} > m_{k-1}$, where $\pi^{(1)} \cdots \pi_k^{(s)}$ contains exactly one letter, say b, $b > \pi_k^{(s+1)}$, and $\pi_k^{(s+1)}$ forms a decreasing sequence (no repeated letters);
- (iii) When $\tau = 110$ and π_k is not empty, we see that π_k can be written as $\beta b^a (m_k)^c > m_{k-1}$ with $a, c \geq 0$, where βb forms a decreasing sequence (no repeated letters).

Now, we ready to define a bijection f between $\mathbf{I}_n(A_{100})$ and $\mathbf{I}_n(A_{110})$. Let $e \in \mathbf{I}_n(A_{100})$ with k LRmax as defined above. If $\pi_k = (m_k)^a$ then $f(e) = e$. Otherwise, let $1 \leq j \leq s$ such that $\pi^{(j)} = b$, so

$$f(e) = \mathbf{m}_1\pi_1 \cdots \mathbf{m}_{k-1}\pi_{k-1}\mathbf{m}_k b \pi_k^{(s+1)} c^{j-1} m_k^s,$$

where c is the minimal letter of $\pi_k^{(s+1)}$. Clearly, $e \in \mathbf{I}_n$ avoids A_{100} if and only if $f(e) \in \mathbf{I}_n$ avoids A_{110}. □

Using a very similar proof to the above theorems, we can state the following result.

Theorem 10. *Let* $A_\tau = \{100, \tau, 102, 120, 201\}$*,* $B_\tau = \{100, \tau, 102, 120, 210\}$*, and* $C_\tau = \{\tau, 102, 120, 201, 210\}$*. Then* $A_{101} \overset{\mathbf{I}}{\sim} A_{110}$*,* $B_{101} \overset{\mathbf{I}}{\sim} B_{110}$*, and* $C_{101} \overset{\mathbf{I}}{\sim} C_{110}$*.*

Theorem 11. *Let* $A_\tau = \{010, 100, 101, 120, \tau\}$*. Then* $A_{201} \overset{\mathbf{I}}{\sim} A_{210}$*.*

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then $e \in \mathbf{I}_n(A_{201})$ (respectively, $e \in \mathbf{I}_n(A_{210})$) if and only if,

- $\pi_i = m_i^a\beta$ for $a > 0$, and β forms an increasing (respectively, decreasing) sequence such that $m_i > \beta > m_{i-1}$ (with $m_0 = 0$), for all $i = 1, 2, \ldots, k$.

By characterizing the set $\mathbf{I}_n(A_\tau)$ with k LRmax, a bijection between $\mathbf{I}_n(A_{201})$ and $\mathbf{I}_n(A_{210})$ can be established. □

Theorem 12. *Let* $A_\tau = \{010, \tau, 120, 201, 210\}$ *and* $B_\tau = \{100, \tau, 120, 201, 210\}$. *Then* $A_{101} \overset{\mathbf{I}}{\sim} A_{110}$ *and* $B_{101} \overset{\mathbf{I}}{\sim} B_{110}$.

Proof. Since the proof of both cases are similar, we present only the proof of $A_{101} \overset{\mathbf{I}}{\sim} A_{110}$. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \ldots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(A_{101})$ if and only if,

   (i) $m_i \geq \pi_i > m_{i-1}$ (here, $m_0 = 0$), for all $i = 1, 2, \ldots, k$;

   (ii) π_i can be written as $m_i \cdots m_i\beta_i$ such that β_i contains at most one letter, for all $i = 1, 2, \ldots, k$.

(2) $e \in \mathbf{I}_n(A_{110})$ if and only if,

   (i) $m_i \geq \pi_i > m_{i-1}$ (here, $m_0 = 0$), for all $i = 1, 2, \ldots, k$;

   (ii) π_i can be written as $\beta_i m_i \cdots m_i$ such that β_i contains at most one letter, for all $i = 1, 2, \ldots, k$.

By characterizing the sets $\mathbf{I}_n(A_{101})$ and $\mathbf{I}_n(A_{110})$ with k LRmax, a bijection between $\mathbf{I}_n(A_{201})$ and $\mathbf{I}_n(A_{210})$ can be established. □

4. 6-Table

In this section, we show that $w_6 = 167$. Moreover, by our procedure, we present the generating function for many of the 167 I-Wilf-equivalences. Actually, we see that there are only 93 sets among $\binom{13}{6} = 1716$ sets of 6 length-3 patterns that cannot be reduced to smaller sets of length-3 patterns. In Table 2, we present all the distinct I-Wilf-equivalences of sets of 6 length-3 patterns. As we mentioned, due to the similarity with the previous section, we only present the cases that the KMY algorithm does not work, that is, we present bijections between some classes of inversion sequences respect to left-right-maxima structure with using the bijections presented in the previous sections; see the theorems at end of this section.

Table 2: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 6$.

Beginning of Table 2			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000, 001, 010, 011, 012, 021 000, 001, 010, 011, 012, 100 000, 001, 010, 011, 012, 101 000, 001, 010, 011, 012, 102 000, 001, 010, 011, 012, 110 000, 001, 010, 011, 012, 120 000, 001, 010, 011, 012, 201 000, 001, 010, 011, 012, 210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000, 001, 010, 012, 021, 100 000, 001, 010, 012, 021, 101 000, 001, 010, 012, 021, 102 000, 001, 010, 012, 021, 110 000, 001, 010, 012, 021, 120 000, 001, 010, 012, 021, 201 000, 001, 010, 012, 021, 210 000, 001, 010, 012, 100, 101 000, 001, 010, 012, 100, 102 000, 001, 010, 012, 100, 110 000, 001, 010, 012, 100, 120 000, 001, 010, 012, 100, 201 000, 001, 010, 012, 100, 210 000, 001, 010, 012, 101, 102 000, 001, 010, 012, 101, 110 000, 001, 010, 012, 101, 120 000, 001, 010, 012, 101, 201 000, 001, 010, 012, 101, 210 000, 001, 010, 012, 102, 110 000, 001, 010, 012, 102, 120 000, 001, 010, 012, 102, 201 000, 001, 010, 012, 102, 210 000, 001, 010, 012, 110, 120 000, 001, 010, 012, 110, 201 000, 001, 010, 012, 110, 210 000, 001, 010, 012, 120, 201 000, 001, 010, 012, 120, 210 000, 001, 010, 012, 201, 210 000, 001, 011, 012, 021, 100 000, 001, 011, 012, 021, 101 000, 001, 011, 012, 021, 102 000, 001, 011, 012, 021, 110 000, 001, 011, 012, 021, 120 000, 001, 011, 012, 021, 201 000, 001, 011, 012, 021, 210 000, 001, 011, 012, 100, 101 000, 001, 011, 012, 100, 102 000, 001, 011, 012, 100, 110 000, 001, 011, 012, 100, 120 000, 001, 011, 012, 100, 201 000, 001, 011, 012, 100, 210 000, 001, 011, 012, 101, 102 000, 001, 011, 012, 101, 110 000, 001, 011, 012, 101, 120 000, 001, 011, 012, 101, 201 000, 001, 011, 012, 101, 210 000, 001, 011, 012, 102, 110 000, 001, 011, 012, 102, 120 000, 001, 011, 012, 102, 201 000, 001, 011, 012, 102, 210 000, 001, 011, 012, 110, 120 000, 001, 011, 012, 110, 201 000, 001, 011, 012, 110, 210 000, 001, 011, 012, 120, 201 000, 001, 011, 012, 120, 210 000, 001, 011, 012, 201, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x(1+x)^2$
3	000, 001, 010, 011, 021, 100 000, 001, 010, 011, 021, 101 000, 001, 010, 011, 021, 102 000, 001, 010, 011, 021, 110 000, 001, 010, 011, 021, 120 000, 001, 010, 011, 021, 201 000, 001, 010, 011, 021, 210 000, 001, 010, 011, 100, 101 000, 001, 010, 011, 100, 102 000, 001, 010, 011, 100, 110 000, 001, 010, 011, 100, 120 000, 001, 010, 011, 100, 201 000, 001, 010, 011, 100, 210 000, 001, 010, 011, 101, 102 000, 001, 010, 011, 101, 110 000, 001, 010, 011, 101, 120 000, 001, 010, 011, 101, 201 000, 001, 010, 011, 101, 210 000, 001, 010, 011, 102, 110 000, 001, 010, 011, 102, 120 000, 001, 010, 011, 102, 201 000, 001, 010, 011, 102, 210 000, 001, 010, 011, 110, 120 000, 001, 010, 011, 110, 201 000, 001, 010, 011, 110, 210 000, 001, 010, 011, 120, 201 000, 001, 010, 011, 120, 210 000, 001, 010, 011, 201, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001, 010, 011, 012, 021, 100 001, 010, 011, 012, 021, 101 001, 010, 011, 012, 021, 102 001, 010, 011, 012, 021, 110 001, 010, 011, 012, 021, 120		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001, 010, 011, 012, 021, 201		
	001, 010, 011, 012, 021, 210		
	001, 010, 011, 012, 100, 101		
	001, 010, 011, 012, 100, 102		
	001, 010, 011, 012, 100, 110		
	001, 010, 011, 012, 100, 120		
	001, 010, 011, 012, 100, 201		
	001, 010, 011, 012, 100, 210		
	001, 010, 011, 012, 101, 102		
	001, 010, 011, 012, 101, 110		
	001, 010, 011, 012, 101, 120		
	001, 010, 011, 012, 101, 201		
	001, 010, 011, 012, 101, 210		
	001, 010, 011, 012, 102, 110		
	001, 010, 011, 012, 102, 120		
	001, 010, 011, 012, 102, 201		
	001, 010, 011, 012, 102, 210		
	001, 010, 011, 012, 110, 120		
	001, 010, 011, 012, 110, 201		
	001, 010, 011, 012, 110, 210		
	001, 010, 011, 012, 120, 201		
	001, 010, 011, 012, 120, 210		
	001, 010, 011, 012, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$x^2 + \frac{x}{1-x}$
4	000, 001, 012, 021, 100, 110		
	000, 001, 012, 021, 101, 110		
	000, 001, 012, 021, 102, 110		
	000, 001, 012, 021, 110, 120		
	000, 001, 012, 021, 110, 201		
	000, 001, 012, 021, 110, 210		
	000, 001, 012, 100, 101, 110		
	000, 001, 012, 100, 102, 110		
	000, 001, 012, 100, 110, 120		
	000, 001, 012, 100, 110, 201		
	000, 001, 012, 100, 110, 210		
	000, 001, 012, 101, 102, 110		
	000, 001, 012, 101, 110, 120		
	000, 001, 012, 101, 110, 201		
	000, 001, 012, 101, 110, 210		
	000, 001, 012, 102, 110, 120		
	000, 001, 012, 102, 110, 201		
	000, 001, 012, 102, 110, 210		
	000, 001, 012, 110, 120, 201		
	000, 001, 012, 110, 120, 210		
	000, 001, 012, 110, 201, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000, 010, 011, 012, 021, 100		
	000, 010, 011, 012, 021, 101		
	000, 010, 011, 012, 021, 102		
	000, 010, 011, 012, 021, 110		
	000, 010, 011, 012, 021, 120		
	000, 010, 011, 012, 021, 201		
	000, 010, 011, 012, 021, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
5	000, 001, 012, 021, 100, 101		
	000, 001, 012, 021, 100, 102		
	000, 001, 012, 021, 100, 120		
	000, 001, 012, 021, 100, 201		
	000, 001, 012, 021, 100, 210		
	000, 001, 012, 021, 101, 102		
	000, 001, 012, 021, 101, 120		
	000, 001, 012, 021, 101, 201		
	000, 001, 012, 021, 101, 210		
	000, 001, 012, 021, 102, 120		
	000, 001, 012, 021, 102, 201		
	000, 001, 012, 021, 102, 210		
	000, 001, 012, 021, 120, 201		
	000, 001, 012, 021, 120, 210		
	000, 001, 012, 021, 201, 210		
	000, 001, 012, 100, 101, 102		
	000, 001, 012, 100, 101, 120		
	000, 001, 012, 100, 101, 201		
	000, 001, 012, 100, 101, 210		
	000, 001, 012, 100, 102, 120		
	000, 001, 012, 100, 102, 201		
	000, 001, 012, 100, 102, 210		
	000, 001, 012, 100, 120, 201		
	000, 001, 012, 100, 120, 210		
	000, 001, 012, 100, 201, 210		
	000, 001, 012, 101, 102, 120		
	000, 001, 012, 101, 102, 201		
	000, 001, 012, 101, 102, 210		
	000, 001, 012, 101, 120, 201		
	000, 001, 012, 101, 120, 210		
	000, 001, 012, 101, 201, 210		
	000, 001, 012, 102, 120, 201		
	000, 001, 012, 102, 120, 210		
	000, 001, 012, 102, 201, 210		
	000, 001, 012, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;\ 011 \rightsquigarrow 00$	
	000, 010, 011, 012, 100, 101		
	000, 010, 011, 012, 100, 102		
	000, 010, 011, 012, 100, 110		
	000, 010, 011, 012, 100, 120		
	000, 010, 011, 012, 100, 201		
	000, 010, 011, 012, 100, 210		
	000, 010, 011, 012, 101, 102		
	000, 010, 011, 012, 101, 110		
	000, 010, 011, 012, 101, 120		
	000, 010, 011, 012, 101, 201		
	000, 010, 011, 012, 101, 210		
	000, 010, 011, 012, 102, 110		
	000, 010, 011, 012, 102, 120		
	000, 010, 011, 012, 102, 201		
	000, 010, 011, 012, 102, 210		
	000, 010, 011, 012, 110, 120		
	000, 010, 011, 012, 110, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 010, 011, 012, 110, 210		
	000, 010, 011, 012, 120, 201		
	000, 010, 011, 012, 120, 210		
	000, 010, 011, 012, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
6	000, 001, 011, 021, 100, 120		
	000, 001, 011, 021, 101, 120		
	000, 001, 011, 021, 102, 120		
	000, 001, 011, 021, 110, 120		
	000, 001, 011, 021, 120, 201		
	000, 001, 011, 021, 120, 210		
	000, 001, 011, 100, 101, 120		
	000, 001, 011, 100, 102, 120		
	000, 001, 011, 100, 110, 120		
	000, 001, 011, 100, 120, 201		
	000, 001, 011, 100, 120, 210		
	000, 001, 011, 101, 102, 120		
	000, 001, 011, 101, 110, 120		
	000, 001, 011, 101, 120, 201		
	000, 001, 011, 101, 120, 210		
	000, 001, 011, 102, 110, 120		
	000, 001, 011, 102, 120, 201		
	000, 001, 011, 102, 120, 210		
	000, 001, 011, 110, 120, 201		
	000, 001, 011, 110, 120, 210		
	000, 001, 011, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 012;\ 012 \rightsquigarrow 012$	
	001, 011, 012, 021, 100, 101		
	001, 011, 012, 021, 100, 102		
	001, 011, 012, 021, 100, 110		
	001, 011, 012, 021, 100, 120		
	001, 011, 012, 021, 100, 201		
	001, 011, 012, 021, 100, 210		
	001, 011, 012, 100, 101, 102		
	001, 011, 012, 100, 101, 110		
	001, 011, 012, 100, 101, 120		
	001, 011, 012, 100, 101, 201		
	001, 011, 012, 100, 101, 210		
	001, 011, 012, 100, 102, 110		
	001, 011, 012, 100, 102, 120		
	001, 011, 012, 100, 102, 201		
	001, 011, 012, 100, 102, 210		
	001, 011, 012, 100, 110, 120		
	001, 011, 012, 100, 110, 201		
	001, 011, 012, 100, 110, 210		
	001, 011, 012, 100, 120, 201		
	001, 011, 012, 100, 120, 210		
	001, 011, 012, 100, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010$	$x^2 + x^3 + \frac{x}{1-x}$
7	000, 001, 010, 021, 100, 101		
	000, 001, 010, 021, 100, 102		
	000, 001, 010, 021, 100, 110		
	000, 001, 010, 021, 100, 120		
	000, 001, 010, 021, 100, 201		
	000, 001, 010, 021, 100, 210		
	000, 001, 010, 021, 101, 102		
	000, 001, 010, 021, 101, 110		
	000, 001, 010, 021, 101, 120		
	000, 001, 010, 021, 101, 201		
	000, 001, 010, 021, 101, 210		
	000, 001, 010, 021, 102, 110		
	000, 001, 010, 021, 102, 120		
	000, 001, 010, 021, 102, 201		
	000, 001, 010, 021, 102, 210		
	000, 001, 010, 021, 110, 120		
	000, 001, 010, 021, 110, 201		
	000, 001, 010, 021, 110, 210		
	000, 001, 010, 021, 120, 201		
	000, 001, 010, 021, 120, 210		
	000, 001, 010, 021, 201, 210		
	000, 001, 010, 100, 101, 102		
	000, 001, 010, 100, 101, 110		
	000, 001, 010, 100, 101, 120		
	000, 001, 010, 100, 101, 201		
	000, 001, 010, 100, 101, 210		
	000, 001, 010, 100, 102, 110		
	000, 001, 010, 100, 102, 120		
	000, 001, 010, 100, 102, 201		
	000, 001, 010, 100, 102, 210		
	000, 001, 010, 100, 110, 120		
	000, 001, 010, 100, 110, 201		
	000, 001, 010, 100, 110, 210		
	000, 001, 010, 100, 120, 201		
	000, 001, 010, 100, 120, 210		
	000, 001, 010, 100, 201, 210		
	000, 001, 010, 101, 102, 110		
	000, 001, 010, 101, 102, 120		
	000, 001, 010, 101, 102, 201		
	000, 001, 010, 101, 102, 210		
	000, 001, 010, 101, 110, 120		
	000, 001, 010, 101, 110, 201		
	000, 001, 010, 101, 110, 210		
	000, 001, 010, 101, 120, 201		
	000, 001, 010, 101, 120, 210		
	000, 001, 010, 101, 201, 210		
	000, 001, 010, 102, 110, 120		
	000, 001, 010, 102, 110, 201		
	000, 001, 010, 102, 110, 210		
	000, 001, 010, 102, 120, 201		
	000, 001, 010, 102, 120, 210		
	000, 001, 010, 102, 201, 210		
	000, 001, 010, 110, 120, 201		
	000, 001, 010, 110, 120, 210		
	000, 001, 010, 110, 201, 210		
	000, 001, 010, 120, 201, 210		
	000, 001, 011, 021, 100, 101		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$	
	000, 001, 011, 021, 100, 102			
	000, 001, 011, 021, 100, 110			
	000, 001, 011, 021, 100, 201			
	000, 001, 011, 021, 100, 210			
	000, 001, 011, 021, 101, 102			
	000, 001, 011, 021, 101, 110			
	000, 001, 011, 021, 101, 201			
	000, 001, 011, 021, 101, 210			
	000, 001, 011, 021, 102, 110			
	000, 001, 011, 021, 102, 201			
	000, 001, 011, 021, 102, 210			
	000, 001, 011, 021, 110, 201			
	000, 001, 011, 021, 110, 210			
	000, 001, 011, 021, 201, 210			
	000, 001, 011, 100, 101, 102			
	000, 001, 011, 100, 101, 110			
	000, 001, 011, 100, 101, 201			
	000, 001, 011, 100, 101, 210			
	000, 001, 011, 100, 102, 110			
	000, 001, 011, 100, 102, 201			
	000, 001, 011, 100, 102, 210			
	000, 001, 011, 100, 110, 201			
	000, 001, 011, 100, 110, 210			
	000, 001, 011, 100, 201, 210			
	000, 001, 011, 101, 102, 110			
	000, 001, 011, 101, 102, 201			
	000, 001, 011, 101, 102, 210			
	000, 001, 011, 101, 110, 201			
	000, 001, 011, 101, 110, 210			
	000, 001, 011, 101, 201, 210			
	000, 001, 011, 102, 110, 201			
	000, 001, 011, 102, 110, 210			
	000, 001, 011, 102, 201, 210			
	000, 001, 011, 110, 201, 210	$0 \rightsquigarrow 00, 0$		
	001, 010, 011, 021, 100, 101			
	001, 010, 011, 021, 100, 102			
	001, 010, 011, 021, 100, 110			
	001, 010, 011, 021, 100, 120			
	001, 010, 011, 021, 100, 201			
	001, 010, 011, 021, 100, 210			
	001, 010, 011, 021, 101, 102			
	001, 010, 011, 021, 101, 110			
	001, 010, 011, 021, 101, 120			
	001, 010, 011, 021, 101, 201			
	001, 010, 011, 021, 101, 210			
	001, 010, 011, 021, 102, 110			
	001, 010, 011, 021, 102, 120			
	001, 010, 011, 021, 102, 201			
	001, 010, 011, 021, 102, 210			
	001, 010, 011, 021, 110, 120			
	001, 010, 011, 021, 110, 201			
	001, 010, 011, 021, 110, 210			
	001, 010, 011, 021, 120, 201			
	001, 010, 011, 021, 120, 210			
	001, 010, 011, 021, 201, 210			
	001, 010, 011, 100, 101, 102			
	001, 010, 011, 100, 101, 110			
	001, 010, 011, 100, 101, 120			
	001, 010, 011, 100, 101, 201			
	001, 010, 011, 100, 101, 210			
	001, 010, 011, 100, 102, 110			
	001, 010, 011, 100, 102, 120			
	001, 010, 011, 100, 102, 201			
	001, 010, 011, 100, 102, 210			
	001, 010, 011, 100, 110, 120			
	001, 010, 011, 100, 110, 201			
	001, 010, 011, 100, 110, 210			
	001, 010, 011, 100, 120, 201			
	001, 010, 011, 100, 120, 210			
	001, 010, 011, 100, 201, 210			
	001, 010, 011, 101, 102, 110			
	001, 010, 011, 101, 102, 120			
	001, 010, 011, 101, 102, 201			
	001, 010, 011, 101, 102, 210			
	001, 010, 011, 101, 110, 120			
	001, 010, 011, 101, 110, 201			
	001, 010, 011, 101, 110, 210			
	001, 010, 011, 101, 120, 201			
	001, 010, 011, 101, 120, 210			
	001, 010, 011, 101, 201, 210			
	001, 010, 011, 102, 110, 120			
	001, 010, 011, 102, 110, 201			
	001, 010, 011, 102, 110, 210			
	001, 010, 011, 102, 120, 201			
	001, 010, 011, 102, 120, 210			
	001, 010, 011, 102, 201, 210			
	001, 010, 011, 110, 120, 201			
	001, 010, 011, 110, 120, 210			
	001, 010, 011, 110, 201, 210			
	001, 010, 011, 120, 201, 210			
	001, 010, 012, 021, 100, 101			
	001, 010, 012, 021, 100, 102			
	001, 010, 012, 021, 100, 110			
	001, 010, 012, 021, 100, 120			
	001, 010, 012, 021, 100, 201			
	001, 010, 012, 021, 100, 210			
	001, 010, 012, 021, 101, 102			
	001, 010, 012, 021, 101, 110			
	001, 010, 012, 021, 101, 120			
	001, 010, 012, 021, 101, 201			
	001, 010, 012, 021, 101, 210			
	001, 010, 012, 021, 102, 110			
	001, 010, 012, 021, 102, 120			
	001, 010, 012, 021, 102, 201			

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001, 010, 012, 021, 102, 210		
	001, 010, 012, 021, 110, 120		
	001, 010, 012, 021, 110, 201		
	001, 010, 012, 021, 110, 210		
	001, 010, 012, 021, 120, 201		
	001, 010, 012, 021, 120, 210		
	001, 010, 012, 021, 201, 210		
	001, 010, 012, 100, 101, 102		
	001, 010, 012, 100, 101, 110		
	001, 010, 012, 100, 101, 120		
	001, 010, 012, 100, 101, 201		
	001, 010, 012, 100, 101, 210		
	001, 010, 012, 100, 102, 110		
	001, 010, 012, 100, 102, 120		
	001, 010, 012, 100, 102, 201		
	001, 010, 012, 100, 102, 210		
	001, 010, 012, 100, 110, 120		
	001, 010, 012, 100, 110, 201		
	001, 010, 012, 100, 110, 210		
	001, 010, 012, 100, 120, 201		
	001, 010, 012, 100, 120, 210		
	001, 010, 012, 100, 201, 210		
	001, 010, 012, 101, 102, 110		
	001, 010, 012, 101, 102, 120		
	001, 010, 012, 101, 102, 201		
	001, 010, 012, 101, 102, 210		
	001, 010, 012, 101, 110, 120		
	001, 010, 012, 101, 110, 201		
	001, 010, 012, 101, 110, 210		
	001, 010, 012, 101, 120, 201		
	001, 010, 012, 101, 120, 210		
	001, 010, 012, 101, 201, 210		
	001, 010, 012, 102, 110, 120		
	001, 010, 012, 102, 110, 201		
	001, 010, 012, 102, 110, 210		
	001, 010, 012, 102, 120, 201		
	001, 010, 012, 102, 120, 210		
	001, 010, 012, 102, 201, 210		
	001, 010, 012, 110, 120, 201		
	001, 010, 012, 110, 120, 210		
	001, 010, 012, 110, 201, 210		
	001, 010, 012, 120, 201, 210		
	001, 011, 012, 021, 101, 102		
	001, 011, 012, 021, 101, 110		
	001, 011, 012, 021, 101, 120		
	001, 011, 012, 021, 101, 201		
	001, 011, 012, 021, 101, 210		
	001, 011, 012, 021, 102, 110		
	001, 011, 012, 021, 102, 120		
	001, 011, 012, 021, 102, 201		
	001, 011, 012, 021, 102, 210		
	001, 011, 012, 021, 110, 120		
	001, 011, 012, 021, 110, 201		
	001, 011, 012, 021, 110, 210		
	001, 011, 012, 021, 120, 201		
	001, 011, 012, 021, 120, 210		
	001, 011, 012, 021, 201, 210		
	001, 011, 012, 101, 102, 110		
	001, 011, 012, 101, 102, 120		
	001, 011, 012, 101, 102, 201		
	001, 011, 012, 101, 102, 210		
	001, 011, 012, 101, 110, 120		
	001, 011, 012, 101, 110, 201		
	001, 011, 012, 101, 110, 210		
	001, 011, 012, 101, 120, 201		
	001, 011, 012, 101, 120, 210		
	001, 011, 012, 101, 201, 210		
	001, 011, 012, 102, 110, 120		
	001, 011, 012, 102, 110, 201		
	001, 011, 012, 102, 110, 210		
	001, 011, 012, 102, 120, 201		
	001, 011, 012, 102, 120, 210		
	001, 011, 012, 102, 201, 210		
	001, 011, 012, 110, 120, 201		
	001, 011, 012, 110, 120, 210		
	001, 011, 012, 110, 201, 210		
	001, 011, 012, 120, 201, 210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
8	000, 011, 012, 021, 100, 101		
	000, 011, 012, 021, 100, 102		
	000, 011, 012, 021, 100, 110		
	000, 011, 012, 021, 100, 120		
	000, 011, 012, 021, 100, 201		
	000, 011, 012, 021, 100, 210		
	000, 011, 012, 021, 101, 102		
	000, 011, 012, 021, 101, 110		
	000, 011, 012, 021, 101, 120		
	000, 011, 012, 021, 101, 201		
	000, 011, 012, 021, 101, 210		
	000, 011, 012, 021, 102, 110		
	000, 011, 012, 021, 102, 120		
	000, 011, 012, 021, 102, 201		
	000, 011, 012, 021, 102, 210		
	000, 011, 012, 021, 110, 120		
	000, 011, 012, 021, 110, 201		
	000, 011, 012, 021, 110, 210		
	000, 011, 012, 021, 120, 201		
	000, 011, 012, 021, 120, 210		
	000, 011, 012, 021, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
9	000, 011, 012, 100, 101, 102		
	000, 011, 012, 100, 101, 110		
	000, 011, 012, 100, 101, 120		
	000, 011, 012, 100, 101, 201		
	000, 011, 012, 100, 101, 210		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 011, 012, 100, 102, 110 000, 011, 012, 100, 102, 120 000, 011, 012, 100, 102, 201 000, 011, 012, 100, 102, 210 000, 011, 012, 100, 110, 120 000, 011, 012, 100, 110, 201 000, 011, 012, 100, 110, 210 000, 011, 012, 100, 120, 201 000, 011, 012, 100, 120, 210 000, 011, 012, 100, 201, 210 000, 011, 012, 101, 102, 110 000, 011, 012, 101, 102, 120 000, 011, 012, 101, 102, 201 000, 011, 012, 101, 102, 210 000, 011, 012, 101, 110, 120 000, 011, 012, 101, 110, 201 000, 011, 012, 101, 110, 210 000, 011, 012, 101, 120, 201 000, 011, 012, 101, 120, 210 000, 011, 012, 101, 201, 210 000, 011, 012, 102, 110, 120 000, 011, 012, 102, 110, 201 000, 011, 012, 102, 110, 210 000, 011, 012, 102, 120, 201 000, 011, 012, 102, 120, 210 000, 011, 012, 102, 201, 210 000, 011, 012, 110, 120, 201 000, 011, 012, 110, 120, 210 000, 011, 012, 110, 201, 210 000, 011, 012, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 01;\ 01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
10	000, 010, 012, 021, 100, 101 000, 010, 012, 021, 100, 102 000, 010, 012, 021, 100, 110 000, 010, 012, 021, 100, 120 000, 010, 012, 021, 100, 201 000, 010, 012, 021, 100, 210 000, 010, 012, 021, 101, 102 000, 010, 012, 021, 101, 110 000, 010, 012, 021, 101, 120 000, 010, 012, 021, 101, 201 000, 010, 012, 021, 101, 210 000, 010, 012, 021, 102, 110 000, 010, 012, 021, 102, 120 000, 010, 012, 021, 102, 201 000, 010, 012, 021, 102, 210 000, 010, 012, 021, 110, 120 000, 010, 012, 021, 110, 201 000, 010, 012, 021, 110, 210 000, 010, 012, 021, 120, 201 000, 010, 012, 021, 120, 210 000, 010, 012, 021, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 01;\ 01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
11	000, 001, 021, 100, 110, 120 000, 001, 021, 101, 110, 120 000, 001, 021, 102, 110, 120 000, 001, 021, 110, 120, 201 000, 001, 021, 110, 120, 210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2, 012;$ $012 \rightsquigarrow 00, 012$	
	001, 011, 021, 100, 101, 120 001, 011, 021, 100, 102, 120 001, 011, 021, 100, 110, 120 001, 011, 021, 100, 120, 201 001, 011, 021, 100, 120, 210 001, 011, 100, 101, 102, 120 001, 011, 100, 101, 110, 120 001, 011, 100, 101, 120, 201 001, 011, 100, 101, 120, 210 001, 011, 100, 102, 110, 120 001, 011, 100, 102, 120, 201 001, 011, 100, 102, 120, 210 001, 011, 100, 110, 120, 201 001, 011, 100, 110, 120, 210 001, 011, 100, 120, 201, 210 001, 012, 021, 100, 101, 110 001, 012, 021, 100, 102, 110 001, 012, 021, 100, 110, 120 001, 012, 021, 100, 110, 201 001, 012, 021, 100, 110, 210 001, 012, 100, 101, 102, 110 001, 012, 100, 101, 110, 120 001, 012, 100, 101, 110, 201 001, 012, 100, 101, 110, 210 001, 012, 100, 102, 110, 120 001, 012, 100, 102, 110, 201 001, 012, 100, 102, 110, 210 001, 012, 100, 110, 120, 201 001, 012, 100, 110, 120, 210 001, 012, 100, 110, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010, 00$	$x + x^3 + \frac{2x^2}{1-x}$
12	000, 010, 012, 100, 101, 110 000, 010, 012, 100, 102, 110 000, 010, 012, 100, 110, 120 000, 010, 012, 100, 110, 201 000, 010, 012, 100, 110, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 01 \rightsquigarrow 011;$ $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 3x^3 + 3x^4$
13	000, 010, 012, 100, 101, 102 000, 010, 012, 100, 101, 120 000, 010, 012, 100, 101, 201 000, 010, 012, 100, 101, 210 000, 010, 012, 100, 102, 120 000, 010, 012, 100, 102, 201 000, 010, 012, 100, 102, 210 000, 010, 012, 100, 120, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 010, 012, 100, 120, 210 000, 010, 012, 100, 201, 210 000, 010, 012, 101, 102, 110 000, 010, 012, 101, 110, 120 000, 010, 012, 101, 110, 201 000, 010, 012, 101, 110, 210 000, 010, 012, 102, 110, 120 000, 010, 012, 102, 110, 201 000, 010, 012, 102, 110, 210 000, 010, 012, 110, 120, 201 000, 010, 012, 110, 120, 210 000, 010, 012, 110, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 011$	$x + 2x^2 + 3x^3 + 3x^4 + x^5$
14	000, 010, 012, 101, 102, 120 000, 010, 012, 101, 102, 201 000, 010, 012, 101, 102, 210 000, 010, 012, 101, 120, 201 000, 010, 012, 101, 120, 210 000, 010, 012, 101, 201, 210 000, 010, 012, 102, 120, 201 000, 010, 012, 102, 120, 210 000, 010, 012, 102, 201, 210 000, 010, 012, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 0022$; $0022 \rightsquigarrow 01$	$x+2x^2+3x^3+3x^4+2x^5+x^6$
15	000, 001, 021, 100, 101, 120 000, 001, 021, 100, 102, 120 000, 001, 021, 100, 120, 201 000, 001, 021, 100, 120, 210 000, 001, 021, 101, 102, 120 000, 001, 021, 101, 120, 201 000, 001, 021, 101, 120, 210 000, 001, 021, 102, 120, 201 000, 001, 021, 102, 120, 210 000, 001, 021, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 012$; $011 \rightsquigarrow 00$; $012 \rightsquigarrow 00, 012$	$x + x^3 + x^4 + \frac{2x^2}{1-x}$
16	000, 001, 021, 100, 101, 110 000, 001, 021, 100, 102, 110 000, 001, 021, 100, 110, 201 000, 001, 021, 100, 110, 210 000, 001, 021, 101, 102, 110 000, 001, 021, 101, 110, 201 000, 001, 021, 101, 110, 210 000, 001, 021, 102, 110, 201 000, 001, 021, 102, 110, 210 000, 001, 021, 110, 201, 210 000, 001, 100, 101, 110, 120 000, 001, 100, 102, 110, 120 000, 001, 100, 110, 120, 201 000, 001, 100, 110, 120, 210 000, 001, 101, 102, 110, 120 000, 001, 101, 110, 120, 201 000, 001, 101, 110, 120, 210 000, 001, 102, 110, 120, 201 000, 001, 102, 110, 120, 210 000, 001, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow (00)^2, 01$	
	001, 011, 021, 100, 101, 102 001, 011, 021, 100, 101, 110 001, 011, 021, 100, 101, 201 001, 011, 021, 100, 101, 210 001, 011, 021, 100, 102, 110 001, 011, 021, 100, 102, 201 001, 011, 021, 100, 102, 210 001, 011, 021, 100, 110, 201 001, 011, 021, 100, 110, 210 001, 011, 021, 100, 201, 210 001, 011, 100, 101, 102, 110 001, 011, 100, 101, 102, 201 001, 011, 100, 101, 102, 210 001, 011, 100, 101, 110, 201 001, 011, 100, 101, 110, 210 001, 011, 100, 101, 201, 210 001, 011, 100, 102, 110, 201 001, 011, 100, 102, 110, 210 001, 011, 100, 102, 201, 210 001, 011, 100, 110, 201, 210 001, 012, 021, 100, 101, 102 001, 012, 021, 100, 101, 120 001, 012, 021, 100, 101, 201 001, 012, 021, 100, 101, 210 001, 012, 021, 100, 102, 120 001, 012, 021, 100, 102, 201 001, 012, 021, 100, 102, 210 001, 012, 021, 100, 120, 201 001, 012, 021, 100, 120, 210 001, 012, 021, 100, 201, 210 001, 012, 100, 101, 102, 120 001, 012, 100, 101, 102, 201 001, 012, 100, 101, 102, 210 001, 012, 100, 101, 120, 201 001, 012, 100, 101, 120, 210 001, 012, 100, 101, 201, 210 001, 012, 100, 102, 120, 201 001, 012, 100, 102, 120, 210 001, 012, 100, 102, 201, 210 001, 012, 100, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 01$	
	001, 011, 021, 101, 102, 120 001, 011, 021, 101, 110, 120 001, 011, 021, 101, 120, 201 001, 011, 021, 101, 120, 210 001, 011, 021, 102, 110, 120 001, 011, 021, 102, 120, 201 001, 011, 021, 102, 120, 210		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001, 011, 021, 110, 120, 201		
	001, 011, 021, 110, 120, 210		
	001, 011, 021, 120, 201, 210		
	001, 011, 101, 102, 110, 120		
	001, 011, 101, 102, 120, 201		
	001, 011, 101, 102, 120, 210		
	001, 011, 101, 110, 120, 201		
	001, 011, 101, 110, 120, 210		
	001, 011, 101, 120, 201, 210		
	001, 011, 102, 110, 120, 201		
	001, 011, 102, 110, 120, 210		
	001, 011, 102, 120, 201, 210		
	001, 011, 110, 120, 201, 210		
	001, 012, 021, 101, 102, 110		
	001, 012, 021, 101, 110, 120		
	001, 012, 021, 101, 110, 201		
	001, 012, 021, 101, 110, 210		
	001, 012, 021, 102, 110, 120		
	001, 012, 021, 102, 110, 201		
	001, 012, 021, 102, 110, 210		
	001, 012, 021, 110, 120, 201		
	001, 012, 021, 110, 120, 210		
	001, 012, 021, 110, 201, 210		
	001, 012, 101, 102, 110, 120		
	001, 012, 101, 102, 110, 201		
	001, 012, 101, 102, 110, 210		
	001, 012, 101, 110, 120, 201		
	001, 012, 101, 110, 120, 210		
	001, 012, 101, 110, 201, 210		
	001, 012, 102, 110, 120, 201		
	001, 012, 102, 110, 120, 210		
	001, 012, 102, 110, 201, 210		
	001, 012, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2$	$x + 2x^2 + \frac{3x^3}{1-x}$
17	000, 001, 021, 100, 101, 102		
	000, 001, 021, 100, 101, 201		
	000, 001, 021, 100, 101, 210		
	000, 001, 021, 100, 102, 201		
	000, 001, 021, 100, 102, 210		
	000, 001, 021, 100, 201, 210		
	000, 001, 021, 101, 102, 201		
	000, 001, 021, 101, 102, 210		
	000, 001, 021, 101, 201, 210		
	000, 001, 021, 102, 201, 210		
	000, 001, 100, 101, 102, 120		
	000, 001, 100, 101, 120, 201		
	000, 001, 100, 101, 120, 210		
	000, 001, 100, 102, 120, 201		
	000, 001, 100, 102, 120, 210		
	000, 001, 100, 120, 201, 210		
	000, 001, 101, 102, 120, 201		
	000, 001, 101, 102, 120, 210		
	000, 001, 101, 120, 201, 210		
	000, 001, 102, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 01$; $011 \rightsquigarrow 00$	$x + 2x^2 + 3x^3 + \frac{4x^4}{1-x}$
18	000, 001, 100, 101, 102, 110		
	000, 001, 100, 101, 110, 201		
	000, 001, 100, 101, 110, 210		
	000, 001, 100, 102, 110, 201		
	000, 001, 100, 102, 110, 210		
	000, 001, 100, 110, 201, 210		
	000, 001, 101, 102, 110, 201		
	000, 001, 101, 102, 110, 210		
	000, 001, 101, 110, 201, 210		
	000, 001, 102, 110, 201, 210	$a_m \rightsquigarrow (00)^m, a_{m+1}$, $a_m = 01 \cdots m$	
	000, 010, 011, 021, 100, 101		
	000, 010, 011, 021, 100, 102		
	000, 010, 011, 021, 100, 110		
	000, 010, 011, 021, 100, 120		
	000, 010, 011, 021, 100, 201		
	000, 010, 011, 021, 100, 210		
	000, 010, 011, 021, 101, 102		
	000, 010, 011, 021, 101, 110		
	000, 010, 011, 021, 101, 120		
	000, 010, 011, 021, 101, 201		
	000, 010, 011, 021, 101, 210		
	000, 010, 011, 021, 102, 110		
	000, 010, 011, 021, 102, 120		
	000, 010, 011, 021, 102, 201		
	000, 010, 011, 021, 102, 210		
	000, 010, 011, 021, 110, 120		
	000, 010, 011, 021, 110, 201		
	000, 010, 011, 021, 110, 210		
	000, 010, 011, 021, 120, 201		
	000, 010, 011, 021, 120, 210		
	000, 010, 011, 021, 201, 210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 01$	
	001, 010, 021, 100, 101, 102		
	001, 010, 021, 100, 101, 110		
	001, 010, 021, 100, 101, 120		
	001, 010, 021, 100, 101, 201		
	001, 010, 021, 100, 101, 210		
	001, 010, 021, 100, 102, 110		
	001, 010, 021, 100, 102, 120		
	001, 010, 021, 100, 102, 201		
	001, 010, 021, 100, 102, 210		
	001, 010, 021, 100, 110, 120		
	001, 010, 021, 100, 110, 201		
	001, 010, 021, 100, 110, 210		
	001, 010, 021, 100, 120, 201		
	001, 010, 021, 100, 120, 210		
	001, 010, 021, 100, 201, 210		
	001, 010, 021, 101, 102, 110		
	001, 010, 021, 101, 102, 120		
	001, 010, 021, 101, 102, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001, 010, 021, 101, 102, 210		
	001, 010, 021, 101, 110, 120		
	001, 010, 021, 101, 110, 201		
	001, 010, 021, 101, 110, 210		
	001, 010, 021, 101, 120, 201		
	001, 010, 021, 101, 120, 210		
	001, 010, 021, 101, 201, 210		
	001, 010, 021, 102, 110, 120		
	001, 010, 021, 102, 110, 201		
	001, 010, 021, 102, 110, 210		
	001, 010, 021, 102, 120, 201		
	001, 010, 021, 102, 120, 210		
	001, 010, 021, 102, 201, 210		
	001, 010, 021, 110, 120, 201		
	001, 010, 021, 110, 120, 210		
	001, 010, 021, 110, 201, 210		
	001, 010, 021, 120, 201, 210		
	001, 010, 100, 101, 102, 110		
	001, 010, 100, 101, 102, 120		
	001, 010, 100, 101, 102, 201		
	001, 010, 100, 101, 102, 210		
	001, 010, 100, 101, 110, 120		
	001, 010, 100, 101, 110, 201		
	001, 010, 100, 101, 110, 210		
	001, 010, 100, 101, 120, 201		
	001, 010, 100, 101, 120, 210		
	001, 010, 100, 101, 201, 210		
	001, 010, 100, 102, 110, 120		
	001, 010, 100, 102, 110, 201		
	001, 010, 100, 102, 110, 210		
	001, 010, 100, 102, 120, 201		
	001, 010, 100, 102, 120, 210		
	001, 010, 100, 102, 201, 210		
	001, 010, 100, 110, 120, 201		
	001, 010, 100, 110, 120, 210		
	001, 010, 100, 110, 201, 210		
	001, 010, 100, 120, 201, 210		
	001, 010, 101, 102, 110, 120		
	001, 010, 101, 102, 110, 201		
	001, 010, 101, 102, 110, 210		
	001, 010, 101, 102, 120, 201		
	001, 010, 101, 102, 120, 210		
	001, 010, 101, 102, 201, 210		
	001, 010, 101, 110, 120, 201		
	001, 010, 101, 110, 120, 210		
	001, 010, 101, 110, 201, 210		
	001, 010, 101, 120, 201, 210		
	001, 010, 102, 110, 120, 201		
	001, 010, 102, 110, 120, 210		
	001, 010, 102, 110, 201, 210		
	001, 010, 102, 120, 201, 210		
	001, 010, 110, 120, 201, 210		
	001, 011, 021, 101, 102, 110		
	001, 011, 021, 101, 102, 201		
	001, 011, 021, 101, 102, 210		
	001, 011, 021, 101, 110, 201		
	001, 011, 021, 101, 110, 210		
	001, 011, 021, 101, 201, 210		
	001, 011, 021, 102, 110, 201		
	001, 011, 021, 102, 110, 210		
	001, 011, 021, 102, 201, 210		
	001, 011, 021, 110, 201, 210		
	001, 011, 101, 102, 110, 201		
	001, 011, 101, 102, 110, 210		
	001, 011, 101, 102, 201, 210		
	001, 011, 101, 110, 201, 210		
	001, 011, 102, 110, 201, 210		
	001, 012, 021, 101, 102, 120		
	001, 012, 021, 101, 102, 201		
	001, 012, 021, 101, 102, 210		
	001, 012, 021, 101, 120, 201		
	001, 012, 021, 101, 120, 210		
	001, 012, 021, 101, 201, 210		
	001, 012, 021, 102, 120, 201		
	001, 012, 021, 102, 120, 210		
	001, 012, 021, 102, 201, 210		
	001, 012, 021, 120, 201, 210		
	001, 012, 101, 102, 120, 201		
	001, 012, 101, 102, 120, 210		
	001, 012, 101, 102, 201, 210		
	001, 012, 101, 120, 201, 210		
	001, 012, 102, 120, 201, 210	$0 \rightsquigarrow 00, 0;\ 00 \rightsquigarrow 00$	
	010, 011, 012, 021, 100, 101		
	010, 011, 012, 021, 100, 102		
	010, 011, 012, 021, 100, 110		
	010, 011, 012, 021, 100, 120		
	010, 011, 012, 021, 100, 201		
	010, 011, 012, 021, 100, 210		
	010, 011, 012, 021, 101, 102		
	010, 011, 012, 021, 101, 110		
	010, 011, 012, 021, 101, 120		
	010, 011, 012, 021, 101, 201		
	010, 011, 012, 021, 101, 210		
	010, 011, 012, 021, 102, 110		
	010, 011, 012, 021, 102, 120		
	010, 011, 012, 021, 102, 201		
	010, 011, 012, 021, 102, 210		
	010, 011, 012, 021, 110, 120		
	010, 011, 012, 021, 110, 201		
	010, 011, 012, 021, 110, 210		
	010, 011, 012, 021, 120, 201		
	010, 011, 012, 021, 120, 210		
	010, 011, 012, 021, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m,\ a_m = 0^m$	$\frac{x}{(1-x)^2}$

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
19	000, 010, 011, 100, 102, 120 000, 010, 011, 101, 102, 120 000, 010, 011, 102, 110, 120 000, 010, 011, 102, 120, 201 000, 010, 011, 102, 120, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
20	000, 001, 100, 101, 102, 210 000, 001, 100, 101, 201, 210 000, 001, 100, 102, 201, 210 000, 001, 101, 102, 201, 210	$a_0 \rightsquigarrow 00, a_1$; $a_m \rightsquigarrow (00)^m, b_m, a_{m+1}$; $b_m \rightsquigarrow (00)^m$, $a_m = 01 \cdots m$, $b_m = a_m m$	
	000, 010, 011, 100, 101, 102 000, 010, 011, 100, 102, 110 000, 010, 011, 100, 102, 201 000, 010, 011, 100, 102, 210 000, 010, 011, 101, 102, 110 000, 010, 011, 101, 102, 201 000, 010, 011, 101, 102, 210 000, 010, 011, 102, 110, 201 000, 010, 011, 102, 110, 210 000, 010, 011, 102, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 002$	$\frac{x(1+x^3)}{(1-x)^2}$
21	010, 011, 012, 100, 101, 210 010, 011, 012, 100, 102, 210 010, 011, 012, 100, 110, 210 010, 011, 012, 100, 120, 210 010, 011, 012, 100, 201, 210 010, 011, 012, 101, 102, 210 010, 011, 012, 101, 110, 210 010, 011, 012, 101, 120, 210 010, 011, 012, 101, 201, 210 010, 011, 012, 102, 110, 210 010, 011, 012, 102, 120, 210 010, 011, 012, 102, 201, 210 010, 011, 012, 110, 120, 210 010, 011, 012, 110, 201, 210 010, 011, 012, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (01)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
22	000, 001, 100, 101, 102, 201	$a_m \rightsquigarrow a_{m+1}, b_0, \ldots, b_m$; $b_m \rightsquigarrow b_0, \ldots \cdot_{m-1}$, $a_m = 01 \cdots m$, $b_m = a_m m$	
	000, 010, 011, 100, 101, 120 000, 010, 011, 100, 110, 120 000, 010, 011, 100, 120, 201 000, 010, 011, 100, 120, 210 000, 010, 011, 101, 110, 120 000, 010, 011, 101, 120, 201 000, 010, 011, 101, 120, 210 000, 010, 011, 110, 120, 201 000, 010, 011, 110, 120, 210 000, 010, 011, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 0$; $01 \rightsquigarrow 01$	
	010, 011, 012, 100, 101, 102 010, 011, 012, 100, 101, 110 010, 011, 012, 100, 101, 120 010, 011, 012, 100, 101, 201 010, 011, 012, 100, 102, 110 010, 011, 012, 100, 102, 120 010, 011, 012, 100, 102, 201 010, 011, 012, 100, 110, 120 010, 011, 012, 100, 110, 201 010, 011, 012, 100, 120, 201 010, 011, 012, 101, 102, 110 010, 011, 012, 101, 102, 120 010, 011, 012, 101, 102, 201 010, 011, 012, 101, 110, 120 010, 011, 012, 101, 110, 201 010, 011, 012, 101, 120, 201 010, 011, 012, 102, 110, 120 010, 011, 012, 102, 110, 201 010, 011, 012, 102, 120, 201 010, 011, 012, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow b_1, \ldots, b_{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1+x)}{1-x-x^2}$
23	000, 010, 011, 100, 101, 110 000, 010, 011, 100, 101, 201 000, 010, 011, 100, 101, 210 000, 010, 011, 100, 110, 201 000, 010, 011, 100, 110, 210 000, 010, 011, 100, 201, 210 000, 010, 011, 101, 110, 201 000, 010, 011, 101, 110, 210 000, 010, 011, 101, 201, 210 000, 010, 011, 110, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00)^2$; $01 \rightsquigarrow 01$	$\frac{x(1-x-x^2)}{(1-x)(1-2x)}$
24	000, 010, 100, 102, 110, 120●		
25	000, 010, 100, 101, 102, 120●		
26	000, 010, 101, 102, 110, 120●		
27	000, 010, 100, 101, 102, 110●		
28	000, 010, 100, 102, 120, 201●		
29	000, 010, 100, 102, 120, 210●		
30	000, 010, 102, 110, 120, 201●		
31	000, 010, 101, 102, 120, 201● 000, 010, 101, 102, 120, 210● 000, 010, 102, 110, 120, 210●		Modification of proof of Theorem 7

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
32	000, 010, 100, 102, 110, 210●		
33	000, 010, 100, 102, 110, 201●		
34	000, 010, 101, 102, 110, 210●		
35	000, 010, 101, 102, 110, 201●		
36	000, 010, 100, 101, 102, 210●		
37	000, 010, 100, 101, 102, 201●		
38	000, 010, 100, 101, 110, 120●		
39	000, 010, 102, 120, 201, 210●		
40	000, 010, 102, 110, 201, 210●		
41	000, 010, 100, 102, 201, 210●		
42	000, 010, 101, 102, 201, 210●		
43	000, 010, 100, 101, 110, 201●	$a_{m,m} \rightsquigarrow b_m, \{a_{m,i}\}_{i=m}^{2m}$; $a_{m,j} \rightsquigarrow b_{2m-j}, \{c_{2m-j+i,2m-j+2i}\}_{i=2}^{j-m}, b_{2m-j}, \{a_{m,i}\}_{i=j}^{2m}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$; $c_{m,j} \rightsquigarrow b_{2m+1-j}, \{c_{2m+1-j+i,2m+1-j+2i}\}_{i=2}^{j-m-1}, \{a_{m,i}\}_{i=j-1}^{2m}$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = a_{m,m}m$, $c_{m,j} = a_{m,j}(j-1)$	
	000, 010, 100, 101, 110, 210●	$a_{m,m} \rightsquigarrow b_m, \{a_{m,i}\}_{i=m}^{2m}$; $a_{m,j} \rightsquigarrow b_{2m-j}, \{c_{2m-j+i,2m-j+2i}\}_{i=2}^{j-m}, b_{2m-j}, \{a_{m,i}\}_{i=j}^{2m}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$; $c_{m,j} \rightsquigarrow b_{2m+1-j}, \{c_{2m+1-j+i,2m+1-j+2i}\}_{i=2}^{j-m-1}, \{a_{m,i}\}_{i=j-1}^{2m}$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = a_{m,m}m$, $c_{m,j} = a_{m,j}m$	
44	000, 010, 100, 101, 120, 201● 000, 010, 100, 101, 120, 210● 000, 010, 100, 110, 120, 201● 000, 010, 100, 110, 120, 210● 000, 010, 101, 110, 120, 201●		Theorem 13
45	000, 010, 101, 110, 120, 210●		
46	000, 010, 100, 120, 201, 210●		
47	000, 010, 100, 110, 201, 210● 000, 010, 101, 110, 201, 210●	$a_{m,j} \rightsquigarrow (a_{2m-j+1,2m-j+1})^{j-m}, b_{2m-j}, \{a_{m,i}\}_{i=j}^{2m}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$, $a_{m,j} = 0^2 1^2 \cdots (m-1)^2 j$, $b_m = 0^2 1^2 \cdots m^2$	
48	000, 010, 101, 120, 201, 210● 000, 010, 110, 120, 201, 210●		Modification of the proof of Theorem 12
49	000, 010, 100, 101, 201, 210●		
50	000, 012, 021, 100, 101, 110 000, 012, 021, 101, 102, 110 000, 012, 021, 101, 110, 120 000, 012, 021, 101, 110, 201 000, 012, 021, 101, 110, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^4$
51	000, 012, 021, 100, 101, 102 000, 012, 021, 100, 101, 120 000, 012, 021, 100, 101, 201 000, 012, 021, 100, 101, 210 000, 012, 021, 101, 102, 120 000, 012, 021, 101, 102, 201 000, 012, 021, 101, 102, 210 000, 012, 021, 101, 120, 201 000, 012, 021, 101, 120, 210 000, 012, 021, 101, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000, 012, 021, 100, 102, 110 000, 012, 021, 100, 110, 120 000, 012, 021, 100, 110, 201 000, 012, 021, 100, 110, 210 000, 012, 021, 102, 110, 120 000, 012, 021, 102, 110, 201 000, 012, 021, 102, 110, 210 000, 012, 021, 110, 120, 201 000, 012, 021, 110, 120, 210 000, 012, 021, 110, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	
	000, 012, 100, 101, 102, 110 000, 012, 100, 101, 110, 120 000, 012, 100, 101, 110, 201 000, 012, 100, 101, 110, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 3x^4$
52	000, 012, 101, 102, 110, 120 000, 012, 101, 102, 110, 201 000, 012, 101, 102, 110, 210 000, 012, 101, 110, 120, 201 000, 012, 101, 110, 120, 210 000, 012, 101, 110, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 010$	$x + 2x^2 + 4x^3 + 3x^4 + x^5$
53	000, 012, 021, 100, 102, 120		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 012, 021, 100, 102, 201 000, 012, 021, 100, 102, 210 000, 012, 021, 100, 120, 201 000, 012, 021, 100, 120, 210 000, 012, 021, 100, 201, 210 000, 012, 021, 102, 120, 201 000, 012, 021, 102, 120, 210 000, 012, 021, 102, 201, 210 000, 012, 021, 120, 201, 210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$	
	000, 012, 100, 102, 110, 120 000, 012, 100, 102, 110, 201 000, 012, 100, 102, 110, 210 000, 012, 100, 110, 120, 201 000, 012, 100, 110, 120, 210 000, 012, 100, 110, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$; $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 4x^3 + 4x^4$
54	000, 012, 100, 101, 102, 120 000, 012, 100, 101, 102, 201 000, 012, 100, 101, 102, 210 000, 012, 100, 101, 120, 201 000, 012, 100, 101, 120, 210 000, 012, 100, 101, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000, 012, 102, 110, 120, 201 000, 012, 102, 110, 120, 210 000, 012, 102, 110, 201, 210 000, 012, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	$x + 2x^2 + 4x^3 + 4x^4 + x^5$
55	000, 012, 101, 102, 120, 201 000, 012, 101, 102, 120, 210 000, 012, 101, 102, 201, 210 000, 012, 101, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 0022$; $0022 \rightsquigarrow 001$	$x+2x^2+4x^3+4x^4+2x^5+x^6$
56	000, 011, 021, 100, 102, 120 000, 011, 021, 101, 102, 120 000, 011, 021, 102, 110, 120 000, 011, 021, 102, 120, 201 000, 011, 021, 102, 120, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	
	001, 021, 100, 101, 110, 120 001, 021, 100, 102, 110, 120 001, 021, 100, 110, 120, 201 001, 021, 100, 110, 120, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2(1-x)^2)}{(1-x)^2}$
57	000, 012, 100, 102, 120, 201 000, 012, 100, 102, 120, 210 000, 012, 100, 102, 201, 210 000, 012, 100, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 001$; $001 \rightsquigarrow 0011$; $002 \rightsquigarrow 0011, 001$	$x + 2x^2 + 4x^3 + 5x^4 + x^5$
58	000, 012, 102, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 001$; $001 \rightsquigarrow 0011$; $002 \rightsquigarrow 001, 0022$; $0022 \rightsquigarrow 001$	$x+2x^2+4x^3+5x^4+2x^5+x^6$
59	000, 011, 021, 100, 101, 102 000, 011, 021, 100, 102, 110 000, 011, 021, 100, 102, 201 000, 011, 021, 100, 102, 210 000, 011, 021, 101, 102, 110 000, 011, 021, 101, 102, 201 000, 011, 021, 101, 102, 210 000, 011, 021, 102, 110, 201 000, 011, 021, 102, 110, 210 000, 011, 021, 102, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 01$; $002 \rightsquigarrow 002$	
	000, 011, 100, 101, 102, 120 000, 011, 100, 102, 110, 120 000, 011, 100, 102, 120, 201 000, 011, 100, 102, 120, 210 000, 011, 101, 102, 110, 120 000, 011, 101, 102, 120, 201 000, 011, 101, 102, 120, 210 000, 011, 102, 110, 120, 201 000, 011, 102, 110, 120, 210 000, 011, 102, 120, 201, 210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 012$; $012 \rightsquigarrow 012$	
	001, 021, 100, 101, 102, 110 001, 021, 100, 101, 110, 201 001, 021, 100, 101, 110, 210 001, 021, 100, 102, 110, 201 001, 021, 100, 102, 110, 210 001, 021, 100, 110, 201, 210 001, 100, 101, 102, 110, 120 001, 100, 101, 110, 120, 201 001, 100, 101, 110, 120, 210 001, 100, 102, 110, 120, 201 001, 100, 102, 110, 120, 210 001, 100, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 01$	
	001, 021, 100, 101, 102, 120 001, 021, 100, 101, 120, 201 001, 021, 100, 101, 120, 210 001, 021, 100, 102, 120, 201 001, 021, 100, 102, 120, 210 001, 021, 100, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 012$; $011 \rightsquigarrow 010, 011$; $012 \rightsquigarrow 00, 012$	
	001, 021, 101, 102, 110, 120 001, 021, 101, 110, 120, 201 001, 021, 101, 110, 120, 210 001, 021, 102, 110, 120, 201 001, 021, 102, 110, 120, 210 001, 021, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2-x^3)}{(1-x)^2}$

Continuation of Table 2			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
60	000, 011, 021, 100, 101, 120 000, 011, 021, 100, 110, 120 000, 011, 021, 100, 120, 201 000, 011, 021, 100, 120, 210 000, 011, 021, 101, 110, 120 000, 011, 021, 101, 120, 201 000, 011, 021, 101, 120, 210 000, 011, 021, 110, 120, 201 000, 011, 021, 110, 120, 210 000, 011, 021, 120, 201, 210	$0 \rightsquigarrow (00)^2;\ 00 \rightsquigarrow 00, 002;\ 002 \rightsquigarrow 002$	
	000, 011, 100, 101, 102, 110 000, 011, 100, 101, 102, 201 000, 011, 100, 101, 102, 210 000, 011, 100, 102, 110, 201 000, 011, 100, 102, 110, 210 000, 011, 100, 102, 201, 210 000, 011, 101, 102, 110, 201 000, 011, 101, 102, 110, 210 000, 011, 101, 102, 201, 210 000, 011, 102, 110, 201, 210	$0 \rightsquigarrow 0, 01;\ 01 \rightsquigarrow 010, 01$	
	001, 021, 100, 101, 102, 201 001, 021, 100, 101, 102, 210 001, 021, 100, 101, 201, 210 001, 021, 100, 102, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010, 011, 01;$ $011 \rightsquigarrow 010, 011$	
	001, 021, 101, 102, 110, 201 001, 021, 101, 102, 110, 210 001, 021, 101, 110, 201, 210 001, 021, 102, 110, 201, 210 001, 101, 102, 110, 120, 201 001, 101, 102, 110, 120, 210 001, 101, 110, 120, 201, 210 001, 102, 110, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow (00)^2, 01$	
	001, 021, 101, 102, 120, 201 001, 021, 101, 102, 120, 210 001, 021, 101, 120, 201, 210 001, 021, 102, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 00, (011)^2;$ $011 \rightsquigarrow 00, 011$	
	001, 100, 101, 102, 110, 201 001, 100, 101, 102, 110, 210 001, 100, 101, 110, 201, 210 001, 100, 102, 110, 201, 210	$a_m \rightsquigarrow (010)^m, 00, a_{m+1};\ 00 \rightsquigarrow 00,$ $a_m = 01 \cdots m$	
	001, 100, 101, 102, 120, 201 001, 100, 101, 102, 120, 210 001, 100, 101, 120, 201, 210 001, 100, 102, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01, 010, 011, 01;$ $011 \rightsquigarrow 010, 011$	
	011, 012, 021, 100, 101, 102 011, 012, 021, 100, 101, 110 011, 012, 021, 100, 101, 120 011, 012, 021, 100, 101, 201 011, 012, 021, 100, 101, 210 011, 012, 021, 100, 102, 110 011, 012, 021, 100, 102, 120 011, 012, 021, 100, 102, 201 011, 012, 021, 100, 102, 210 011, 012, 021, 100, 110, 120 011, 012, 021, 100, 110, 201 011, 012, 021, 100, 110, 210 011, 012, 021, 100, 120, 201 011, 012, 021, 100, 120, 210 011, 012, 021, 100, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m;\ 01 \rightsquigarrow 010,$ $a_m = 0^m$	$\frac{x(1+x^2)}{(1-x)^2}$
61	000, 011, 021, 100, 101, 110 000, 011, 021, 100, 101, 201 000, 011, 021, 100, 101, 210 000, 011, 021, 100, 110, 201 000, 011, 021, 100, 110, 210 000, 011, 021, 100, 201, 210 000, 011, 021, 101, 110, 201 000, 011, 021, 101, 110, 210 000, 011, 021, 101, 201, 210 000, 011, 021, 110, 201, 210	$0 \rightsquigarrow 00, 0;\ 00 \rightsquigarrow 00, 002;\ 002 \rightsquigarrow 002$	
	001, 021, 101, 102, 201, 210 001, 101, 102, 120, 201, 210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 00, 011, 01;$ $011 \rightsquigarrow 00, 011$	
	001, 100, 101, 102, 201, 210	$a_m \rightsquigarrow (010)^m, b_m, a_{m+1};$ $b_m \rightsquigarrow (010)^m, b_m,\ a_m = 01 \cdots m,$ $b_m = a_m m$	
	001, 101, 102, 110, 201, 210	$a_m \rightsquigarrow (00)^{m+1}, a_{m+1};\ 00 \rightsquigarrow 00,$ $a_m = 01 \cdots m$	
	010, 012, 021, 100, 101, 102 010, 012, 021, 100, 101, 110 010, 012, 021, 100, 101, 120 010, 012, 021, 100, 101, 201 010, 012, 021, 100, 101, 210 010, 012, 021, 100, 102, 110 010, 012, 021, 100, 102, 120 010, 012, 021, 100, 102, 201 010, 012, 021, 100, 102, 210 010, 012, 021, 100, 110, 120 010, 012, 021, 100, 110, 201 010, 012, 021, 100, 110, 210 010, 012, 021, 100, 120, 201 010, 012, 021, 100, 120, 210 010, 012, 021, 100, 201, 210 010, 012, 021, 101, 102, 110 010, 012, 021, 101, 102, 120 010, 012, 021, 101, 102, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010, 012, 021, 101, 102, 210 010, 012, 021, 101, 110, 120 010, 012, 021, 101, 110, 201 010, 012, 021, 101, 110, 210 010, 012, 021, 101, 120, 201 010, 012, 021, 101, 120, 210 010, 012, 021, 101, 201, 210 010, 012, 021, 102, 110, 120 010, 012, 021, 102, 110, 201 010, 012, 021, 102, 110, 210 010, 012, 021, 102, 120, 201 010, 012, 021, 102, 120, 210 010, 012, 021, 102, 201, 210 010, 012, 021, 110, 120, 201 010, 012, 021, 110, 120, 210 010, 012, 021, 110, 201, 210 010, 012, 021, 120, 201, 210 011, 012, 021, 101, 102, 110 011, 012, 021, 101, 102, 120 011, 012, 021, 101, 102, 201 011, 012, 021, 101, 102, 210 011, 012, 021, 101, 110, 120 011, 012, 021, 101, 110, 201 011, 012, 021, 101, 110, 210 011, 012, 021, 101, 120, 201 011, 012, 021, 101, 120, 210 011, 012, 021, 101, 201, 210 011, 012, 021, 102, 110, 120 011, 012, 021, 102, 110, 201 011, 012, 021, 102, 110, 210 011, 012, 021, 102, 120, 201 011, 012, 021, 102, 120, 210 011, 012, 021, 102, 201, 210 011, 012, 021, 110, 120, 201 011, 012, 021, 110, 120, 210 011, 012, 021, 110, 201, 210 011, 012, 021, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01$, $a_m = 0^m$	
	011, 012, 100, 101, 201, 210 011, 012, 100, 102, 201, 210 011, 012, 100, 110, 201, 210 011, 012, 100, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^2)}{(1-x)^3}$
62	011, 012, 100, 101, 102, 210 011, 012, 100, 101, 110, 210 011, 012, 100, 101, 120, 210 011, 012, 100, 102, 110, 210 011, 012, 100, 102, 120, 210 011, 012, 100, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow c_1$; $b_2 \rightsquigarrow b_1, c_1$; $b_m \rightsquigarrow c_m, (010)^{m-1}$; $c_m \rightsquigarrow (010)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x+x^2+x^4)}{(1-x)^3}$
63	000, 011, 100, 101, 110, 120 000, 011, 100, 101, 120, 201 000, 011, 100, 101, 120, 210 000, 011, 100, 110, 120, 201 000, 011, 100, 110, 120, 210 000, 011, 100, 120, 201, 210 000, 011, 101, 110, 120, 201 000, 011, 101, 110, 120, 210 000, 011, 101, 120, 201, 210 000, 011, 110, 120, 201, 210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 0, 012$; $012 \rightsquigarrow 012$	
	011, 012, 100, 101, 102, 201 011, 012, 100, 101, 110, 201 011, 012, 100, 101, 120, 201 011, 012, 100, 102, 110, 201 011, 012, 100, 102, 120, 201 011, 012, 100, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x}{(1-x)(1-x-x^2)}$
64	011, 012, 100, 101, 102, 110 011, 012, 100, 101, 102, 120 011, 012, 100, 101, 110, 120 011, 012, 100, 102, 110, 120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow (b_1)^2$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m-1}$; $c_m \rightsquigarrow 010, b_1, \{c_i\}_{i=3}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x^2-x^3)}{(1-x-x^2)^2}$
65	010, 012, 100, 101, 110, 210 010, 012, 100, 102, 110, 210 010, 012, 100, 110, 120, 210 010, 012, 100, 110, 201, 210 011, 012, 101, 102, 201, 210 011, 012, 101, 110, 201, 210 011, 012, 101, 120, 201, 210 011, 012, 102, 110, 201, 210 011, 012, 102, 120, 201, 210 011, 012, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (0021)^{m-1}, b_1$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+x^2-x+1)}{(1-x)^3}$
66	010, 012, 100, 101, 102, 110 010, 012, 100, 101, 110, 120 010, 012, 100, 101, 110, 201 010, 012, 100, 102, 110, 120		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010, 012, 100, 102, 110, 201 010, 012, 100, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=2}^{m}, b_1$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x((1-x)^2+x^3-2x^4)}{(1-x)^3(1-x-x^2)}$
67	010, 012, 100, 101, 102, 210 010, 012, 100, 101, 120, 210 010, 012, 100, 101, 201, 210 010, 012, 100, 102, 120, 210 010, 012, 100, 102, 201, 210 010, 012, 100, 120, 201, 210 011, 012, 101, 102, 110, 210 011, 012, 101, 102, 120, 210 011, 012, 101, 110, 120, 210 011, 012, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (0021)^{m-1}, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	010, 012, 101, 102, 110, 210 010, 012, 101, 110, 120, 210 010, 012, 101, 110, 201, 210 010, 012, 102, 110, 120, 210 010, 012, 102, 110, 201, 210 010, 012, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (b_1)^m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-2x+2x^2)}{(1-x)^4}$
68	010, 012, 100, 101, 102, 120 010, 012, 100, 101, 102, 201 010, 012, 100, 101, 120, 201 010, 012, 100, 102, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=2}^{m}, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	
	010, 012, 101, 102, 110, 120 010, 012, 101, 102, 110, 201 010, 012, 101, 110, 120, 201 010, 012, 102, 110, 120, 201 011, 012, 101, 102, 110, 201 011, 012, 101, 102, 120, 201 011, 012, 101, 110, 120, 201 011, 012, 102, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}, b_1$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3-x+1)}{(1-x)^2(1-x-x^2)}$
69	010, 012, 101, 102, 120, 210 010, 012, 101, 102, 201, 210 010, 012, 101, 120, 201, 210 010, 012, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (b_1)^{m-1}, b_m$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-3x+4x^2-2x^3+x^4)}{(1-x)^5}$
70	000, 011, 100, 101, 110, 201 000, 011, 100, 101, 110, 210 000, 011, 100, 101, 201, 210 000, 011, 100, 110, 201, 210 000, 011, 101, 110, 201, 210	$0 \rightsquigarrow (0)^2$	
	010, 011, 021, 100, 101, 102 010, 011, 021, 100, 101, 110 010, 011, 021, 100, 101, 120 010, 011, 021, 100, 101, 201 010, 011, 021, 100, 101, 210 010, 011, 021, 100, 102, 110 010, 011, 021, 100, 102, 120 010, 011, 021, 100, 102, 201 010, 011, 021, 100, 102, 210 010, 011, 021, 100, 110, 120 010, 011, 021, 100, 110, 201 010, 011, 021, 100, 110, 210 010, 011, 021, 100, 120, 201 010, 011, 021, 100, 120, 210 010, 011, 021, 100, 201, 210 010, 011, 021, 101, 102, 110 010, 011, 021, 101, 102, 120 010, 011, 021, 101, 102, 201 010, 011, 021, 101, 102, 210 010, 011, 021, 101, 110, 120 010, 011, 021, 101, 110, 201 010, 011, 021, 101, 110, 210 010, 011, 021, 101, 120, 201 010, 011, 021, 101, 120, 210 010, 011, 021, 101, 201, 210 010, 011, 021, 102, 110, 120 010, 011, 021, 102, 110, 201 010, 011, 021, 102, 110, 210 010, 011, 021, 102, 120, 201 010, 011, 021, 102, 120, 210 010, 011, 021, 102, 201, 210 010, 011, 021, 110, 120, 201 010, 011, 021, 110, 120, 210 010, 011, 021, 110, 201, 210 010, 011, 021, 120, 201, 210 010, 012, 101, 102, 120, 201 011, 012, 101, 102, 110, 120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$	$\frac{x}{1-2x}$

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
71	010, 011, 102, 120, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(0021)^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x((1-x)^2+x^3)}{(1-2x)(1-x)^2}$
72	010, 011, 100, 102, 120, 201• 010, 011, 101, 102, 120, 201 010, 011, 102, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow c_2, \ldots, c_{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	
	010, 011, 100, 102, 120, 210• 010, 011, 101, 102, 120, 210 010, 011, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m1$	$\frac{x(2x^3-2x+1)}{(1-x)(1-2x)(1-x-x^2)}$
73	010, 011, 100, 101, 102, 120 010, 011, 100, 102, 110, 120 010, 011, 101, 102, 110, 120	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_{j,i}\}_{i=1}^{j-2}, c_{j,1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_{m,1} \rightsquigarrow \{c_{m-1,i}\}_{i=1}^{m-3}, c_{m-1,1}$; $c_{m,j} \rightsquigarrow$ $\{c_{j,i}\}_{i=1}^{j-2}, c_{j,1}, \{c_{m-j,i}\}_{i=1}^{m-2-j}$, $c_{m-j,1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_{m,j} = a_m mj$	$\frac{x(3x^3-x^2-2x+1)}{(1-x^2)(1-2x)^2}$
74	010, 011, 100, 102, 201, 210• 010, 011, 101, 102, 201, 210 010, 011, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, \{b_{m,i}\}_{i=j}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x}{1-x} + \frac{x^2(1-x)}{(1-2x)^2}$
75	010, 011, 100, 101, 102, 210 010, 011, 100, 102, 110, 210 010, 011, 101, 102, 110, 210	$\mathcal{T}(\{010, 011, 102, 210\})$, [4]	$\frac{x(3x^3+x^2-3x+1)}{(1-2x)^2(1-x-x^2)}$
76	010, 011, 100, 101, 102, 201 010, 011, 100, 102, 110, 201 010, 011, 101, 102, 110, 201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, \{b_{m,i}\}_{i=j}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	$\frac{x(1-x-x^2)}{(1-x)(1-2x-x^2)}$
77	010, 011, 100, 101, 102, 110	$\mathcal{T}(\{010, 011, 102\})$	
78	000, 010, 021, 100, 101, 102 000, 010, 021, 100, 101, 110 000, 010, 021, 100, 101, 120 000, 010, 021, 100, 101, 201 000, 010, 021, 100, 101, 210 000, 010, 021, 100, 102, 110 000, 010, 021, 100, 102, 120 000, 010, 021, 100, 102, 201 000, 010, 021, 100, 102, 210 000, 010, 021, 100, 110, 120 000, 010, 021, 100, 110, 201 000, 010, 021, 100, 110, 210 000, 010, 021, 100, 120, 201 000, 010, 021, 100, 120, 210 000, 010, 021, 100, 201, 210 000, 010, 021, 101, 102, 110 000, 010, 021, 101, 102, 120 000, 010, 021, 101, 102, 201 000, 010, 021, 101, 102, 210 000, 010, 021, 101, 110, 120 000, 010, 021, 101, 110, 201 000, 010, 021, 101, 110, 210 000, 010, 021, 101, 120, 201 000, 010, 021, 101, 120, 210 000, 010, 021, 101, 201, 210 000, 010, 021, 102, 110, 120 000, 010, 021, 102, 110, 201 000, 010, 021, 102, 110, 210 000, 010, 021, 102, 120, 201 000, 010, 021, 102, 120, 210 000, 010, 021, 102, 201, 210 000, 010, 021, 110, 120, 201 000, 010, 021, 110, 120, 210 000, 010, 021, 110, 201, 210 000, 010, 021, 120, 201, 210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^{m}$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$; $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1$
79	010, 011, 100, 120, 201, 210• 010, 011, 101, 120, 201, 210 010, 011, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(b_{m+2-j,1})^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$	$\frac{x(1-2x)}{(1-3x+x^2)(1-x)}$

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
80	010, 011, 100, 101, 120, 201 010, 011, 100, 101, 120, 210 010, 011, 100, 110, 120, 201 010, 011, 100, 110, 120, 210 010, 011, 101, 110, 120, 201 010, 011, 101, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $\{b_{m+2-j,i}\}_{i=1}^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $a_m = 0^m$, $b_{m,j} = a_m j$	
81	010, 011, 100, 101, 110, 120	$\mathcal{T}(\{010, 011, 120\})$	
82	010, 011, 100, 101, 201, 210 010, 011, 100, 110, 201, 210 010, 011, 101, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (b_{m+2-j,1}^{j-1}, \{b_{m,i}\}_{i=j}^m$; $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-3x+x^2)}{(1-3x)(1-x)^2}$
83	010, 011, 100, 101, 110, 201 010, 011, 100, 101, 110, 210	$\mathcal{T}(\{010, 011, 201\})$	
84	012, 021, 100, 101, 102, 120 012, 021, 100, 101, 102, 201 012, 021, 100, 101, 102, 210 012, 021, 100, 101, 120, 201 012, 021, 100, 101, 120, 210 012, 021, 100, 101, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$, $a_m = 0^m$	
	012, 021, 100, 102, 110, 120 012, 021, 100, 102, 110, 201 012, 021, 100, 102, 110, 210 012, 021, 100, 110, 120, 201 012, 021, 100, 110, 120, 210 012, 021, 100, 110, 201, 210 012, 021, 101, 102, 110, 120 012, 021, 101, 102, 110, 201 012, 021, 101, 102, 110, 210 012, 021, 101, 110, 120, 201 012, 021, 101, 110, 120, 210 012, 021, 101, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow (010)^2$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012, 100, 101, 110, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+2x^2)}{(1-x)^3}$
85	012, 100, 101, 102, 110, 210 012, 100, 101, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow c_m, (010)^{m-1}, 011$; $c_m \rightsquigarrow (010)^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x+2x^2+x^4)}{(1-x)^3}$
86	012, 100, 101, 102, 110, 201 012, 100, 101, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 011$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x(1-2x+2x^2-x^3-x^4)}{(1-x)^3(1-x-x^2)}$
87	012, 100, 101, 102, 110, 120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow c_1, 011$; $b_m \rightsquigarrow c_m, c_2, \{d_i\}_{i=3}^m, 011$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $d_m \rightsquigarrow c_m, c_2, \{d_i\}_{i=3}^{m-1}$; $011 \rightsquigarrow 011$, $a=0^m$, $b_m = a_m m$, $c_m = a_m m0$, $d_m = a_m m(m-1)$	$\frac{x(2x^6-x^4+(1-x)^3)}{(1-x)^3(1-x-x^2)^2}$
88	011, 021, 100, 101, 102, 120 011, 021, 100, 102, 110, 120 011, 021, 100, 102, 120, 201 011, 021, 100, 102, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^m$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-2x+2x^2-2x^3)}{(1-x)^2(1-2x)}$
89	000, 021, 100, 101, 102, 120 000, 021, 101, 102, 120, 201 000, 021, 101, 102, 120, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 010, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	
	000, 021, 100, 102, 110, 120 000, 021, 102, 110, 120, 201 000, 021, 102, 110, 120, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 002$; $010 \rightsquigarrow 0101$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3 + x^4$
90	000, 021, 100, 101, 102, 110 000, 021, 101, 102, 110, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 021, 101, 102, 110, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 01$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + \frac{x^3}{1-x}$
91	012, 100, 102, 110, 201, 210 012, 100, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^2, (0021)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+2x^2-x+1)}{(1-x)^3}$
92	012, 100, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (0021)^{m-1}, 010$; $c_m \rightsquigarrow (0021)^{m-1}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^4+x^3+2x^2-x+1)}{(1-x)^3}$
93	012, 021, 100, 102, 120, 201 012, 021, 100, 102, 120, 210 012, 021, 100, 102, 201, 210 012, 021, 100, 120, 201, 210 012, 021, 101, 102, 120, 201 012, 021, 101, 102, 120, 210 012, 021, 101, 102, 201, 210 012, 021, 101, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012, 021, 102, 110, 120, 201 012, 021, 102, 110, 120, 210 012, 021, 102, 110, 201, 210 012, 021, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$	
	012, 100, 101, 102, 201, 210 012, 100, 101, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^m, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	012, 101, 102, 110, 201, 210 012, 101, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^{m+1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-2x+3x^2-x^3)}{(1-x)^4}$
94	012, 100, 102, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow 00210, \{c_i\}_{i=2}^{m}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x(x^2(1-2x^2-x^3)+(1-x)^2)}{(1-x)^3(1-x-x^2)}$
95	000, 021, 100, 102, 120, 201 000, 021, 100, 102, 120, 210 000, 021, 102, 120, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 0101, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3 + 2x^4$
96	012, 100, 101, 102, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (010)^{m-1}, b_m$; $c_m \rightsquigarrow (010)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^4-x^3+3x^2-2x+1)}{(1-x)^4}$
97	012, 101, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, (010)^m$; $c_m \rightsquigarrow c_m, (010)^{m-1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(x^2-x+1)(2x^2-2x+1)}{(1-x)^5}$
98	012, 100, 101, 102, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=1}^{m}, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	
	012, 101, 102, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}, (010)^2$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+x^2-x+1)}{(1-x)^2(1-x-x^2)}$
99	011, 021, 101, 102, 110, 120 011, 021, 101, 102, 120, 201 011, 021, 101, 102, 120, 210 011, 021, 102, 110, 120, 201 011, 021, 102, 110, 120, 210 011, 021, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^2, \{c_i\}_{i=2}^{m}$; $c_m \rightsquigarrow 010, \{c_i\}_{i=2}^{m}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-3x+4x^2-3x^3)}{(1-x)^3(1-2x)}$
100	000, 021, 100, 102, 110, 201 000, 021, 100, 102, 110, 210		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000, 021, 102, 110, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 012$; $010 \rightsquigarrow 0101$; $012 \rightsquigarrow 0101, b_0, 012$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $x^4 + \frac{x^3}{1-x}$
101	011, 021, 100, 101, 102, 110 011, 021, 100, 101, 102, 201 011, 021, 100, 101, 102, 210 011, 021, 100, 102, 110, 201 011, 021, 100, 102, 110, 210 011, 021, 100, 102, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$	
	011, 100, 102, 120, 201, 210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(010)^j, c_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $c_m \rightsquigarrow c_m, \{b_{m-1,i}\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 12$	$\frac{x(1-x+x^2)}{(1-x)(1-2x)}$
102	011, 100, 101, 102, 120, 201 011, 100, 102, 110, 120, 201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^{j}, d_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m m0$, $d_m = 0^m 12$	
	011, 100, 101, 102, 120, 210 011, 100, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^{j}, d_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m m0$, $d_m = 0^m 12$	$\frac{x(1-x)}{(1-2x)(1-x-x^2)}$
103	011, 100, 101, 102, 110, 120	$\mathcal{T}(\{011, 100, 102, 120\})$	$\frac{x(1-x-x^2)}{(1+x)(1-2x)^2}$
104	000, 021, 100, 101, 102, 201 000, 021, 100, 101, 102, 210 000, 021, 101, 102, 201, 210	$\mathcal{T}(\{000, 021, 101, 102\})$	
105	000, 021, 100, 101, 110, 120 000, 021, 101, 110, 120, 201 000, 021, 101, 110, 120, 210	$\mathcal{T}(\{000, 021, 101, 110, 120\})$	
106	000, 100, 101, 102, 110, 120●		
107	000, 101, 102, 110, 120, 201●		
108	000, 101, 102, 110, 120, 210●		
109	012, 100, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, (0021)^{m-1}, b_m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	
	012, 102, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow b_1, (011)^m$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(3x^2-2x+1)}{(1-x)^4}$
110	000, 021, 100, 102, 201, 210	$\mathcal{T}(\{000, 021, 102\})$	
111	012, 101, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-3x+5x^2-3x^3+x^4)}{(1-x)^5}$
112	011, 101, 102, 120, 201, 210 011, 102, 110, 120, 201, 210	$\mathcal{T}(\{011, 102, 120, 201, 210\})$	
113	011, 021, 100, 101, 110, 120 011, 021, 100, 101, 120, 201 011, 021, 100, 101, 120, 210 011, 021, 100, 110, 120, 201 011, 021, 100, 110, 120, 210 011, 021, 100, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, 012$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_m = a_m 1$	
	011, 021, 101, 102, 110, 201 011, 021, 101, 102, 110, 210 011, 021, 101, 102, 201, 210 011, 021, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$	
	011, 101, 102, 110, 120, 201	$\mathcal{T}(\{011, 102, 120, 201\})$	Theorem 14
	011, 101, 102, 110, 120, 210	$\mathcal{T}(\{011, 102, 120, 210\})$	Theorem 14
	012, 021, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow (01)^2$, $a_m = 0^m$	$\frac{x(2x^2-2x+1)}{(1-2x)(1-x)^2}$
114	011, 100, 101, 102, 201, 210 011, 100, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, \{b_{m,i}\}_{i=j}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-x)^2}{(1-2x)^2}$
115	011, 100, 101, 102, 110, 210	$\mathcal{T}(\{011, 100, 102, 210\})$	

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
116	011, 100, 101, 102, 110, 201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=1}^{j}, \{b_{m,i}\}_{i=j}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	$\frac{x}{1-2x-x^2}$
117	000, 021, 100, 101, 110, 201 000, 021, 100, 101, 110, 210 000, 021, 101, 110, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (b_0)^2, 01$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{(1+x)(1-x-\sqrt{1-2x-3x^2})}{2x^2}$ $-\frac{1}{1-x}$
118	000, 021, 100, 101, 120, 201 000, 021, 100, 101, 120, 210 000, 021, 100, 110, 120, 201 000, 021, 100, 110, 120, 210 000, 021, 101, 120, 201, 210 000, 021, 110, 120, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow b_0, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{(2+x)(1-2x-x^2)}{2x^2}$ − $\frac{(2+x)(1-x)\sqrt{1-2x-3x^2}}{2x^2}$ + x
119	000, 100, 102, 110, 120, 201●		
120	000, 100, 101, 102, 120, 201● 000, 100, 101, 102, 120, 210● 000, 100, 102, 110, 120, 210●		By the proof of Theorem 7
121	000, 101, 102, 120, 201, 210● 000, 102, 110, 120, 201, 210●		By the proof of Theorem 12
122	000, 100, 101, 102, 110, 210●		
123	000, 100, 101, 102, 110, 201●		
124	000, 101, 102, 110, 201, 210●		
125	000, 100, 102, 120, 201, 210●		
126	011, 101, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow 010, (0021)^{j-1}, \{b_{m,i}\}_{i=j}^{m}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-4x+6x^2-3x^3-x^4)}{(1-x)^2(1-2x)^2}$
127	000, 021, 100, 120, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (a_1)^2, 002$; $002 \rightsquigarrow b_0, 002$, $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \ldots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $\frac{4x^2(1-\sqrt{1-2x-3x^2})}{(1-x+\sqrt{1-2x-3x^2})^2}$
128	000, 100, 102, 110, 201, 210●		
129	011, 021, 100, 101, 110, 201 011, 021, 100, 101, 110, 210 011, 021, 100, 101, 201, 210 011, 021, 100, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}, 0103$; $0103 \rightsquigarrow 0103$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 10$	$\frac{x(1-3x+3x^2)}{(1-x)(1-2x)^2}$
	011, 021, 101, 110, 120, 201 011, 021, 101, 110, 120, 210 011, 021, 101, 120, 201, 210 011, 021, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	
130	000, 021, 100, 110, 201, 210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow a_1, b_0, 01$; $002 \rightsquigarrow b_0, 002$, $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \ldots (m-1)^2 m$, $b_m = a_m m$	$\frac{3x^3-3x^2-4x+2}{2x^2(1-x)}$ + $\frac{(x^2+2x-2)\sqrt{1-2x-3x^2}}{2x^2(1-x)}$
131	011, 100, 101, 120, 201, 210 011, 100, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (c_{m+1-j})^j, c_{m-j}$, $\{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow c_m, \{b_{m-1,i}\}_{i=1}^{m-1}$; $013 \rightsquigarrow 013$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 10$	$\frac{x(1-x)}{1-3x+x^2}$
132	000, 021, 100, 101, 201, 210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m=1}, 002$; $c_m \rightsquigarrow b_{m-1}, d_m, \{a_i\}_{i=2}^{m}, c_1$; $d_m \rightsquigarrow b_m, \{a_i\}_{i=2}^{m+1}, c_1$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c_m = 01^2 \cdots (m-1)^2 m$, $d_m = c_m m$	
	000, 100, 101, 102, 201, 210●	$a_{m,j} \rightsquigarrow (010)^{j-m}, b_{m,j}, \{a_{i,j+1}\}_{i=0}^{m}$; $b_{m,j} \rightsquigarrow (01)^{j-m}, \{a_{i,j+1}\}_{i=0}^{m+1}$, $a_{m,j} = 0^2 \cdots (m-1)^2 m \cdots j$, $b_{m,j} = a_{m,j} j$	$\frac{2x}{3x-1+\sqrt{1-2x-3x^2}} - 1$
133	011, 100, 101, 110, 120, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	011, 100, 101, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow c_{m+1-j}, \{b_{m+1+i-j,i}\}_{i=1}^{j-1}, c_{m-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $c_m \rightsquigarrow c_m, \{b_{m,i}\}_{i=1}^{m}$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 10$	Theorem 3.4 in [4]
134	000, 100, 101, 110, 120, 201● 000, 100, 101, 110, 120, 210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,m} \rightsquigarrow c_1, \{b_{i+1,i}\}_{i=1}^{m-1}, 012$; $b_{m,j} \rightsquigarrow c_{m+1-j}, \{b_{m+1+i-j,i}\}_{i=1}^{j-1}, c_{m-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $c_m \rightsquigarrow c_m, \{b_{m,i}\}_{i=1}^{m}$; $c_1 \rightsquigarrow c_1, b_{1,1}$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m 10$	
135	000, 101, 110, 120, 201, 210●		
136	011, 101, 110, 120, 201, 210		
137	011, 100, 101, 110, 201, 210		
138	000, 100, 101, 110, 201, 210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (b_{m+1-j,1})^j, \{b_{m,i}\}_{i=j}^{m}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
	010, 021, 100, 101, 102, 110 010, 021, 100, 101, 102, 120 010, 021, 100, 101, 102, 201 010, 021, 100, 101, 102, 210 010, 021, 100, 101, 110, 120 010, 021, 100, 101, 110, 201 010, 021, 100, 101, 110, 210 010, 021, 100, 101, 120, 201 010, 021, 100, 101, 120, 210 010, 021, 100, 101, 201, 210 010, 021, 100, 102, 110, 120 010, 021, 100, 102, 110, 201 010, 021, 100, 102, 110, 210 010, 021, 100, 102, 120, 201 010, 021, 100, 102, 120, 210 010, 021, 100, 102, 201, 210 010, 021, 100, 110, 120, 201 010, 021, 100, 110, 120, 210 010, 021, 100, 110, 201, 210 010, 021, 100, 120, 201, 210 010, 021, 101, 102, 110, 120 010, 021, 101, 102, 110, 201 010, 021, 101, 102, 110, 210 010, 021, 101, 102, 120, 201 010, 021, 101, 102, 120, 210 010, 021, 101, 102, 201, 210 010, 021, 101, 110, 120, 201 010, 021, 101, 110, 120, 210 010, 021, 101, 110, 201, 210 010, 021, 101, 120, 201, 210 010, 021, 102, 110, 120, 201 010, 021, 102, 110, 120, 210 010, 021, 102, 110, 201, 210 010, 021, 102, 120, 201, 210 010, 021, 110, 120, 201, 210 011, 021, 101, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \ldots, a_1$, $a_m = 0^m$	$\frac{1-2x-sqrt1-4x}{2x}$
139	000, 100, 101, 120, 201, 210●		
140	000, 100, 110, 120, 201, 210●		
141	010, 100, 101, 102, 110, 120●		
142	010, 100, 102, 110, 120, 201●		
143	010, 100, 101, 102, 120, 201● 010, 100, 101, 102, 120, 210● 010, 100, 102, 110, 120, 210● 010, 101, 102, 110, 120, 201●		Theorem 15
144	010, 101, 102, 110, 120, 210●		
145	010, 100, 101, 102, 110, 210●		
146	010, 100, 101, 102, 110, 201●		
147	010, 100, 102, 120, 201, 210● 010, 101, 102, 120, 201, 210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, b_{m+1,j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $0021 \rightsquigarrow 0021$, $a_m = 0^m$, $b_{m,j} = a_m j$	By the proof of Theorem 12
	010, 102, 110, 120, 201, 210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $0021 \rightsquigarrow 0021, 00212$; $00212 \rightsquigarrow 00212$, $a_m = 0^m$, $b_{m,j} = a_m j$	
148	010, 100, 102, 110, 201, 210●		
149	010, 101, 102, 110, 201, 210●		
150	010, 100, 101, 102, 201, 210●		
151	010, 100, 101, 110, 120, 201● 010, 100, 101, 110, 120, 210●		By the proof of Theorem 9
152	010, 100, 101, 110, 201, 210●		
153	010, 100, 101, 120, 201, 210● 010, 100, 110, 120, 201, 210● 010, 101, 110, 120, 201, 210●		By the proof of Theorem 12
154	012, 021, 100, 101, 102, 110 012, 021, 100, 101, 110, 120 012, 021, 100, 101, 110, 201		

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	012, 021, 100, 101, 110, 210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 011$; $011 \rightsquigarrow 011$	$\frac{x(1-x(1-x)^2)}{(1-x)^3}$
155	000, 021, 101, 102, 110, 120●	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 002$; $002 \rightsquigarrow_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-2)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3$
156	021, 100, 101, 102, 110, 120●	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x}{(1-x)^2} -$ $\frac{2x^2}{1-x}$
157	021, 100, 101, 102, 120, 201 021, 100, 101, 102, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$,; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, b_{m+1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	
	021, 100, 102, 110, 120, 201 021, 100, 102, 110, 120, 210 021, 101, 102, 110, 120, 201 021, 101, 102, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x^3}{(1-x)^3}$
158	021, 100, 101, 102, 110, 201 021, 100, 101, 102, 110, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^m, 010, c_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-\sqrt{1-4x}}{2x} - 1 +$ $\frac{x^3}{(1-x)(1-2x)}$
159	021, 100, 102, 120, 201, 210 021, 101, 102, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, b_{m+1}, \{c_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$; $00 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	
	021, 102, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^m, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $010 \rightsquigarrow 010, 0101$; $012 \rightsquigarrow c_1, 012$; $0101 \rightsquigarrow 0101$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-2x-\sqrt{1-4x}}{2x} + \frac{x^3}{(1-x)^4}$
160	021, 100, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, c_m, \{d_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $d_m \rightsquigarrow \{d_i\}_{i=1}^m, 0120$; $010 \rightsquigarrow 010$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$, $d_m = a_m 12$	$\frac{1-2x-\sqrt{1-4x}}{2x} +$ $\frac{x^3(1-x-x^2)}{(1-x)^3(1-2x)}$
161	021, 101, 102, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, c_m, \{b_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $010 \rightsquigarrow 010$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-2x-\sqrt{1-4x}}{2x} +$ $\frac{x^3}{(1-x)^2(1-2x)}$
162	021, 100, 101, 102, 201, 210 021, 100, 101, 110, 120, 201 021, 100, 101, 110, 120, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, c_{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 010$; $010 \rightsquigarrow c_1, 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{(1+x)(1-\sqrt{1-4x})}{2x} - 1 -$ $\frac{x}{1-x}$
163	100, 101, 102, 110, 120, 201●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^j, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	
	100, 101, 102, 110, 120, 210●	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^j, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m0$	
164	021, 100, 101, 110, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010, c_1, b_1$; $b_m \rightsquigarrow c_{m-1}, c_m, \{b_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-3x+x^2}{2x(1-2x)}$ $+ \frac{(x^2+x-1)\sqrt{1-4x}}{2x(1-2x)}$

Continuation of Table 2

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
165	021, 100, 101, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
	021, 100, 110, 120, 201, 210 021, 101, 110, 120, 201, 210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{c_i\}_{i=1}^{m}, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m1$, $c_m = a_m10$	
	100, 101, 102, 120, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(010)^j, b_{m+1,j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_mj$	
	100, 102, 110, 120, 201, 210• 101, 102, 110, 120, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(010)^j, a_{m+2-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_mj$	$\frac{4x^2-7x+2+(3x-2)\sqrt{1-4x}}{2x(1-x)}$
166	100, 101, 102, 110, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(010)^j, a_{m+2-j}, \{b_{m+1,i}\}_{i=j}^{m+1}$, $a_m = 0^m$, $b_{m,j} = a_mj$	
167	100, 101, 110, 120, 201, 210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (a_{m+1-j})^j, a_{m+2-j}$, $\{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_mj$	

End of Table 2

Theorem 13. *Let* $A_\tau = \{000, 010, 100, 101, 120, \tau\}$, $B_\tau = \{000, 010, 100, 110, 120, \tau\}$, *and* $C = \{000, 010, 101, 110, 120, 201\}$. *Then* $A_{201} \overset{\mathbf{I}}{\sim} A_{210} \overset{\mathbf{I}}{\sim} B_{201} \overset{\mathbf{I}}{\sim} B_{210} \overset{\mathbf{I}}{\sim} C$.

Proof. By simple modifications of the proof of Theorems 5 and 8, we obtain that $A_{201} \overset{\mathbf{I}}{\sim} A_{210} \overset{\mathbf{I}}{\sim} B_{201} \overset{\mathbf{I}}{\sim} B_{210}$. By our procedure, we see that the rules of the generating trees $\mathcal{T}(A_{201})$ and $\mathcal{T}(C)$ are given by

$$a_{m,j} \rightsquigarrow b_{2m-j}, \{a_{2m-j+i,2m-j+2i-1}\}_{i=1}^{j-m}, \{a_{2m-j,i}\}_{i=2m-j}^{4m-2j},$$
$$b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2},$$

where $a_{m,j} = 0^2 \cdots (m-1)^2 j$ and $b_m = a_{m,m}m$. Hence, $A_{201} \overset{\mathbf{I}}{\sim} C$. □

Theorem 14. *Let* $A_\tau = \{011, 101, 102, 110, 120, \tau\}$ *and* $B = \{011, 021, 102, 110, 201, 210\}$. *Then* $A_{201} \overset{\mathbf{I}}{\sim} A_{210} \overset{\mathbf{I}}{\sim} B$.

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \ldots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(A_{201})$ (respectively, $e \in \mathbf{I}_n(A_{210})$) if and only if,

  (i) $\pi_1 = 0^a$ for some $a \geq 0$ and $\pi_i = \emptyset$ for $i = 2, \ldots, k-1$;

  (ii) $m_k > \pi_k > m_{k-1}$ (here $m_0 = 0$) and π_k is decreasing (respectively, increasing) sequence (without repeated letters).

(2) $e \in \mathbf{I}_n(B)$ if and only if $\pi_1 = 0^a$ and $\pi_k = 0^b$ for some $a, b \geq 0$ and $\pi_i = \emptyset$ for all $i = 2, \ldots, k-1$;

Clearly, for each k, the number of inversion sequences in either $\mathbf{I}_n(A_{201})$, $\mathbf{I}_n(A_{210})$, or $\mathbf{I}_n(B)$ with k LRmax are equal, which completes the proof. □

Theorem 15. *Let* $A_\tau = \{010, 100, 101, 102, 120, \tau\}$*,* $B = \{010, 100, 110, 102, 120, 201\}$*, and* $C = \{010, 101, 110, 102, 120, 210\}$*. Then* $A_{201} \overset{\mathbf{I}}{\sim} A_{210} \overset{\mathbf{I}}{\sim} B \overset{\mathbf{I}}{\sim} C$*.*

Proof. Suppose $e \in \mathbf{I}_n$ and $\mathbf{m}_1\pi_1\mathbf{m}_2\pi_2 \dots \mathbf{m}_k\pi_k$ is the LRmax decomposition of e. Then

(1) $e \in \mathbf{I}_n(A_{201})$ (respectively, $e \in \mathbf{I}_n(A_{210})$) if and only if $\pi_i = m_i^{a_i}$ for some $a_i \geq 0$, for $i = 1, \dots, k-1$, and $\pi_k = m_k^a\beta > m_{k-1}$ (here $m_0 = 0$) for some $a \geq 0$ and β forms a decreasing (respectively, increasing) sequence.

(2) $e \in \mathbf{I}_n(B)$ if and only if $\pi_i = m_i^{a_i}$ for some $a_i \geq 0$ and $\pi_k = \beta m_k^a > m_{k-1}$ (here $m_0 = 0$) for some $a \geq 0$ and β forms a decreasing sequence.

(3) $e \in \mathbf{I}_n(C)$ if and only if $\pi_i = m_i^{a_i}$ for some $a_i \geq 0$ and either $\pi_k = m_k^a$ or $m_k > \pi_k > m_{k-1}$ (here $m_0 = 0$) such that π_k forms a nondecreasing sequence.

Clearly, for each k, the number of inversion sequences in either $\mathbf{I}_n(A_{201})$, $\mathbf{I}_n(A_{210})$, $\mathbf{I}_n(B)$, or $\mathbf{I}_n(C)$ with k LRmax are equal, which completes the proof. □

5. 7-Table

In this section, we show that $w_7 = 105$. Moreover, by our procedure, we present the generating function for many of the 105 I-Wilf-equivalences. Actually, we see that there are only 37 sets among $\binom{13}{7} = 1716$ sets of 7 length-3 patterns that cannot be reduced to smaller sets of length-3 patterns. In Table 3, we present all the I-Wilf-equivalences of sets of 7 length-3 patterns. As we mentioned, due to the similarity with the previous sections, we only present the cases that the KMY algorithm does not work, that is, we present bijections between some classes of inversion sequences respect to left-right-maxima structure with using the bijections presented in the previous sections; see the theorems at end of this section.

Table 3: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \in\subset \mathcal{P}_3$ and $|B| = 7$.

Beginning of Table 3			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100 000,001,010,011,012,021,101 000,001,010,011,012,021,102 000,001,010,011,012,021,110 000,001,010,011,012,021,120 000,001,010,011,012,021,201 000,001,010,011,012,021,210 000,001,010,011,012,100,101 000,001,010,011,012,100,102 000,001,010,011,012,100,110 000,001,010,011,012,100,120 000,001,010,011,012,100,201 000,001,010,011,012,100,210 000,001,010,011,012,101,102 000,001,010,011,012,101,110 000,001,010,011,012,101,120		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,011,012,101,201		
	000,001,010,011,012,101,210		
	000,001,010,011,012,102,110		
	000,001,010,011,012,102,120		
	000,001,010,011,012,102,201		
	000,001,010,011,012,102,210		
	000,001,010,011,012,110,120		
	000,001,010,011,012,110,201		
	000,001,010,011,012,110,210		
	000,001,010,011,012,120,201		
	000,001,010,011,012,120,210		
	000,001,010,011,012,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101		
	000,001,010,012,021,100,102		
	000,001,010,012,021,100,110		
	000,001,010,012,021,100,120		
	000,001,010,012,021,100,201		
	000,001,010,012,021,100,210		
	000,001,010,012,021,101,102		
	000,001,010,012,021,101,110		
	000,001,010,012,021,101,120		
	000,001,010,012,021,101,201		
	000,001,010,012,021,101,210		
	000,001,010,012,021,102,110		
	000,001,010,012,021,102,120		
	000,001,010,012,021,102,201		
	000,001,010,012,021,102,210		
	000,001,010,012,021,110,120		
	000,001,010,012,021,110,201		
	000,001,010,012,021,110,210		
	000,001,010,012,021,120,201		
	000,001,010,012,021,120,210		
	000,001,010,012,021,201,210		
	000,001,010,012,100,101,102		
	000,001,010,012,100,101,110		
	000,001,010,012,100,101,120		
	000,001,010,012,100,101,201		
	000,001,010,012,100,101,210		
	000,001,010,012,100,102,110		
	000,001,010,012,100,102,120		
	000,001,010,012,100,102,201		
	000,001,010,012,100,102,210		
	000,001,010,012,100,110,120		
	000,001,010,012,100,110,201		
	000,001,010,012,100,110,210		
	000,001,010,012,100,120,201		
	000,001,010,012,100,120,210		
	000,001,010,012,100,201,210		
	000,001,010,012,101,102,110		
	000,001,010,012,101,102,120		
	000,001,010,012,101,102,201		
	000,001,010,012,101,102,210		
	000,001,010,012,101,110,120		
	000,001,010,012,101,110,201		
	000,001,010,012,101,110,210		
	000,001,010,012,101,120,201		
	000,001,010,012,101,120,210		
	000,001,010,012,101,201,210		
	000,001,010,012,102,110,120		
	000,001,010,012,102,110,201		
	000,001,010,012,102,110,210		
	000,001,010,012,102,120,201		
	000,001,010,012,102,120,210		
	000,001,010,012,102,201,210		
	000,001,010,012,110,120,201		
	000,001,010,012,110,120,210		
	000,001,010,012,110,201,210		
	000,001,010,012,120,201,210		
	000,001,011,012,021,100,101		
	000,001,011,012,021,100,102		
	000,001,011,012,021,100,110		
	000,001,011,012,021,100,120		
	000,001,011,012,021,100,201		
	000,001,011,012,021,100,210		
	000,001,011,012,021,101,102		
	000,001,011,012,021,101,110		
	000,001,011,012,021,101,120		
	000,001,011,012,021,101,201		
	000,001,011,012,021,101,210		
	000,001,011,012,021,102,110		
	000,001,011,012,021,102,120		
	000,001,011,012,021,102,201		
	000,001,011,012,021,102,210		
	000,001,011,012,021,110,120		
	000,001,011,012,021,110,201		
	000,001,011,012,021,110,210		
	000,001,011,012,021,120,201		
	000,001,011,012,021,120,210		
	000,001,011,012,021,201,210		
	000,001,011,012,100,101,102		
	000,001,011,012,100,101,110		
	000,001,011,012,100,101,120		
	000,001,011,012,100,101,201		
	000,001,011,012,100,101,210		
	000,001,011,012,100,102,110		
	000,001,011,012,100,102,120		
	000,001,011,012,100,102,201		
	000,001,011,012,100,102,210		
	000,001,011,012,100,110,120		
	000,001,011,012,100,110,201		
	000,001,011,012,100,110,210		
	000,001,011,012,100,120,201		
	000,001,011,012,100,120,210		
	000,001,011,012,100,201,210		
	000,001,011,012,101,102,110		
	000,001,011,012,101,102,120		
	000,001,011,012,101,102,201		
	000,001,011,012,101,102,210		
	000,001,011,012,101,110,120		
	000,001,011,012,101,110,201		
	000,001,011,012,101,110,210		
	000,001,011,012,101,120,201		
	000,001,011,012,101,120,210		
	000,001,011,012,101,201,210		
	000,001,011,012,102,110,120		
	000,001,011,012,102,110,201		
	000,001,011,012,102,110,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,011,012,102,120,201		
	000,001,011,012,102,120,210		
	000,001,011,012,102,201,210		
	000,001,011,012,110,120,201		
	000,001,011,012,110,120,210		
	000,001,011,012,110,201,210		
	000,001,011,012,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x(1+x)^2$
3	000,001,010,011,021,100,101		
	000,001,010,011,021,100,102		
	000,001,010,011,021,100,110		
	000,001,010,011,021,100,120		
	000,001,010,011,021,100,201		
	000,001,010,011,021,100,210		
	000,001,010,011,021,101,102		
	000,001,010,011,021,101,110		
	000,001,010,011,021,101,120		
	000,001,010,011,021,101,201		
	000,001,010,011,021,101,210		
	000,001,010,011,021,102,110		
	000,001,010,011,021,102,120		
	000,001,010,011,021,102,201		
	000,001,010,011,021,102,210		
	000,001,010,011,021,110,120		
	000,001,010,011,021,110,201		
	000,001,010,011,021,110,210		
	000,001,010,011,021,120,201		
	000,001,010,011,021,120,210		
	000,001,010,011,021,201,210		
	000,001,010,011,100,101,102		
	000,001,010,011,100,101,110		
	000,001,010,011,100,101,120		
	000,001,010,011,100,101,201		
	000,001,010,011,100,101,210		
	000,001,010,011,100,102,110		
	000,001,010,011,100,102,120		
	000,001,010,011,100,102,201		
	000,001,010,011,100,102,210		
	000,001,010,011,100,110,120		
	000,001,010,011,100,110,201		
	000,001,010,011,100,110,210		
	000,001,010,011,100,120,201		
	000,001,010,011,100,120,210		
	000,001,010,011,100,201,210		
	000,001,010,011,101,102,110		
	000,001,010,011,101,102,120		
	000,001,010,011,101,102,201		
	000,001,010,011,101,102,210		
	000,001,010,011,101,110,120		
	000,001,010,011,101,110,201		
	000,001,010,011,101,110,210		
	000,001,010,011,101,120,201		
	000,001,010,011,101,120,210		
	000,001,010,011,101,201,210		
	000,001,010,011,102,110,120		
	000,001,010,011,102,110,201		
	000,001,010,011,102,110,210		
	000,001,010,011,102,120,201		
	000,001,010,011,102,120,210		
	000,001,010,011,102,201,210		
	000,001,010,011,110,120,201		
	000,001,010,011,110,120,210		
	000,001,010,011,110,201,210		
	000,001,010,011,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101		
	001,010,011,012,021,100,102		
	001,010,011,012,021,100,110		
	001,010,011,012,021,100,120		
	001,010,011,012,021,100,201		
	001,010,011,012,021,100,210		
	001,010,011,012,021,101,102		
	001,010,011,012,021,101,110		
	001,010,011,012,021,101,120		
	001,010,011,012,021,101,201		
	001,010,011,012,021,101,210		
	001,010,011,012,021,102,110		
	001,010,011,012,021,102,120		
	001,010,011,012,021,102,201		
	001,010,011,012,021,102,210		
	001,010,011,012,021,110,120		
	001,010,011,012,021,110,201		
	001,010,011,012,021,110,210		
	001,010,011,012,021,120,201		
	001,010,011,012,021,120,210		
	001,010,011,012,021,201,210		
	001,010,011,012,100,101,102		
	001,010,011,012,100,101,110		
	001,010,011,012,100,101,120		
	001,010,011,012,100,101,201		
	001,010,011,012,100,101,210		
	001,010,011,012,100,102,110		
	001,010,011,012,100,102,120		
	001,010,011,012,100,102,201		
	001,010,011,012,100,102,210		
	001,010,011,012,100,110,120		
	001,010,011,012,100,110,201		
	001,010,011,012,100,110,210		
	001,010,011,012,100,120,201		
	001,010,011,012,100,120,210		
	001,010,011,012,100,201,210		
	001,010,011,012,101,102,110		
	001,010,011,012,101,102,120		
	001,010,011,012,101,102,201		
	001,010,011,012,101,102,210		
	001,010,011,012,101,110,120		
	001,010,011,012,101,110,201		
	001,010,011,012,101,110,210		
	001,010,011,012,101,120,201		
	001,010,011,012,101,120,210		
	001,010,011,012,101,201,210		
	001,010,011,012,102,110,120		
	001,010,011,012,102,110,201		
	001,010,011,012,102,110,210		
	001,010,011,012,102,120,201		
	001,010,011,012,102,120,210		
	001,010,011,012,102,201,210		
	001,010,011,012,110,120,201		
	001,010,011,012,110,120,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,010,011,012,110,201,210		
	001,010,011,012,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$\frac{x}{1-x} + x^2$
4	000,001,012,021,100,101,110		
	000,001,012,021,100,102,110		
	000,001,012,021,100,110,120		
	000,001,012,021,100,110,201		
	000,001,012,021,100,110,210		
	000,001,012,021,101,102,110		
	000,001,012,021,101,110,120		
	000,001,012,021,101,110,201		
	000,001,012,021,101,110,210		
	000,001,012,021,102,110,120		
	000,001,012,021,102,110,201		
	000,001,012,021,102,110,210		
	000,001,012,021,110,120,201		
	000,001,012,021,110,120,210		
	000,001,012,021,110,201,210		
	000,001,012,100,101,102,110		
	000,001,012,100,101,110,120		
	000,001,012,100,101,110,201		
	000,001,012,100,101,110,210		
	000,001,012,100,102,110,120		
	000,001,012,100,102,110,201		
	000,001,012,100,102,110,210		
	000,001,012,100,110,120,201		
	000,001,012,100,110,120,210		
	000,001,012,100,110,201,210		
	000,001,012,101,102,110,120		
	000,001,012,101,102,110,201		
	000,001,012,101,102,110,210		
	000,001,012,101,110,120,201		
	000,001,012,101,110,120,210		
	000,001,012,101,110,201,210		
	000,001,012,102,110,120,201		
	000,001,012,102,110,120,210		
	000,001,012,102,110,201,210		
	000,001,012,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (00)^2$	
	000,010,011,012,021,100,101		
	000,010,011,012,021,100,102		
	000,010,011,012,021,100,110		
	000,010,011,012,021,100,120		
	000,010,011,012,021,100,201		
	000,010,011,012,021,100,210		
	000,010,011,012,021,101,102		
	000,010,011,012,021,101,110		
	000,010,011,012,021,101,120		
	000,010,011,012,021,101,201		
	000,010,011,012,021,101,210		
	000,010,011,012,021,102,110		
	000,010,011,012,021,102,120		
	000,010,011,012,021,102,201		
	000,010,011,012,021,102,210		
	000,010,011,012,021,110,120		
	000,010,011,012,021,110,201		
	000,010,011,012,021,110,210		
	000,010,011,012,021,120,201		
	000,010,011,012,021,120,210		
	000,010,011,012,021,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
5	000,001,012,021,100,101,102		
	000,001,012,021,100,101,120		
	000,001,012,021,100,101,201		
	000,001,012,021,100,101,210		
	000,001,012,021,100,102,120		
	000,001,012,021,100,102,201		
	000,001,012,021,100,102,210		
	000,001,012,021,100,120,201		
	000,001,012,021,100,120,210		
	000,001,012,021,100,201,210		
	000,001,012,021,101,102,120		
	000,001,012,021,101,102,201		
	000,001,012,021,101,102,210		
	000,001,012,021,101,120,201		
	000,001,012,021,101,120,210		
	000,001,012,021,101,201,210		
	000,001,012,021,102,120,201		
	000,001,012,021,102,120,210		
	000,001,012,021,102,201,210		
	000,001,012,021,120,201,210		
	000,001,012,100,101,102,120		
	000,001,012,100,101,102,201		
	000,001,012,100,101,102,210		
	000,001,012,100,101,120,201		
	000,001,012,100,101,120,210		
	000,001,012,100,101,201,210		
	000,001,012,100,102,120,201		
	000,001,012,100,102,120,210		
	000,001,012,100,102,201,210		
	000,001,012,100,120,201,210		
	000,001,012,101,102,120,201		
	000,001,012,101,102,120,210		
	000,001,012,101,102,201,210		
	000,001,012,101,120,201,210		
	000,001,012,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;\ 011 \rightsquigarrow 00$	
	000,010,011,012,100,101,102		
	000,010,011,012,100,101,110		
	000,010,011,012,100,101,120		
	000,010,011,012,100,101,201		
	000,010,011,012,100,101,210		
	000,010,011,012,100,102,110		
	000,010,011,012,100,102,120		
	000,010,011,012,100,102,201		
	000,010,011,012,100,102,210		
	000,010,011,012,100,110,120		
	000,010,011,012,100,110,201		
	000,010,011,012,100,110,210		
	000,010,011,012,100,120,201		
	000,010,011,012,100,120,210		
	000,010,011,012,100,201,210		
	000,010,011,012,101,102,110		
	000,010,011,012,101,102,120		
	000,010,011,012,101,102,201		
	000,010,011,012,101,102,210		
	000,010,011,012,101,110,120		
	000,010,011,012,101,110,201		
	000,010,011,012,101,110,210		
	000,010,011,012,101,120,201		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,011,012,101,120,210		
	000,010,011,012,101,201,210		
	000,010,011,012,102,110,120		
	000,010,011,012,102,110,201		
	000,010,011,012,102,110,210		
	000,010,011,012,102,120,201		
	000,010,011,012,102,120,210		
	000,010,011,012,102,201,210		
	000,010,011,012,110,120,201		
	000,010,011,012,110,120,210		
	000,010,011,012,110,201,210		
	000,010,011,012,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
6	000,001,011,021,100,101,120		
	000,001,011,021,100,102,120		
	000,001,011,021,100,110,120		
	000,001,011,021,100,120,201		
	000,001,011,021,100,120,210		
	000,001,011,021,101,102,120		
	000,001,011,021,101,110,120		
	000,001,011,021,101,120,201		
	000,001,011,021,101,120,210		
	000,001,011,021,102,110,120		
	000,001,011,021,102,120,201		
	000,001,011,021,102,120,210		
	000,001,011,021,110,120,201		
	000,001,011,021,110,120,210		
	000,001,011,021,120,201,210		
	000,001,011,100,101,102,120		
	000,001,011,100,101,110,120		
	000,001,011,100,101,120,201		
	000,001,011,100,101,120,210		
	000,001,011,100,102,110,120		
	000,001,011,100,102,120,201		
	000,001,011,100,102,120,210		
	000,001,011,100,110,120,201		
	000,001,011,100,110,120,210		
	000,001,011,100,120,201,210		
	000,001,011,101,102,110,120		
	000,001,011,101,102,120,201		
	000,001,011,101,102,120,210		
	000,001,011,101,110,120,201		
	000,001,011,101,110,120,210		
	000,001,011,101,120,201,210		
	000,001,011,102,110,120,201		
	000,001,011,102,110,120,210		
	000,001,011,102,120,201,210		
	000,001,011,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 012;\ 012 \rightsquigarrow 012$	
	001,011,012,021,100,101,102		
	001,011,012,021,100,101,110		
	001,011,012,021,100,101,120		
	001,011,012,021,100,101,201		
	001,011,012,021,100,101,210		
	001,011,012,021,100,102,110		
	001,011,012,021,100,102,120		
	001,011,012,021,100,102,201		
	001,011,012,021,100,102,210		
	001,011,012,021,100,110,120		
	001,011,012,021,100,110,201		
	001,011,012,021,100,110,210		
	001,011,012,021,100,120,201		
	001,011,012,021,100,120,210		
	001,011,012,021,100,201,210		
	001,011,012,100,101,102,110		
	001,011,012,100,101,102,120		
	001,011,012,100,101,102,201		
	001,011,012,100,101,102,210		
	001,011,012,100,101,110,120		
	001,011,012,100,101,110,201		
	001,011,012,100,101,110,210		
	001,011,012,100,101,120,201		
	001,011,012,100,101,120,210		
	001,011,012,100,101,201,210		
	001,011,012,100,102,110,120		
	001,011,012,100,102,110,201		
	001,011,012,100,102,110,210		
	001,011,012,100,102,120,201		
	001,011,012,100,102,120,210		
	001,011,012,100,102,201,210		
	001,011,012,100,110,120,201		
	001,011,012,100,110,120,210		
	001,011,012,100,110,201,210		
	001,011,012,100,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010$	$\frac{x}{1-x} + x^2 + x^3$
7	000,001,010,021,100,101,102		
	000,001,010,021,100,101,110		
	000,001,010,021,100,101,120		
	000,001,010,021,100,101,201		
	000,001,010,021,100,101,210		
	000,001,010,021,100,102,110		
	000,001,010,021,100,102,120		
	000,001,010,021,100,102,201		
	000,001,010,021,100,102,210		
	000,001,010,021,100,110,120		
	000,001,010,021,100,110,201		
	000,001,010,021,100,110,210		
	000,001,010,021,100,120,201		
	000,001,010,021,100,120,210		
	000,001,010,021,100,201,210		
	000,001,010,021,101,102,110		
	000,001,010,021,101,102,120		
	000,001,010,021,101,102,201		
	000,001,010,021,101,102,210		
	000,001,010,021,101,110,120		
	000,001,010,021,101,110,201		
	000,001,010,021,101,110,210		
	000,001,010,021,101,120,201		
	000,001,010,021,101,120,210		
	000,001,010,021,101,201,210		
	000,001,010,021,102,110,120		
	000,001,010,021,102,110,201		
	000,001,010,021,102,110,210		
	000,001,010,021,102,120,201		
	000,001,010,021,102,120,210		
	000,001,010,021,102,201,210		
	000,001,010,021,110,120,201		
	000,001,010,021,110,120,210		
	000,001,010,021,110,201,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,021,120,201,210		
	000,001,010,100,101,102,110		
	000,001,010,100,101,102,120		
	000,001,010,100,101,102,201		
	000,001,010,100,101,102,210		
	000,001,010,100,101,110,120		
	000,001,010,100,101,110,201		
	000,001,010,100,101,110,210		
	000,001,010,100,101,120,201		
	000,001,010,100,101,120,210		
	000,001,010,100,101,201,210		
	000,001,010,100,102,110,120		
	000,001,010,100,102,110,201		
	000,001,010,100,102,110,210		
	000,001,010,100,102,120,201		
	000,001,010,100,102,120,210		
	000,001,010,100,102,201,210		
	000,001,010,100,110,120,201		
	000,001,010,100,110,120,210		
	000,001,010,100,110,201,210		
	000,001,010,100,120,201,210		
	000,001,010,101,102,110,120		
	000,001,010,101,102,110,201		
	000,001,010,101,102,110,210		
	000,001,010,101,102,120,201		
	000,001,010,101,102,120,210		
	000,001,010,101,102,201,210		
	000,001,010,101,110,120,201		
	000,001,010,101,110,120,210		
	000,001,010,101,110,201,210		
	000,001,010,101,120,201,210		
	000,001,010,102,110,120,201		
	000,001,010,102,110,120,210		
	000,001,010,102,110,201,210		
	000,001,010,102,120,201,210		
	000,001,010,110,120,201,210		
	000,001,011,021,100,101,102		
	000,001,011,021,100,101,110		
	000,001,011,021,100,101,201		
	000,001,011,021,100,101,210		
	000,001,011,021,100,102,110		
	000,001,011,021,100,102,201		
	000,001,011,021,100,102,210		
	000,001,011,021,100,110,201		
	000,001,011,021,100,110,210		
	000,001,011,021,100,201,210		
	000,001,011,021,101,102,110		
	000,001,011,021,101,102,201		
	000,001,011,021,101,102,210		
	000,001,011,021,101,110,201		
	000,001,011,021,101,110,210		
	000,001,011,021,101,201,210		
	000,001,011,021,102,110,201		
	000,001,011,021,102,110,210		
	000,001,011,021,102,201,210		
	000,001,011,021,110,201,210		
	000,001,011,100,101,102,110		
	000,001,011,100,101,102,201		
	000,001,011,100,101,102,210		
	000,001,011,100,101,110,201		
	000,001,011,100,101,110,210		
	000,001,011,100,101,201,210		
	000,001,011,100,102,110,201		
	000,001,011,100,102,110,210		
	000,001,011,100,102,201,210		
	000,001,011,100,110,201,210		
	000,001,011,101,102,110,201		
	000,001,011,101,102,110,210		
	000,001,011,101,102,201,210		
	000,001,011,101,110,201,210		
	000,001,011,102,110,201,210	$\underline{0} \rightsquigarrow \underline{00}, \underline{0}$	
	001,010,011,021,100,101,102		
	001,010,011,021,100,101,110		
	001,010,011,021,100,101,120		
	001,010,011,021,100,101,201		
	001,010,011,021,100,101,210		
	001,010,011,021,100,102,110		
	001,010,011,021,100,102,120		
	001,010,011,021,100,102,201		
	001,010,011,021,100,102,210		
	001,010,011,021,100,110,120		
	001,010,011,021,100,110,201		
	001,010,011,021,100,110,210		
	001,010,011,021,100,120,201		
	001,010,011,021,100,120,210		
	001,010,011,021,100,201,210		
	001,010,011,021,101,102,110		
	001,010,011,021,101,102,120		
	001,010,011,021,101,102,201		
	001,010,011,021,101,102,210		
	001,010,011,021,101,110,120		
	001,010,011,021,101,110,201		
	001,010,011,021,101,110,210		
	001,010,011,021,101,120,201		
	001,010,011,021,101,120,210		
	001,010,011,021,101,201,210		
	001,010,011,021,102,110,120		
	001,010,011,021,102,110,201		
	001,010,011,021,102,110,210		
	001,010,011,021,102,120,201		
	001,010,011,021,102,120,210		
	001,010,011,021,102,201,210		
	001,010,011,021,110,120,201		
	001,010,011,021,110,120,210		
	001,010,011,021,110,201,210		
	001,010,011,021,120,201,210		
	001,010,011,100,101,102,110		
	001,010,011,100,101,102,120		
	001,010,011,100,101,102,201		
	001,010,011,100,101,102,210		
	001,010,011,100,101,110,120		
	001,010,011,100,101,110,201		
	001,010,011,100,101,110,210		
	001,010,011,100,101,120,201		
	001,010,011,100,101,120,210		
	001,010,011,100,101,201,210		
	001,010,011,100,102,110,120		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,010,011,100,102,110,201		
	001,010,011,100,102,110,210		
	001,010,011,100,102,120,201		
	001,010,011,100,102,120,210		
	001,010,011,100,102,201,210		
	001,010,011,100,110,120,201		
	001,010,011,100,110,120,210		
	001,010,011,100,110,201,210		
	001,010,011,100,120,201,210		
	001,010,011,101,102,110,120		
	001,010,011,101,102,110,201		
	001,010,011,101,102,110,210		
	001,010,011,101,102,120,201		
	001,010,011,101,102,120,210		
	001,010,011,101,102,201,210		
	001,010,011,101,110,120,201		
	001,010,011,101,110,120,210		
	001,010,011,101,110,201,210		
	001,010,011,101,120,201,210		
	001,010,011,102,110,120,201		
	001,010,011,102,110,120,210		
	001,010,011,102,110,201,210		
	001,010,011,102,120,201,210		
	001,010,011,110,120,201,210		
	001,010,012,021,100,101,102		
	001,010,012,021,100,101,110		
	001,010,012,021,100,101,120		
	001,010,012,021,100,101,201		
	001,010,012,021,100,101,210		
	001,010,012,021,100,102,110		
	001,010,012,021,100,102,120		
	001,010,012,021,100,102,201		
	001,010,012,021,100,102,210		
	001,010,012,021,100,110,120		
	001,010,012,021,100,110,201		
	001,010,012,021,100,110,210		
	001,010,012,021,100,120,201		
	001,010,012,021,100,120,210		
	001,010,012,021,100,201,210		
	001,010,012,021,101,102,110		
	001,010,012,021,101,102,120		
	001,010,012,021,101,102,201		
	001,010,012,021,101,102,210		
	001,010,012,021,101,110,120		
	001,010,012,021,101,110,201		
	001,010,012,021,101,110,210		
	001,010,012,021,101,120,201		
	001,010,012,021,101,120,210		
	001,010,012,021,101,201,210		
	001,010,012,021,102,110,120		
	001,010,012,021,102,110,201		
	001,010,012,021,102,110,210		
	001,010,012,021,102,120,201		
	001,010,012,021,102,120,210		
	001,010,012,021,102,201,210		
	001,010,012,021,110,120,201		
	001,010,012,021,110,120,210		
	001,010,012,021,110,201,210		
	001,010,012,021,120,201,210		
	001,010,012,100,101,102,110		
	001,010,012,100,101,102,120		
	001,010,012,100,101,102,201		
	001,010,012,100,101,102,210		
	001,010,012,100,101,110,120		
	001,010,012,100,101,110,201		
	001,010,012,100,101,110,210		
	001,010,012,100,101,120,201		
	001,010,012,100,101,120,210		
	001,010,012,100,101,201,210		
	001,010,012,100,102,110,120		
	001,010,012,100,102,110,201		
	001,010,012,100,102,110,210		
	001,010,012,100,102,120,201		
	001,010,012,100,102,120,210		
	001,010,012,100,102,201,210		
	001,010,012,100,110,120,201		
	001,010,012,100,110,120,210		
	001,010,012,100,110,201,210		
	001,010,012,100,120,201,210		
	001,010,012,101,102,110,120		
	001,010,012,101,102,110,201		
	001,010,012,101,102,110,210		
	001,010,012,101,102,120,201		
	001,010,012,101,102,120,210		
	001,010,012,101,102,201,210		
	001,010,012,101,110,120,201		
	001,010,012,101,110,120,210		
	001,010,012,101,110,201,210		
	001,010,012,101,120,201,210		
	001,010,012,102,110,120,201		
	001,010,012,102,110,120,210		
	001,010,012,102,110,201,210		
	001,010,012,102,120,201,210		
	001,010,012,110,120,201,210		
	001,011,012,021,101,102,110		
	001,011,012,021,101,102,120		
	001,011,012,021,101,102,201		
	001,011,012,021,101,102,210		
	001,011,012,021,101,110,120		
	001,011,012,021,101,110,201		
	001,011,012,021,101,110,210		
	001,011,012,021,101,120,201		
	001,011,012,021,101,120,210		
	001,011,012,021,101,201,210		
	001,011,012,021,102,110,120		
	001,011,012,021,102,110,201		
	001,011,012,021,102,110,210		
	001,011,012,021,102,120,201		
	001,011,012,021,102,120,210		
	001,011,012,021,102,201,210		
	001,011,012,021,110,120,201		
	001,011,012,021,110,120,210		
	001,011,012,021,110,201,210		
	001,011,012,021,120,201,210		
	001,011,012,101,102,110,120		
	001,011,012,101,102,110,201		
	001,011,012,101,102,110,210		
	001,011,012,101,102,120,201		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,011,012,101,102,120,210		
	001,011,012,101,102,201,210		
	001,011,012,101,110,120,201		
	001,011,012,101,110,120,210		
	001,011,012,101,110,201,210		
	001,011,012,101,120,201,210		
	001,011,012,102,110,120,201		
	001,011,012,102,110,120,210		
	001,011,012,102,110,201,210		
	001,011,012,102,120,201,210		
	001,011,012,110,120,201,210	$0 \rightsquigarrow (00)^2;\ 00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
8	000,011,012,021,100,101,102		
	000,011,012,021,100,101,110		
	000,011,012,021,100,101,120		
	000,011,012,021,100,101,201		
	000,011,012,021,100,101,210		
	000,011,012,021,100,102,110		
	000,011,012,021,100,102,120		
	000,011,012,021,100,102,201		
	000,011,012,021,100,102,210		
	000,011,012,021,100,110,120		
	000,011,012,021,100,110,201		
	000,011,012,021,100,110,210		
	000,011,012,021,100,120,201		
	000,011,012,021,100,120,210		
	000,011,012,021,100,201,210		
	000,011,012,021,101,102,110		
	000,011,012,021,101,102,120		
	000,011,012,021,101,102,201		
	000,011,012,021,101,102,210		
	000,011,012,021,101,110,120		
	000,011,012,021,101,110,201		
	000,011,012,021,101,110,210		
	000,011,012,021,101,120,201		
	000,011,012,021,101,120,210		
	000,011,012,021,101,201,210		
	000,011,012,021,102,110,120		
	000,011,012,021,102,110,201		
	000,011,012,021,102,110,210		
	000,011,012,021,102,120,201		
	000,011,012,021,102,120,210		
	000,011,012,021,102,201,210		
	000,011,012,021,110,120,201		
	000,011,012,021,110,120,210		
	000,011,012,021,110,201,210		
	000,011,012,021,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (001)^2;\ 01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
9	000,011,012,100,101,102,110		
	000,011,012,100,101,102,120		
	000,011,012,100,101,102,201		
	000,011,012,100,101,102,210		
	000,011,012,100,101,110,120		
	000,011,012,100,101,110,201		
	000,011,012,100,101,110,210		
	000,011,012,100,101,120,201		
	000,011,012,100,101,120,210		
	000,011,012,100,101,201,210		
	000,011,012,100,102,110,120		
	000,011,012,100,102,110,201		
	000,011,012,100,102,110,210		
	000,011,012,100,102,120,201		
	000,011,012,100,102,120,210		
	000,011,012,100,102,201,210		
	000,011,012,100,110,120,201		
	000,011,012,100,110,120,210		
	000,011,012,100,110,201,210		
	000,011,012,100,120,201,210		
	000,011,012,101,102,110,120		
	000,011,012,101,102,110,201		
	000,011,012,101,102,110,210		
	000,011,012,101,102,120,201		
	000,011,012,101,102,120,210		
	000,011,012,101,102,201,210		
	000,011,012,101,110,120,201		
	000,011,012,101,110,120,210		
	000,011,012,101,110,201,210		
	000,011,012,101,120,201,210		
	000,011,012,102,110,120,201		
	000,011,012,102,110,120,210		
	000,011,012,102,110,201,210		
	000,011,012,102,120,201,210		
	000,011,012,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 01;\ 01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
10	000,010,012,021,100,101,102		
	000,010,012,021,100,101,110		
	000,010,012,021,100,101,120		
	000,010,012,021,100,101,201		
	000,010,012,021,100,101,210		
	000,010,012,021,100,102,110		
	000,010,012,021,100,102,120		
	000,010,012,021,100,102,201		
	000,010,012,021,100,102,210		
	000,010,012,021,100,110,120		
	000,010,012,021,100,110,201		
	000,010,012,021,100,110,210		
	000,010,012,021,100,120,201		
	000,010,012,021,100,120,210		
	000,010,012,021,100,201,210		
	000,010,012,021,101,102,110		
	000,010,012,021,101,102,120		
	000,010,012,021,101,102,201		
	000,010,012,021,101,102,210		
	000,010,012,021,101,110,120		
	000,010,012,021,101,110,201		
	000,010,012,021,101,110,210		
	000,010,012,021,101,120,201		
	000,010,012,021,101,120,210		
	000,010,012,021,101,201,210		
	000,010,012,021,102,110,120		
	000,010,012,021,102,110,201		
	000,010,012,021,102,110,210		
	000,010,012,021,102,120,201		
	000,010,012,021,102,120,210		
	000,010,012,021,102,201,210		
	000,010,012,021,110,120,201		
	000,010,012,021,110,120,210		
	000,010,012,021,110,201,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,012,021,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2;\ 01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
11	000,001,021,100,101,110,120		
	000,001,021,100,102,110,120		
	000,001,021,100,110,120,201		
	000,001,021,100,110,120,210		
	000,001,021,101,102,110,120		
	000,001,021,101,110,120,201		
	000,001,021,101,110,120,210		
	000,001,021,102,110,120,201		
	000,001,021,102,110,120,210		
	000,001,021,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2, 012;$	
		$012 \rightsquigarrow 00, 012$	
	001,011,021,100,101,102,120		
	001,011,021,100,101,110,120		
	001,011,021,100,101,120,201		
	001,011,021,100,101,120,210		
	001,011,021,100,102,110,120		
	001,011,021,100,102,120,201		
	001,011,021,100,102,120,210		
	001,011,021,100,110,120,201		
	001,011,021,100,110,120,210		
	001,011,021,100,120,201,210		
	001,011,100,101,102,110,120		
	001,011,100,101,102,120,201		
	001,011,100,101,102,120,210		
	001,011,100,101,110,120,201		
	001,011,100,101,110,120,210		
	001,011,100,101,120,201,210		
	001,011,100,102,110,120,201		
	001,011,100,102,110,120,210		
	001,011,100,102,120,201,210		
	001,011,100,110,120,201,210		
	001,012,021,100,101,102,110		
	001,012,021,100,101,110,120		
	001,012,021,100,101,110,201		
	001,012,021,100,101,110,210		
	001,012,021,100,102,110,120		
	001,012,021,100,102,110,201		
	001,012,021,100,102,110,210		
	001,012,021,100,110,120,201		
	001,012,021,100,110,120,210		
	001,012,021,100,110,201,210		
	001,012,100,101,102,110,120		
	001,012,100,101,102,110,201		
	001,012,100,101,102,110,210		
	001,012,100,101,110,120,201		
	001,012,100,101,110,120,210		
	001,012,100,101,110,201,210		
	001,012,100,102,110,120,201		
	001,012,100,102,110,120,210		
	001,012,100,102,110,201,210		
	001,012,100,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010, 00$	$x + x^3 + \frac{2x^2}{1-x}$
12	000,010,012,100,101,102,110		
	000,010,012,100,101,110,120		
	000,010,012,100,101,110,201		
	000,010,012,100,101,110,210		
	000,010,012,100,102,110,120		
	000,010,012,100,102,110,201		
	000,010,012,100,102,110,210		
	000,010,012,100,110,120,201		
	000,010,012,100,110,120,210		
	000,010,012,100,110,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 01 \rightsquigarrow 011;$	$x + 2x^2 + 3x^3 + 3x^4$
		$002 \rightsquigarrow (011)^2$	
13	000,010,012,100,101,102,120		
	000,010,012,100,101,102,201		
	000,010,012,100,101,102,210		
	000,010,012,100,101,120,201		
	000,010,012,100,101,120,210		
	000,010,012,100,101,201,210		
	000,010,012,100,102,120,201		
	000,010,012,100,102,120,210		
	000,010,012,100,102,201,210		
	000,010,012,100,120,201,210		
	000,010,012,101,102,110,120		
	000,010,012,101,102,110,201		
	000,010,012,101,102,110,210		
	000,010,012,101,110,120,201		
	000,010,012,101,110,120,210		
	000,010,012,101,110,201,210		
	000,010,012,102,110,120,201		
	000,010,012,102,110,120,210		
	000,010,012,102,110,201,210		
	000,010,012,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 01 \rightsquigarrow 011;$	$x + 2x^2 + 3x^3 + 3x^4 + x^5$
		$002 \rightsquigarrow 01, 011$	
14	000,010,012,101,102,120,201		
	000,010,012,101,102,120,210		
	000,010,012,101,102,201,210		
	000,010,012,101,120,201,210		
	000,010,012,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 01 \rightsquigarrow 011;$	$x+2x^2+3x^3+3x^4+2x^5+x^6$
		$002 \rightsquigarrow 01, 0022;\ 0022 \rightsquigarrow 01$	
15	000,001,021,100,101,102,120		
	000,001,021,100,101,120,201		
	000,001,021,100,101,120,210		
	000,001,021,100,102,120,201		
	000,001,021,100,102,120,210		
	000,001,021,100,120,201,210		
	000,001,021,101,102,120,201		
	000,001,021,101,102,120,210		
	000,001,021,101,120,201,210		
	000,001,021,102,120,201,210	$R1)0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011, 012;$	$x + x^3 + x^4 + \frac{2x^2}{1-x}$
		$011 \rightsquigarrow 00;\ 012 \rightsquigarrow 00, 012$	
16	000,001,021,100,101,102,110		
	000,001,021,100,101,110,201		
	000,001,021,100,101,110,210		
	000,001,021,100,102,110,201		
	000,001,021,100,102,110,210		
	000,001,021,100,110,201,210		
	000,001,021,101,102,110,201		
	000,001,021,101,102,110,210		
	000,001,021,101,110,201,210		
	000,001,021,102,110,201,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,100,101,102,110,120 000,001,100,101,110,120,201 000,001,100,101,110,120,210 000,001,100,102,110,120,201 000,001,100,102,110,120,210 000,001,100,110,120,201,210 000,001,101,102,110,120,201 000,001,101,102,110,120,210 000,001,101,110,120,201,210 000,001,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 00, 01$	
	001,011,021,100,101,102,110 001,011,021,100,101,102,201 001,011,021,100,101,102,210 001,011,021,100,101,110,201 001,011,021,100,101,110,210 001,011,021,100,101,201,210 001,011,021,100,102,110,201 001,011,021,100,102,110,210 001,011,021,100,102,201,210 001,011,021,100,110,201,210 001,011,100,101,102,110,201 001,011,100,101,102,110,210 001,011,100,101,102,201,210 001,011,100,101,110,201,210 001,011,100,102,110,201,210 001,012,021,100,101,102,120 001,012,021,100,101,102,201 001,012,021,100,101,102,210 001,012,021,100,101,120,201 001,012,021,100,101,120,210 001,012,021,100,101,201,210 001,012,021,100,102,120,201 001,012,021,100,102,120,210 001,012,021,100,102,201,210 001,012,021,100,120,201,210 001,012,100,101,102,120,201 001,012,100,101,102,120,210 001,012,100,101,102,201,210 001,012,100,101,120,201,210 001,012,100,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 01$	
	001,011,021,101,102,110,120 001,011,021,101,102,120,201 001,011,021,101,102,120,210 001,011,021,101,110,120,201 001,011,021,101,110,120,210 001,011,021,101,120,201,210 001,011,021,102,110,120,201 001,011,021,102,110,120,210 001,011,021,102,120,201,210 001,011,021,110,120,201,210 001,011,101,102,110,120,201 001,011,101,102,110,120,210 001,011,101,102,120,201,210 001,011,101,110,120,201,210 001,011,102,110,120,201,210 001,012,021,101,102,110,120 001,012,021,101,102,110,201 001,012,021,101,102,110,210 001,012,021,101,110,120,201 001,012,021,101,110,120,210 001,012,021,101,110,201,210 001,012,021,102,110,120,201 001,012,021,102,110,120,210 001,012,021,102,110,201,210 001,012,021,110,120,201,210 001,012,101,102,110,120,201 001,012,101,102,110,120,210 001,012,101,102,110,201,210 001,012,101,110,120,201,210 001,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 00, 00$	$x + 2x^2 + \frac{3x^3}{1-x}$
17	000,001,021,100,101,102,201 000,001,021,100,101,102,210 000,001,021,100,101,201,210 000,001,021,100,102,201,210 000,001,021,101,102,201,210 000,001,100,101,102,120,201 000,001,100,101,102,120,210 000,001,100,101,120,201,210 000,001,100,102,120,201,210 000,001,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 01$; $011 \rightsquigarrow 00$	$x + 2x^2 + 3x^3 + \frac{4x^4}{1-x}$
18	000,001,100,101,102,110,201 000,001,100,101,102,110,210 000,001,100,101,110,201,210 000,001,100,102,110,201,210 000,001,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, (00)^{m+1}$, $a_m = 01 \cdots m$	
	000,010,011,021,100,101,102 000,010,011,021,100,101,110 000,010,011,021,100,101,120 000,010,011,021,100,101,201 000,010,011,021,100,101,210 000,010,011,021,100,102,110 000,010,011,021,100,102,120 000,010,011,021,100,102,201 000,010,011,021,100,102,210 000,010,011,021,100,110,120 000,010,011,021,100,110,201 000,010,011,021,100,110,210 000,010,011,021,100,120,201 000,010,011,021,100,120,210 000,010,011,021,100,201,210 000,010,011,021,101,102,110 000,010,011,021,101,102,120 000,010,011,021,101,102,201 000,010,011,021,101,102,210 000,010,011,021,101,110,120 000,010,011,021,101,110,201 000,010,011,021,101,110,210 000,010,011,021,101,120,201 000,010,011,021,101,120,210 000,010,011,021,101,201,210 000,010,011,021,102,110,120 000,010,011,021,102,110,201 000,010,011,021,102,110,210 000,010,011,021,102,120,201		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,011,021,102,120,210		
	000,010,011,021,102,201,210		
	000,010,011,021,110,120,201		
	000,010,011,021,110,120,210		
	000,010,011,021,110,201,210		
	000,010,011,021,120,201,210	$0 \rightsquigarrow 0, 01;\ 01 \rightsquigarrow 01$	
	001,010,021,100,101,102,110		
	001,010,021,100,101,102,120		
	001,010,021,100,101,102,201		
	001,010,021,100,101,102,210		
	001,010,021,100,101,110,120		
	001,010,021,100,101,110,201		
	001,010,021,100,101,110,210		
	001,010,021,100,101,120,201		
	001,010,021,100,101,120,210		
	001,010,021,100,101,201,210		
	001,010,021,100,102,110,120		
	001,010,021,100,102,110,201		
	001,010,021,100,102,110,210		
	001,010,021,100,102,120,201		
	001,010,021,100,102,120,210		
	001,010,021,100,102,201,210		
	001,010,021,100,110,120,201		
	001,010,021,100,110,120,210		
	001,010,021,100,110,201,210		
	001,010,021,100,120,201,210		
	001,010,021,101,102,110,120		
	001,010,021,101,102,110,201		
	001,010,021,101,102,110,210		
	001,010,021,101,102,120,201		
	001,010,021,101,102,120,210		
	001,010,021,101,102,201,210		
	001,010,021,101,110,120,201		
	001,010,021,101,110,120,210		
	001,010,021,101,110,201,210		
	001,010,021,101,120,201,210		
	001,010,021,102,110,120,201		
	001,010,021,102,110,120,210		
	001,010,021,102,110,201,210		
	001,010,021,102,120,201,210		
	001,010,021,110,120,201,210		
	001,010,100,101,102,110,120		
	001,010,100,101,102,110,201		
	001,010,100,101,102,110,210		
	001,010,100,101,102,120,201		
	001,010,100,101,102,120,210		
	001,010,100,101,102,201,210		
	001,010,100,101,110,120,201		
	001,010,100,101,110,120,210		
	001,010,100,101,110,201,210		
	001,010,100,101,120,201,210		
	001,010,100,102,110,120,201		
	001,010,100,102,110,120,210		
	001,010,100,102,110,201,210		
	001,010,100,102,120,201,210		
	001,010,100,110,120,201,210		
	001,010,101,102,110,120,201		
	001,010,101,102,110,120,210		
	001,010,101,102,110,201,210		
	001,010,101,102,120,201,210		
	001,010,101,110,120,201,210		
	001,010,102,110,120,201,210		
	001,011,021,101,102,110,201		
	001,011,021,101,102,110,210		
	001,011,021,101,102,201,210		
	001,011,021,101,110,201,210		
	001,011,021,102,110,201,210		
	001,011,101,102,110,201,210		
	001,012,021,101,102,120,201		
	001,012,021,101,102,120,210		
	001,012,021,101,102,201,210		
	001,012,021,101,120,201,210		
	001,012,021,102,120,201,210		
	001,012,101,102,120,201,210	$0 \rightsquigarrow 00, 0;\ 00 \rightsquigarrow 00$	
	010,011,012,021,100,101,102		
	010,011,012,021,100,101,110		
	010,011,012,021,100,101,120		
	010,011,012,021,100,101,201		
	010,011,012,021,100,101,210		
	010,011,012,021,100,102,110		
	010,011,012,021,100,102,120		
	010,011,012,021,100,102,201		
	010,011,012,021,100,102,210		
	010,011,012,021,100,110,120		
	010,011,012,021,100,110,201		
	010,011,012,021,100,110,210		
	010,011,012,021,100,120,201		
	010,011,012,021,100,120,210		
	010,011,012,021,100,201,210		
	010,011,012,021,101,102,110		
	010,011,012,021,101,102,120		
	010,011,012,021,101,102,201		
	010,011,012,021,101,102,210		
	010,011,012,021,101,110,120		
	010,011,012,021,101,110,201		
	010,011,012,021,101,110,210		
	010,011,012,021,101,120,201		
	010,011,012,021,101,120,210		
	010,011,012,021,101,201,210		
	010,011,012,021,102,110,120		
	010,011,012,021,102,110,201		
	010,011,012,021,102,110,210		
	010,011,012,021,102,120,201		
	010,011,012,021,102,120,210		
	010,011,012,021,102,201,210		
	010,011,012,021,110,120,201		
	010,011,012,021,110,120,210		
	010,011,012,021,110,201,210		
	010,011,012,021,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m,\ a_m = 0^m$	$\frac{x}{(1-x)^2}$
19	000,010,011,100,101,102,120		
	000,010,011,100,102,110,120		
	000,010,011,100,102,120,201		
	000,010,011,100,102,120,210		
	000,010,011,101,102,110,120		
	000,010,011,101,102,120,201		
	000,010,011,101,102,120,210		
	000,010,011,102,110,120,201		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,011,102,110,120,210		
	000,010,011,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
20	000,001,100,101,102,201,210	$a_m \rightsquigarrow (b_0)^m, b_m, a_{m+1}$; $a_m \rightsquigarrow (b_0)^m$, $a_m = 01 \cdots m$, $b_m = a_m m$	
	000,010,011,100,101,102,110		
	000,010,011,100,101,102,201		
	000,010,011,100,101,102,210		
	000,010,011,100,102,110,201		
	000,010,011,100,102,110,210		
	000,010,011,100,102,201,210		
	000,010,011,101,102,110,201		
	000,010,011,101,102,110,210		
	000,010,011,101,102,201,210		
	000,010,011,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 002$	$\frac{x(1+x^3)}{(1-x)^2}$
21	010,011,012,100,101,102,210		
	010,011,012,100,101,110,210		
	010,011,012,100,101,120,210		
	010,011,012,100,101,201,210		
	010,011,012,100,102,110,210		
	010,011,012,100,102,120,210		
	010,011,012,100,102,201,210		
	010,011,012,100,110,120,210		
	010,011,012,100,110,201,210		
	010,011,012,100,120,201,210		
	010,011,012,101,102,110,210		
	010,011,012,101,102,120,210		
	010,011,012,101,102,201,210		
	010,011,012,101,110,120,210		
	010,011,012,101,110,201,210		
	010,011,012,101,120,201,210		
	010,011,012,102,110,120,210		
	010,011,012,102,110,201,210		
	010,011,012,102,120,201,210		
	010,011,012,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (b_1)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
22	000,010,011,100,101,110,120		
	000,010,011,100,101,120,201		
	000,010,011,100,101,120,210		
	000,010,011,100,110,120,201		
	000,010,011,100,110,120,210		
	000,010,011,100,120,201,210		
	000,010,011,101,110,120,201		
	000,010,011,101,110,120,210		
	000,010,011,101,120,201,210		
	000,010,011,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 0$; $01 \rightsquigarrow 01$	
	010,011,012,100,101,102,110		
	010,011,012,100,101,102,120		
	010,011,012,100,101,102,201		
	010,011,012,100,101,110,120		
	010,011,012,100,101,110,201		
	010,011,012,100,101,120,201		
	010,011,012,100,102,110,120		
	010,011,012,100,102,110,201		
	010,011,012,100,102,120,201		
	010,011,012,100,110,120,201		
	010,011,012,101,102,110,120		
	010,011,012,101,102,110,201		
	010,011,012,101,102,120,201		
	010,011,012,101,110,120,201		
	010,011,012,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow b_1, \ldots, b_{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1+x)}{1-x-x^2}$
23	000,010,011,100,101,110,201		
	000,010,011,100,101,110,210		
	000,010,011,100,101,201,210		
	000,010,011,100,110,201,210		
	000,010,011,101,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 00$; $01 \rightsquigarrow 01$	$\frac{x(1-x-x^2)}{(1-x)(1-2x)}$
24	000,010,100,101,102,110,120•		
25	000,010,100,102,110,120,201•		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
26	000,010,100,101,102,120,201● 000,010,100,101,102,120,210●	$a_{m,j} \rightsquigarrow$ $\{d_i\}_{i=1}^{j-m}, c_{m,j}, \{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $c_{m,j} \rightsquigarrow$ $\{d_i\}_{i=1}^{j-m}, \{a_{2m+1-j,i}\}_{i=2m+1-j}^{4m+2-2j}$; $d_m \rightsquigarrow \{d_i\}_{i=1}^{m-1}$, $d_m = 0^2 \cdots (m-1)^2(2m)(2m-1)$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $c_{m,j} = a_{m,j} j$	
	000,010,100,102,110,120,210●	$a_{m,j} \rightsquigarrow$ $\{b_i\}_{i=1}^{j-m}, d_{2m-j}, \{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}, 00212$; $d_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$; $d_m = 0^2 \cdots m^2$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = a_{m,2m} m$	
	000,010,101,102,110,120,201●	$a_{m,j} \rightsquigarrow$ $\{b_i\}_{i=1}^{j-m}, d_{2m-j}, \{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}, 00211$; $d_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$; $d_m = 0^2 \cdots m^2$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = a_{m,2m}(2m-1)$	$\frac{(1+x)(1-x-2x^2-2x^3)}{2x^3(x^2+2x+2)}$ $-\frac{\sqrt{1-2x-3x^2})}{2x^3(x^2+2x+2)}$
27	000,010,101,102,110,120,210●		
28	000,010,100,101,102,110,210●		
29	000,010,100,101,102,110,201●		
30	000,010,100,102,120,201,210●		
31	000,010,101,102,120,201,210●	$a_{m,j} \rightsquigarrow (0021)^{j-m}, b_{m,j}$, $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_{m,j} \rightsquigarrow$ $(0021)^{j-m}$, $\{a_{2m+1-j,i}\}_{i=2m+1-j}^{4m+2-2j}$; $0021 \rightsquigarrow 00211$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_{m,j} = a_{m,j} j$	
	000,010,102,110,120,201,210●	$a_{m,j} \rightsquigarrow (0021)^{j-m}, b_{2m-j}$, $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$; $0021 \rightsquigarrow 00211, 00212$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = 0^2 \cdots m^2$	$\frac{2-3x-5x^2+2x^3+2x^4}{2x^2}$ + $\frac{(2x^2+x-2)\sqrt{1-2x-3x^2}}{2x^2}$
32	000,010,100,102,110,201,210●		
33	000,010,101,102,110,201,210●		
34	000,010,100,101,102,201,210●		
35	000,010,100,101,110,120,201● 000,010,100,101,110,120,210●		By the proof of Theorem 5
36	000,010,100,101,110,201,210●		
37	000,010,100,101,120,201,210● 000,010,100,110,120,201,210● 000,010,101,110,120,201,210●		By the proof of Theorem 12
38	000,012,021,100,101,102,110 000,012,021,100,101,110,120 000,012,021,100,101,110,201 000,012,021,100,101,110,210 000,012,021,101,102,110,120 000,012,021,101,102,110,201 000,012,021,101,102,110,210 000,012,021,101,110,120,201 000,012,021,101,110,120,210 000,012,021,101,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 001$; $01 \rightsquigarrow 010, 010$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^5$
39	000,012,021,100,101,102,120 000,012,021,100,101,102,201 000,012,021,100,101,102,210 000,012,021,100,101,120,201 000,012,021,100,101,120,210 000,012,021,100,101,201,210 000,012,021,101,102,120,201 000,012,021,101,102,120,210 000,012,021,101,102,201,210 000,012,021,101,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000,012,021,100,102,110,120 000,012,021,100,102,110,201 000,012,021,100,102,110,210 000,012,021,100,110,120,201 000,012,021,100,110,120,210 000,012,021,100,110,201,210 000,012,021,102,110,120,201 000,012,021,102,110,120,210 000,012,021,102,110,201,210 000,012,021,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	
	000,012,100,101,102,110,120 000,012,100,101,102,110,201 000,012,100,101,102,110,210 000,012,100,101,110,120,201 000,012,100,101,110,120,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,012,100,101,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 010, 010$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 3x^5$
40	000,012,101,102,110,120,201 000,012,101,102,110,120,210 000,012,101,102,110,201,210 000,012,101,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 010$	$x + 2x^2 + 4x^3 + 3x^5 + x^6$
41	000,012,021,100,102,120,201 000,012,021,100,102,120,210 000,012,021,100,102,201,210 000,012,021,100,120,201,210 000,012,021,102,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 001, 001)62$; $001 \rightsquigarrow 0011$	
	000,012,100,102,110,120,201 000,012,100,102,110,120,210 000,012,100,102,110,201,210 000,012,100,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$; $002 \rightsquigarrow 011, 011$	$x + 2x^2 + 4x^3 + 4x^5$
42	000,012,100,101,102,120,201 000,012,100,101,102,120,210 000,012,100,101,102,201,210 000,012,100,101,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$	$x + 2x^2 + 4x^3 + 4x^5 + x^6$
43	000,012,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 0022$; $0022 \rightsquigarrow 001$	$x+2x^2+4x^3+4x^5+2x^6+x^7$
44	000,011,021,100,101,102,120 000,011,021,100,102,110,120 000,011,021,100,102,120,201 000,011,021,100,102,120,210 000,011,021,101,102,110,120 000,011,021,101,102,120,201 000,011,021,101,102,120,210 000,011,021,102,110,120,201 000,011,021,102,110,120,210 000,011,021,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	
	001,021,100,101,102,110,120 001,021,100,101,110,120,201 001,021,100,101,110,120,210 001,021,100,102,110,120,201 001,021,100,102,110,120,210 001,021,100,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 012$; $012 \rightsquigarrow 00, 012$	$x^3 + \frac{x}{(1-x)^2}$
45	000,012,100,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$; $002 \rightsquigarrow 0011, 001$	$x + 2x^2 + 4x^3 + 5x^5 + x^6$
46	000,011,021,100,101,102,110 000,011,021,100,101,102,201 000,011,021,100,101,102,210 000,011,021,100,102,110,201 000,011,021,100,102,110,210 000,011,021,100,102,201,210 000,011,021,101,102,110,201 000,011,021,101,102,110,210 000,011,021,101,102,201,210 000,011,021,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 01$; $002 \rightsquigarrow 002$	
	000,011,100,101,102,110,120 000,011,100,101,102,120,201 000,011,100,101,102,120,210 000,011,100,102,110,120,201 000,011,100,102,110,120,210 000,011,100,102,120,201,210 000,011,101,102,110,120,201 000,011,101,102,110,120,210 000,011,101,102,120,201,210 000,011,102,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 012$; $012 \rightsquigarrow 012$	
	001,021,100,101,102,110,201 001,021,100,101,102,110,210 001,021,100,101,110,201,210 001,021,100,102,110,201,210 001,100,101,102,110,120,201 001,100,101,102,110,120,210 001,100,101,110,120,201,210 001,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 01$	
	001,021,100,101,102,120,201 001,021,100,101,102,120,210 001,021,100,101,120,201,210 001,021,100,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 012$; $011 \rightsquigarrow 010, 011$; $012 \rightsquigarrow 00, 012$	
	001,021,101,102,110,120,201 001,021,101,102,110,120,210 001,021,101,110,120,201,210 001,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2-x^3)}{(1-x)^2}$
47	000,011,021,100,101,110,120 000,011,021,100,101,120,201 000,011,021,100,101,120,210 000,011,021,100,110,120,201 000,011,021,100,110,120,210 000,011,021,100,120,201,210 000,011,021,101,110,120,201 000,011,021,101,110,120,210 000,011,021,101,120,201,210 000,011,021,110,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	000,011,100,101,102,110,201 000,011,100,101,102,110,210 000,011,100,101,102,201,210 000,011,100,102,110,201,210 000,011,101,102,110,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 01$	
	001,021,100,101,102,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 01$; $011 \rightsquigarrow 010, 011$	
	001,021,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 00, 00, 01$	
	001,021,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 00, (011)^2$; $011 \rightsquigarrow 00, 011$	
	001,100,101,102,110,201,210	$a_m \rightsquigarrow (010)^m, 00, a_{m+1}$; $00 \rightsquigarrow 00$, $a_m = 01 \cdots m$	
	001,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 01$; $011 \rightsquigarrow 010, 011$	

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 01$	
	011,012,021,100,101,102,110		
	011,012,021,100,101,102,120		
	011,012,021,100,101,102,201		
	011,012,021,100,101,102,210		
	011,012,021,100,101,110,120		
	011,012,021,100,101,110,201		
	011,012,021,100,101,110,210		
	011,012,021,100,101,120,201		
	011,012,021,100,101,120,210		
	011,012,021,100,101,201,210		
	011,012,021,100,102,110,120		
	011,012,021,100,102,110,201		
	011,012,021,100,102,110,210		
	011,012,021,100,102,120,201		
	011,012,021,100,102,120,210		
	011,012,021,100,102,201,210		
	011,012,021,100,110,120,201		
	011,012,021,100,110,120,210		
	011,012,021,100,110,201,210		
	011,012,021,100,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010$; $a_m = 0^m$	$\frac{x(1+x^2)}{(1-x)^2}$
48	000,011,021,100,101,110,201		
	000,011,021,100,101,110,210		
	000,011,021,100,101,201,210		
	000,011,021,100,110,201,210		
	000,011,021,101,110,201,210	$0 \rightsquigarrow 00, 0$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	010,012,021,100,101,102,110		
	010,012,021,100,101,102,120		
	010,012,021,100,101,102,201		
	010,012,021,100,101,102,210		
	010,012,021,100,101,110,120		
	010,012,021,100,101,110,201		
	010,012,021,100,101,110,210		
	010,012,021,100,101,120,201		
	010,012,021,100,101,120,210		
	010,012,021,100,101,201,210		
	010,012,021,100,102,110,120		
	010,012,021,100,102,110,201		
	010,012,021,100,102,110,210		
	010,012,021,100,102,120,201		
	010,012,021,100,102,120,210		
	010,012,021,100,102,201,210		
	010,012,021,100,110,120,201		
	010,012,021,100,110,120,210		
	010,012,021,100,110,201,210		
	010,012,021,100,120,201,210		
	010,012,021,101,102,110,120		
	010,012,021,101,102,110,201		
	010,012,021,101,102,110,210		
	010,012,021,101,102,120,201		
	010,012,021,101,102,120,210		
	010,012,021,101,102,201,210		
	010,012,021,101,110,120,201		
	010,012,021,101,110,120,210		
	010,012,021,101,110,201,210		
	010,012,021,101,120,201,210		
	010,012,021,102,110,120,201		
	010,012,021,102,110,120,210		
	010,012,021,102,110,201,210		
	010,012,021,102,120,201,210		
	010,012,021,110,120,201,210		
	011,012,021,101,102,110,120		
	011,012,021,101,102,110,201		
	011,012,021,101,102,110,210		
	011,012,021,101,102,120,201		
	011,012,021,101,102,120,210		
	011,012,021,101,102,201,210		
	011,012,021,101,110,120,201		
	011,012,021,101,110,120,210		
	011,012,021,101,110,201,210		
	011,012,021,101,120,201,210		
	011,012,021,102,110,120,201		
	011,012,021,102,110,120,210		
	011,012,021,102,110,201,210		
	011,012,021,102,120,201,210		
	011,012,021,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01$, $a_m = 0^m$	
	011,012,100,101,102,201,210		
	011,012,100,101,110,201,210		
	011,012,100,101,120,201,210		
	011,012,100,102,110,201,210		
	011,012,100,102,120,201,210		
	011,012,100,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01$, $a_m = 0^m$	$\frac{x(1-x+x^2)}{(1-x)^3}$
49	011,012,100,101,102,110,210		
	011,012,100,101,102,120,210		
	011,012,100,101,110,120,210		
	011,012,100,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow 01, 010$; $b_m \rightsquigarrow c_m, (010)^{m-1}$; $c_m \rightsquigarrow (010)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x+x^2+x^4)}{(1-x)^3}$
50	000,011,100,101,110,120,201		
	000,011,100,101,110,120,210		
	000,011,100,101,120,201,210		
	000,011,100,110,120,201,210		
	000,011,101,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 0, 012$; $012 \rightsquigarrow 012$	
	011,012,100,101,102,110,201		
	011,012,100,101,102,120,201		
	011,012,100,101,110,120,201		
	011,012,100,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x}{(1-x)(1-x-x^2)}$

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
51	011,012,100,101,102,110,120	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow b_1^2$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m-1}$; $c_m \rightsquigarrow 010, 01, c_3, \ldots, c_{m-1}$; $a_m = 0^m$, $b_m = a_m m$; $c_m = a_m m0$	$\frac{x(1-x^2-x^3)}{(1-x-x^2)^2}$
52	010,012,100,101,102,110,210 010,012,100,101,110,120,210 010,012,100,101,110,201,210 010,012,100,102,110,120,210 010,012,100,102,110,201,210 010,012,100,110,120,201,210 011,012,101,102,110,201,210 011,012,101,102,120,201,210 011,012,101,110,120,201,210 011,012,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (0021)^{m-1}, b_1$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^2+x^3)}{(1-x)^3}$
53	010,012,100,101,102,110,120 010,012,100,101,102,110,201 010,012,100,101,110,120,201 010,012,100,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=2}^{m}, b_1$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$, $c_m = 0^m m(m-1)$	$\frac{x((1-x)^2+x^3-2x^4)}{(1-x)^3(1-x-x^2)}$
54	010,012,100,101,102,120,210 010,012,100,101,102,201,210 010,012,100,101,120,201,210 010,012,100,102,120,201,210 011,012,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (0021)^{m-1}, b_m$, $a_m = 0^m$, $b_m = 0^m m$	
	010,012,101,102,110,120,210 010,012,101,102,110,201,210 010,012,101,110,120,201,210 010,012,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (01)^m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-2x+2x^2)}{(1-x)^4}$
55	010,012,100,101,102,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{c_i\}_{i=2}^{m}, b_m$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$, $c_m = 0^m m(m-1)$	
	010,012,101,102,110,120,201 011,012,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}, b_1$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(x^3-x+1)}{(1-x)^2(1-x-x^2)}$
56	010,012,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (01)^{m-1}, b_m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-3x+4x^2-2x^3+x^4)}{(1-x)^5}$
57	000,011,100,101,110,201,210	$0 \rightsquigarrow (0)^2$	
	010,011,021,100,101,102,110 010,011,021,100,101,102,120 010,011,021,100,101,102,201 010,011,021,100,101,102,210 010,011,021,100,101,110,120 010,011,021,100,101,110,201 010,011,021,100,101,110,210 010,011,021,100,101,120,201 010,011,021,100,101,120,210 010,011,021,100,101,201,210 010,011,021,100,102,110,120 010,011,021,100,102,110,201 010,011,021,100,102,110,210 010,011,021,100,102,120,201 010,011,021,100,102,120,210 010,011,021,100,102,201,210 010,011,021,100,110,120,201 010,011,021,100,110,120,210 010,011,021,100,110,201,210 010,011,021,100,120,201,210 010,011,021,101,102,110,120 010,011,021,101,102,110,201 010,011,021,101,102,110,210 010,011,021,101,102,120,201 010,011,021,101,102,120,210 010,011,021,101,102,201,210 010,011,021,101,110,120,201 010,011,021,101,110,120,210 010,011,021,101,110,201,210 010,011,021,101,120,201,210 010,011,021,102,110,120,201 010,011,021,102,110,120,210 010,011,021,102,110,201,210 010,011,021,102,120,201,210 010,011,021,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$	$\frac{x}{1-2x}$
58	010,011,100,102,120,201,210 010,011,101,102,120,201,210 010,011,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(0021)^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x((1-x)^2+x^3)}{(1-2x)(1-x)^2}$
59	010,011,100,101,102,120,201 010,011,100,102,110,120,201		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,011,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^j, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	
	010,011,100,101,102,120,210 010,011,100,102,110,120,210 010,011,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^j, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m1$	$\frac{x(2x^3-2x+1)}{(1-x)(1-2x)(1-x-x^2)}$
60	010,011,100,101,102,110,120		
61	010,011,100,101,102,201,210 010,011,100,102,110,201,210 010,011,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, \{b_{m,i}\}_{i=j}^m$, $a_m = 0^m$, $b_{m,j} = a_m j$	Theorem 16
62	010,011,100,101,102,110,210		
63	010,011,100,101,102,110,201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^j, \{b_{m,i}\}_{i=j}^m$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	$\frac{x(1-x-x^2)}{(1-x)(1-2x-x^2)}$
64	000,010,021,100,101,102,110 000,010,021,100,101,102,120 000,010,021,100,101,102,201 000,010,021,100,101,102,210 000,010,021,100,101,110,120 000,010,021,100,101,110,201 000,010,021,100,101,110,210 000,010,021,100,101,120,201 000,010,021,100,101,120,210 000,010,021,100,101,201,210 000,010,021,100,102,110,120 000,010,021,100,102,110,201 000,010,021,100,102,110,210 000,010,021,100,102,120,201 000,010,021,100,102,120,210 000,010,021,100,102,201,210 000,010,021,100,110,120,201 000,010,021,100,110,120,210 000,010,021,100,110,201,210 000,010,021,100,120,201,210 000,010,021,101,102,110,120 000,010,021,101,102,110,201 000,010,021,101,102,110,210 000,010,021,101,102,120,201 000,010,021,101,102,120,210 000,010,021,101,102,201,210 000,010,021,101,110,120,201 000,010,021,101,110,120,210 000,010,021,101,110,201,210 000,010,021,101,120,201,210 000,010,021,102,110,120,201 000,010,021,102,110,120,210 000,010,021,102,110,201,210 000,010,021,102,120,201,210 000,010,021,110,120,201,210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^m$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$; $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1$
65	010,011,100,101,120,201,210 010,011,100,110,120,201,210 010,011,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $(b_{m+2-j,1})^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-2x)}{(1-3x+x^2)(1-x)}$
66	010,011,100,101,110,120,201 010,011,100,101,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow \{b_{m+1+i-j,i}\}_{i=1}^{j-1}$, $\{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $a_m = 0^m$, $b_{m,j} = a_m j$	
67	010,011,100,101,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (b_{m+2-j,1}^{j-1}, \{b_{m,i}\}_{i=j}^m$; $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x(1-3x+x^2)}{(1-3x)(1-x)^2}$
68	012,021,100,101,102,120,201 012,021,100,101,102,120,210 012,021,100,101,102,201,210 012,021,100,101,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$, $a_m = 0^m$	
	012,021,100,102,110,120,201 012,021,100,102,110,120,210 012,021,100,102,110,201,210 012,021,100,110,120,201,210 012,021,101,102,110,120,201 012,021,101,102,110,120,210 012,021,101,102,110,201,210 012,021,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010^2$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012,100,101,102,110,201,210		

Continuation of Table 3			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	012,100,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (010)^m, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+2x^2)}{(1-x)^3}$
69	012,100,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow c_m, (010)^{m-1}, 011$; $c_m \rightsquigarrow (010)^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m0$	$\frac{x(1-x+2x^2+x^4)}{(1-x)^3}$
70	012,100,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 011$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$; $011 \rightsquigarrow 011$, $a_m = 0^m$, $b_m = a_m m$, $c_m = a_m m(m-1)$	$\frac{x(1-2x+2x^2-x^3-x^4)}{(1-x)^3(1-x-x^2)}$
71	011,021,100,101,102,110,120 011,021,100,101,102,120,201 011,021,100,101,102,120,210 011,021,100,102,110,120,201 011,021,100,102,110,120,210 011,021,100,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^m$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-2x+2x^2-2x^3)}{(1-x)^2(1-2x)}$
72	000,021,100,101,102,120,201 000,021,100,101,102,120,210 000,021,101,102,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 010, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	
	000,021,100,102,110,120,201 000,021,100,102,110,120,210 000,021,102,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 002$; $010 \rightsquigarrow 0101$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $x^3 + x^4$
73	000,021,100,101,102,110,201 000,021,100,101,102,110,210 000,021,101,102,110,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 01$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $\frac{x^3}{1-x}$
74	012,100,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^2, (0021)^{m-1}$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(x^3+2x^2-x+1)}{(1-x)^3}$
75	012,021,100,102,120,201,210 012,021,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012,021,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 011, 01$; $011 \rightsquigarrow 011$, $a_m = 0^m$	
	012,100,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m, b_m$, $a_m = 0^m$, $b_m = a_m m$	
	012,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^{m+1}$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-2x+3x^2-x^3)}{(1-x)^4}$
76	000,021,100,102,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, 011, 002$; $011 \rightsquigarrow 0101, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $x^3 + 2x^4$
77	011,021,101,102,110,120,201 011,021,101,102,110,120,210 011,021,101,102,120,201,210 011,021,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^2, \{c_i\}_{i=2}^m$; $c_m \rightsquigarrow 010, \{c_i\}_{i=2}^m$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-3x+4x^2-3x^3)}{(1-x)^3(1-2x)}$
78	000,021,100,102,110,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 012$; $010 \rightsquigarrow 0101$; $012 \rightsquigarrow 0101, b_0, 012$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 +$ $x^4 + \frac{x^3}{1-x}$
79	011,021,100,101,102,110,201 011,021,100,101,102,110,210 011,021,100,101,102,201,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	011,021,100,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = 0^m 1$	
	011,100,101,102,120,201,210 011,100,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $(010)^j, c_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $c_m \rightsquigarrow c_m, \{b_{m-1,i}\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m 12$	$\frac{x(1-x+x^2)}{(1-x)(1-2x)}$
80	011,100,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^{j}, d_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m m(m-1)$, $d_m = 0^m 12$	
	011,100,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow$ $\{c_i\}_{i=1}^{j}, d_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $d_m \rightsquigarrow d_m, \{b_{m-1,i}\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m m0$, $d_m = 0^m 12$	$\frac{x(1-x)}{(1-2x)(1-x-x^2)}$
81	000,021,100,101,102,201,210	$a_0 \rightsquigarrow b_0, c_1$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $c_m \rightsquigarrow 010, d_m, \{c_i\}_{i=1}^{m}$; $d_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m+1}$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c_m = 01^2 \cdots (m-1)^2 m$, $d_m = c_m m$	$\frac{(1+x^2)(1-x-\sqrt{1-2x-3x^2})}{2x^2}$ $-1-x^2$
82	000,021,100,101,110,120,201 000,021,100,101,110,120,210 000,021,101,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (b_0)^2, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{(1+x-x^2)(1-x)}{2x^2} -$ $\frac{(1+x-x^2)\sqrt{1-2x-3x^2}}{2x^2} -$ $1-x$
83	000,100,101,102,110,120,201•	$a_m \rightsquigarrow b_m, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $b_m \rightsquigarrow a_{m+1}, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $c_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m-1}$; $d_{m,j} \rightsquigarrow 010, \{c_i\}_{i=1}^{j-1-m}, b_{2m+1-j}$, $\{d_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c_m = a + m(2m+1)(2m)$, $d_{m,j} = a_m j$	
	000,100,101,102,110,120,210•	$a_m \rightsquigarrow b_m, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $b_m \rightsquigarrow a_{m+1}, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $c_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m-1}$; $d_{m,j} \rightsquigarrow 010, \{c_i\}_{i=1}^{j-1-m}, b_{2m+1-j}$, $\{d_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c_m = a + m(2m+1)m$, $d_{m,j} = a_m j$	
84	000,101,102,110,120,201,210•		
85	011,101,102,110,120,201,210		
86	011,021,100,101,110,120,201 011,021,100,101,110,120,210 011,021,100,101,120,201,210 011,021,100,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, 012$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_m = 0^m 1$	
	011,021,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = 0^m 1$	$\frac{x(2x^2-2x+1)}{(1-2x)(1-x)^2}$
87	011,100,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, \{b_{m,i}\}_{i=j}^{m}$	$\frac{x(1-x)^2}{(1-2x)^2}$
88	000,021,100,101,110,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow (b_0)^2, 01$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{(1+x)(1-x-\sqrt{1-2x-3x^2})}{2x^2} -$ $\frac{1}{1-x}$
89	000,021,100,101,120,201,210		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,021,100,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow b_0, a_1, 002$; $002 \rightsquigarrow b_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	
	000,100,101,102,120,201,210●	$a_m \rightsquigarrow c_m, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $c_m \rightsquigarrow a_{m+1}, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $d_{m,j} \rightsquigarrow (010)^{j-m}, b_{m,j}$, $\{d_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$; $b_{m,j} \rightsquigarrow (010)^{j-m}, a_{2m+2-j}$, $\{d_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_{m,j} = a_m jj$, $c_m = a_m m$, $d_{m,j} = a_{m,j} j$	
	000,100,102,110,120,201,210●	$a_m \rightsquigarrow c_m, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $c_m \rightsquigarrow a_{m+1}, \{d_{m,i}\}_{i=m+1}^{2m+1}$; $d_{m,j} \rightsquigarrow (010)^{j-m}, c_{2m+1-j}$, $\{d_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$; $010 \rightsquigarrow 0101$, $a_m = 0^2 \cdots (m-1)^2 m$, $c_m = a_m m$, $d_{m,j} = a_{m,j} j$	$\frac{(2+x)(1-2x-x^2)}{2x^2} -$ $\frac{(2+x)(1-x)\sqrt{1-2x-3x^2}}{2x^2} + x$
90	000,100,101,102,110,201,210●	$a_{m,j} \rightsquigarrow (010)^{j-m}, b_j, \{a_{m+1,i}\}_{i=0}^{j}$; $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=0}^{m+1}$, $a_{m,j} = 0^2 \cdots (m-1)^2 m \cdots j$, $b_m = 0^2 \cdots m^2$	$\frac{4x}{(1-x+\sqrt{1-2x-3x^2})^2}$
91	011,021,100,101,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow c_m, \{b_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}, 0103$; $0103 \rightsquigarrow 0103$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 10$	
	011,021,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow b_{m+1}, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{x(1-3x+3x^2)}{(1-x)(1-2x)^2}$
92	011,100,101,110,120,201,210		
93	000,100,101,110,120,201,210●	$a_m \rightsquigarrow b_m, \{c_{m,i}\}_{i=m+1}^{2m+1}$; $b_m \rightsquigarrow a_{m+1}, \{c_{m,i}\}_{i=m+1}^{2m+1}$; $c_{m,j} \rightsquigarrow (b_{2m+1-j})^{j+1-m}$, $\{c_{2m+1-j,i}\}_{i=2m+2-j}^{4m+3-2j}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c_{m,j} = a_m j$	
94	010,021,100,101,102,110,120 010,021,100,101,102,110,201 010,021,100,101,102,110,210 010,021,100,101,102,120,201 010,021,100,101,102,120,210 010,021,100,101,102,201,210 010,021,100,101,110,120,201 010,021,100,101,110,120,210 010,021,100,101,110,201,210 010,021,100,101,120,201,210 010,021,100,102,110,120,201 010,021,100,102,110,120,210 010,021,100,102,110,201,210 010,021,100,102,120,201,210 010,021,100,110,120,201,210 010,021,101,102,110,120,201 010,021,101,102,110,120,210 010,021,101,102,110,201,210 010,021,101,102,120,201,210 010,021,101,110,120,201,210 010,021,102,110,120,201,210	$a_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}$, $a_m = 0^m$	$\frac{1-2x-\sqrt{1-4x}}{2x}$
95	010,100,101,102,110,120,201●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, a_{m+2-j}, a_{m+1-j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m(m-1)$	
	010,100,101,102,110,120,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow \{c_i\}_{i=2}^{j}, a_{m+2-j}, a_{m+1-j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = a_m m1$	$\frac{2x}{\frac{1-3x+x^2}{1-x-x^2}+\sqrt{1-4x}}$
96	010,100,101,102,120,201,210●	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, b_{m+1,j}, a_{m+1-j}$, $\{b_{m+1-j,i}\}_{i=2}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	
	010,100,102,110,120,201,210●		

Continuation of Table 3

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,101,102,110,120,201,210•	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^m$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $0021 \rightsquigarrow 0021$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{2-8x+7x^2}{2x(1-x)} - \frac{(2-4x+x^2)\sqrt{1-4x}}{2x(1-x)}$
97	010,100,101,102,110,201,210•	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^m$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, a_{m+2-j}, \{b_{m,i}\}_{i=j}^m$; $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{1-5x+7x^2-2x^3}{2x^2} - \frac{(1-3x+3x^2)\sqrt{1-4x}}{2x^2}$
98	010,100,101,110,120,201,210•	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^m$; $b_{m,j} \rightsquigarrow (c_{m+1-j})^{j-1}, a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$; $c_m \rightsquigarrow a_m, \{b_{m,i}\}_{i=2}^m$, $a_m = 0^m$, $b_{m,j} = a_m j$, $c_m = 0^m 21$	
99	012,021,100,101,102,110,120 012,021,100,101,102,110,201 012,021,100,101,102,110,210 012,021,100,101,110,120,201 012,021,100,101,110,120,210 012,021,100,101,110,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$	$\frac{x(1-x(1-x)^2)}{(1-x)^3}$
100	000,021,100,101,102,110,120 000,021,101,102,110,120,201 000,021,101,102,110,120,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, 002$; $002 \rightsquigarrow a_0, 002$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$, $a_m = 0^2 \cdots (m-2)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3$
101	021,100,101,102,110,120,201 021,100,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x}{(1-x)^2} - \frac{2x^2}{1-x}$
102	021,100,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, b_{m+1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 12$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x^3}{(1-x)^3}$
	021,100,102,110,120,201,210 021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	
103	021,100,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^m, 010, c_m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 0113$; $0113 \rightsquigarrow c_1, 0113$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x^3}{(1-x)(1-2x)}$
104	021,100,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=1}^m, 010, c_{m-1}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 010$; $010 \rightsquigarrow c_1, 010$, $a_m = 0^m$, $b_m = a_m 1$, $c_m = a_m 11$	$\frac{(1+x)(1-\sqrt{1-4x})}{2x} - 1 - \frac{x}{1-x}$
105	100,101,102,110,120,201,210•	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (010)^j, a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$	$\frac{1-3x+x^2-(1-x)\sqrt{1-4x}}{x}$

End of Table 3

Theorem 16. *Let* $B \in \{\{010, 011, 100, 101, 102, 201, 210\}, \{010, 011, 100, 102, 110, 201, 210\}, \{010, 011, 101, 102, 110, 201, 210\}\}$. *Then*

$$F_B(x) = \frac{x}{1-x} + \frac{x^2(1-x)}{(1-2x)^2}.$$

Proof. From the fact that the generating trees are isomorphic, it remains to find the generating function $F_B(x)$. According to Class 61 in Table 3, define $A_m(x)$ (respectively, $B_{m,j}(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B; a_m)$ (respectively, $\mathcal{T}(B; b_{m,j})$), where its root stays at level 0. Thus,

$$A_m(x) = x + xA_{m+1} + x\sum_{i=1}^{m} B_{m,i}(x),$$
$$B_{m,j}(x) = x + (j-1)x^2 + x\sum_{i=j}^{m} B_{m,i}(x).$$

Define $A(v) = \sum_{m\geq 1} A_m(x)v^{m-1}$ and $B(v,u) = \sum_{m\geq 1}\sum_{i=1}^{m} B_{m,i}u^{i-1}v^{m-1}$. Thus, the recurrences can be written as

$$A(v) = \frac{x}{1-v} + \frac{x}{v}(A(v) - A(0)) + xB(v,1),$$
$$B(v,u) = \frac{x(uvx - uv + 1)}{(1-v)(1-uv)^2} + \frac{x}{1-u}(B(v,1) - uB(v,u)).$$

By taking $u = \frac{1}{1-x}$, we obtain that $B(v,1) = \frac{x(vx-v+1-x)}{(1-v)(1-x-v)^2}$. Now, by taking $v = x$, we have that $A(0) = \frac{x}{1-x} + \frac{x^2(1-x)}{(1-2x)^2}$, as claimed. □

6. 8-Table

In this section, we show that $w_8 = 61$. Moreover, by our procedure, we present the generating function for many of the 61 I-Wilf-equivalences. Actually, we see that there are only 9 sets among $\binom{13}{8} = 1287$ sets of 8 length-3 patterns that cannot be reduced to smaller sets of length-3 patterns. In Table 4, we present all the I-Wilf-equivalences of sets of 8 length-3 patterns. Again, due to the similarity with the work of previous sections, we only present the cases that the KMY algorithm does not work, that is, we present bijections between some classes of inversion sequences respect to left-right-maxima structure with using the bijections presented in the previous sections; see the theorems at end of this section.

Table 4: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 8$.

Beginning of Table 4			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100,101 000,001,010,011,012,021,100,102 000,001,010,011,012,021,100,110 000,001,010,011,012,021,100,120 000,001,010,011,012,021,100,201 000,001,010,011,012,021,100,210 000,001,010,011,012,021,101,102 000,001,010,011,012,021,101,110 000,001,010,011,012,021,101,120 000,001,010,011,012,021,101,201 000,001,010,011,012,021,101,210 000,001,010,011,012,021,102,110 000,001,010,011,012,021,102,120 000,001,010,011,012,021,102,201 000,001,010,011,012,021,102,210 000,001,010,011,012,021,110,120 000,001,010,011,012,021,110,201 000,001,010,011,012,021,110,210 000,001,010,011,012,021,120,201		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,011,012,021,120,210		
	000,001,010,011,012,021,201,210		
	000,001,010,011,012,100,101,102		
	000,001,010,011,012,100,101,110		
	000,001,010,011,012,100,101,120		
	000,001,010,011,012,100,101,201		
	000,001,010,011,012,100,101,210		
	000,001,010,011,012,100,102,110		
	000,001,010,011,012,100,102,120		
	000,001,010,011,012,100,102,201		
	000,001,010,011,012,100,102,210		
	000,001,010,011,012,100,110,120		
	000,001,010,011,012,100,110,201		
	000,001,010,011,012,100,110,210		
	000,001,010,011,012,100,120,201		
	000,001,010,011,012,100,120,210		
	000,001,010,011,012,100,201,210		
	000,001,010,011,012,101,102,110		
	000,001,010,011,012,101,102,120		
	000,001,010,011,012,101,102,201		
	000,001,010,011,012,101,102,210		
	000,001,010,011,012,101,110,120		
	000,001,010,011,012,101,110,201		
	000,001,010,011,012,101,110,210		
	000,001,010,011,012,101,120,201		
	000,001,010,011,012,101,120,210		
	000,001,010,011,012,101,201,210		
	000,001,010,011,012,102,110,120		
	000,001,010,011,012,102,110,201		
	000,001,010,011,012,102,110,210		
	000,001,010,011,012,102,120,201		
	000,001,010,011,012,102,120,210		
	000,001,010,011,012,102,201,210		
	000,001,010,011,012,110,120,201		
	000,001,010,011,012,110,120,210		
	000,001,010,011,012,110,201,210		
	000,001,010,011,012,120,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101,102		
	000,001,010,012,021,100,101,110		
	000,001,010,012,021,100,101,120		
	000,001,010,012,021,100,101,201		
	000,001,010,012,021,100,101,210		
	000,001,010,012,021,100,102,110		
	000,001,010,012,021,100,102,120		
	000,001,010,012,021,100,102,201		
	000,001,010,012,021,100,102,210		
	000,001,010,012,021,100,110,120		
	000,001,010,012,021,100,110,201		
	000,001,010,012,021,100,110,210		
	000,001,010,012,021,100,120,201		
	000,001,010,012,021,100,120,210		
	000,001,010,012,021,100,201,210		
	000,001,010,012,021,101,102,110		
	000,001,010,012,021,101,102,120		
	000,001,010,012,021,101,102,201		
	000,001,010,012,021,101,102,210		
	000,001,010,012,021,101,110,120		
	000,001,010,012,021,101,110,201		
	000,001,010,012,021,101,110,210		
	000,001,010,012,021,101,120,201		
	000,001,010,012,021,101,120,210		
	000,001,010,012,021,101,201,210		
	000,001,010,012,021,102,110,120		
	000,001,010,012,021,102,110,201		
	000,001,010,012,021,102,110,210		
	000,001,010,012,021,102,120,201		
	000,001,010,012,021,102,120,210		
	000,001,010,012,021,102,201,210		
	000,001,010,012,021,110,120,201		
	000,001,010,012,021,110,120,210		
	000,001,010,012,021,110,201,210		
	000,001,010,012,021,120,201,210		
	000,001,010,012,100,101,102,110		
	000,001,010,012,100,101,102,120		
	000,001,010,012,100,101,102,201		
	000,001,010,012,100,101,102,210		
	000,001,010,012,100,101,110,120		
	000,001,010,012,100,101,110,201		
	000,001,010,012,100,101,110,210		
	000,001,010,012,100,101,120,201		
	000,001,010,012,100,101,120,210		
	000,001,010,012,100,101,201,210		
	000,001,010,012,100,102,110,120		
	000,001,010,012,100,102,110,201		
	000,001,010,012,100,102,110,210		
	000,001,010,012,100,102,120,201		
	000,001,010,012,100,102,120,210		
	000,001,010,012,100,102,201,210		
	000,001,010,012,100,110,120,201		
	000,001,010,012,100,110,120,210		
	000,001,010,012,100,110,201,210		
	000,001,010,012,100,120,201,210		
	000,001,010,012,101,102,110,120		
	000,001,010,012,101,102,110,201		
	000,001,010,012,101,102,110,210		
	000,001,010,012,101,102,120,201		
	000,001,010,012,101,102,120,210		
	000,001,010,012,101,102,201,210		
	000,001,010,012,101,110,120,201		
	000,001,010,012,101,110,120,210		
	000,001,010,012,101,110,201,210		
	000,001,010,012,101,120,201,210		
	000,001,010,012,102,110,120,201		
	000,001,010,012,102,110,120,210		
	000,001,010,012,102,110,201,210		
	000,001,010,012,102,120,201,210		
	000,001,010,012,110,120,201,210		
	000,001,011,012,021,100,101,102		
	000,001,011,012,021,100,101,110		
	000,001,011,012,021,100,101,120		
	000,001,011,012,021,100,101,201		
	000,001,011,012,021,100,101,210		
	000,001,011,012,021,100,102,110		
	000,001,011,012,021,100,102,120		
	000,001,011,012,021,100,102,201		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,011,012,021,100,102,210		
	000,001,011,012,021,100,110,120		
	000,001,011,012,021,100,110,201		
	000,001,011,012,021,100,110,210		
	000,001,011,012,021,100,120,201		
	000,001,011,012,021,100,120,210		
	000,001,011,012,021,100,201,210		
	000,001,011,012,021,101,102,110		
	000,001,011,012,021,101,102,120		
	000,001,011,012,021,101,102,201		
	000,001,011,012,021,101,102,210		
	000,001,011,012,021,101,110,120		
	000,001,011,012,021,101,110,201		
	000,001,011,012,021,101,110,210		
	000,001,011,012,021,101,120,201		
	000,001,011,012,021,101,120,210		
	000,001,011,012,021,101,201,210		
	000,001,011,012,021,102,110,120		
	000,001,011,012,021,102,110,201		
	000,001,011,012,021,102,110,210		
	000,001,011,012,021,102,120,201		
	000,001,011,012,021,102,120,210		
	000,001,011,012,021,102,201,210		
	000,001,011,012,021,110,120,201		
	000,001,011,012,021,110,120,210		
	000,001,011,012,021,110,201,210		
	000,001,011,012,021,120,201,210		
	000,001,011,012,100,101,102,110		
	000,001,011,012,100,101,102,120		
	000,001,011,012,100,101,102,201		
	000,001,011,012,100,101,102,210		
	000,001,011,012,100,101,110,120		
	000,001,011,012,100,101,110,201		
	000,001,011,012,100,101,110,210		
	000,001,011,012,100,101,120,201		
	000,001,011,012,100,101,120,210		
	000,001,011,012,100,101,201,210		
	000,001,011,012,100,102,110,120		
	000,001,011,012,100,102,110,201		
	000,001,011,012,100,102,110,210		
	000,001,011,012,100,102,120,201		
	000,001,011,012,100,102,120,210		
	000,001,011,012,100,102,201,210		
	000,001,011,012,100,110,120,201		
	000,001,011,012,100,110,120,210		
	000,001,011,012,100,110,201,210		
	000,001,011,012,100,120,201,210		
	000,001,011,012,101,102,110,120		
	000,001,011,012,101,102,110,201		
	000,001,011,012,101,102,110,210		
	000,001,011,012,101,102,120,201		
	000,001,011,012,101,102,120,210		
	000,001,011,012,101,102,201,210		
	000,001,011,012,101,110,120,201		
	000,001,011,012,101,110,120,210		
	000,001,011,012,101,110,201,210		
	000,001,011,012,101,120,201,210		
	000,001,011,012,102,110,120,201		
	000,001,011,012,102,110,120,210		
	000,001,011,012,102,110,201,210		
	000,001,011,012,102,120,201,210		
	000,001,011,012,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00$	$x(1+x)^2$
3	000,001,010,011,021,100,101,102		
	000,001,010,011,021,100,101,110		
	000,001,010,011,021,100,101,120		
	000,001,010,011,021,100,101,201		
	000,001,010,011,021,100,101,210		
	000,001,010,011,021,100,102,110		
	000,001,010,011,021,100,102,120		
	000,001,010,011,021,100,102,201		
	000,001,010,011,021,100,102,210		
	000,001,010,011,021,100,110,120		
	000,001,010,011,021,100,110,201		
	000,001,010,011,021,100,110,210		
	000,001,010,011,021,100,120,201		
	000,001,010,011,021,100,120,210		
	000,001,010,011,021,100,201,210		
	000,001,010,011,021,101,102,110		
	000,001,010,011,021,101,102,120		
	000,001,010,011,021,101,102,201		
	000,001,010,011,021,101,102,210		
	000,001,010,011,021,101,110,120		
	000,001,010,011,021,101,110,201		
	000,001,010,011,021,101,110,210		
	000,001,010,011,021,101,120,201		
	000,001,010,011,021,101,120,210		
	000,001,010,011,021,101,201,210		
	000,001,010,011,021,102,110,120		
	000,001,010,011,021,102,110,201		
	000,001,010,011,021,102,110,210		
	000,001,010,011,021,102,120,201		
	000,001,010,011,021,102,120,210		
	000,001,010,011,021,102,201,210		
	000,001,010,011,021,110,120,201		
	000,001,010,011,021,110,120,210		
	000,001,010,011,021,110,201,210		
	000,001,010,011,021,120,201,210		
	000,001,010,011,100,101,102,110		
	000,001,010,011,100,101,102,120		
	000,001,010,011,100,101,102,201		
	000,001,010,011,100,101,102,210		
	000,001,010,011,100,101,110,120		
	000,001,010,011,100,101,110,201		
	000,001,010,011,100,101,110,210		
	000,001,010,011,100,101,120,201		
	000,001,010,011,100,101,120,210		
	000,001,010,011,100,101,201,210		
	000,001,010,011,100,102,110,120		
	000,001,010,011,100,102,110,201		
	000,001,010,011,100,102,110,210		
	000,001,010,011,100,102,120,201		
	000,001,010,011,100,102,120,210		
	000,001,010,011,100,102,201,210		
	000,001,010,011,100,110,120,201		
	000,001,010,011,100,110,120,210		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,011,100,110,201,210		
	000,001,010,011,100,120,201,210		
	000,001,010,011,101,102,110,120		
	000,001,010,011,101,102,110,201		
	000,001,010,011,101,102,110,210		
	000,001,010,011,101,102,120,201		
	000,001,010,011,101,102,120,210		
	000,001,010,011,101,102,201,210		
	000,001,010,011,101,110,120,201		
	000,001,010,011,101,110,120,210		
	000,001,010,011,101,110,201,210		
	000,001,010,011,101,120,201,210		
	000,001,010,011,102,110,120,201		
	000,001,010,011,102,110,120,210		
	000,001,010,011,102,110,201,210		
	000,001,010,011,102,120,201,210		
	000,001,010,011,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101,102		
	001,010,011,012,021,100,101,110		
	001,010,011,012,021,100,101,120		
	001,010,011,012,021,100,101,201		
	001,010,011,012,021,100,101,210		
	001,010,011,012,021,100,102,110		
	001,010,011,012,021,100,102,120		
	001,010,011,012,021,100,102,201		
	001,010,011,012,021,100,102,210		
	001,010,011,012,021,100,110,120		
	001,010,011,012,021,100,110,201		
	001,010,011,012,021,100,110,210		
	001,010,011,012,021,100,120,201		
	001,010,011,012,021,100,120,210		
	001,010,011,012,021,100,201,210		
	001,010,011,012,021,101,102,110		
	001,010,011,012,021,101,102,120		
	001,010,011,012,021,101,102,201		
	001,010,011,012,021,101,102,210		
	001,010,011,012,021,101,110,120		
	001,010,011,012,021,101,110,201		
	001,010,011,012,021,101,110,210		
	001,010,011,012,021,101,120,201		
	001,010,011,012,021,101,120,210		
	001,010,011,012,021,101,201,210		
	001,010,011,012,021,102,110,120		
	001,010,011,012,021,102,110,201		
	001,010,011,012,021,102,110,210		
	001,010,011,012,021,102,120,201		
	001,010,011,012,021,102,120,210		
	001,010,011,012,021,102,201,210		
	001,010,011,012,021,110,120,201		
	001,010,011,012,021,110,120,210		
	001,010,011,012,021,110,201,210		
	001,010,011,012,021,120,201,210		
	001,010,011,012,100,101,102,110		
	001,010,011,012,100,101,102,120		
	001,010,011,012,100,101,102,201		
	001,010,011,012,100,101,102,210		
	001,010,011,012,100,101,110,120		
	001,010,011,012,100,101,110,201		
	001,010,011,012,100,101,110,210		
	001,010,011,012,100,101,120,201		
	001,010,011,012,100,101,120,210		
	001,010,011,012,100,101,201,210		
	001,010,011,012,100,102,110,120		
	001,010,011,012,100,102,110,201		
	001,010,011,012,100,102,110,210		
	001,010,011,012,100,102,120,201		
	001,010,011,012,100,102,120,210		
	001,010,011,012,100,102,201,210		
	001,010,011,012,100,110,120,201		
	001,010,011,012,100,110,120,210		
	001,010,011,012,100,110,201,210		
	001,010,011,012,100,120,201,210		
	001,010,011,012,101,102,110,120		
	001,010,011,012,101,102,110,201		
	001,010,011,012,101,102,110,210		
	001,010,011,012,101,102,120,201		
	001,010,011,012,101,102,120,210		
	001,010,011,012,101,102,201,210		
	001,010,011,012,101,110,120,201		
	001,010,011,012,101,110,120,210		
	001,010,011,012,101,110,201,210		
	001,010,011,012,101,120,201,210		
	001,010,011,012,102,110,120,201		
	001,010,011,012,102,110,120,210		
	001,010,011,012,102,110,201,210		
	001,010,011,012,102,120,201,210		
	001,010,011,012,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$x^2 + \frac{x}{1-x}$
4	000,001,012,021,100,101,102,110		
	000,001,012,021,100,101,110,120		
	000,001,012,021,100,101,110,201		
	000,001,012,021,100,101,110,210		
	000,001,012,021,100,102,110,120		
	000,001,012,021,100,102,110,201		
	000,001,012,021,100,102,110,210		
	000,001,012,021,100,110,120,201		
	000,001,012,021,100,110,120,210		
	000,001,012,021,100,110,201,210		
	000,001,012,021,101,102,110,120		
	000,001,012,021,101,102,110,201		
	000,001,012,021,101,102,110,210		
	000,001,012,021,101,110,120,201		
	000,001,012,021,101,110,120,210		
	000,001,012,021,101,110,201,210		
	000,001,012,021,102,110,120,201		
	000,001,012,021,102,110,120,210		
	000,001,012,021,102,110,201,210		
	000,001,012,021,110,120,201,210		
	000,001,012,100,101,102,110,120		
	000,001,012,100,101,102,110,201		
	000,001,012,100,101,102,110,210		
	000,001,012,100,101,110,120,201		
	000,001,012,100,101,110,120,210		
	000,001,012,100,101,110,201,210		
	000,001,012,100,102,110,120,201		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,012,100,102,110,120,210		
	000,001,012,100,102,110,201,210		
	000,001,012,100,110,120,201,210		
	000,001,012,101,102,110,120,201		
	000,001,012,101,102,110,120,210		
	000,001,012,101,102,110,201,210		
	000,001,012,101,110,120,201,210		
	000,001,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000,010,011,012,021,100,101,102		
	000,010,011,012,021,100,101,110		
	000,010,011,012,021,100,101,120		
	000,010,011,012,021,100,101,201		
	000,010,011,012,021,100,101,210		
	000,010,011,012,021,100,102,110		
	000,010,011,012,021,100,102,120		
	000,010,011,012,021,100,102,201		
	000,010,011,012,021,100,102,210		
	000,010,011,012,021,100,110,120		
	000,010,011,012,021,100,110,201		
	000,010,011,012,021,100,110,210		
	000,010,011,012,021,100,120,201		
	000,010,011,012,021,100,120,210		
	000,010,011,012,021,100,201,210		
	000,010,011,012,021,101,102,110		
	000,010,011,012,021,101,102,120		
	000,010,011,012,021,101,102,201		
	000,010,011,012,021,101,102,210		
	000,010,011,012,021,101,110,120		
	000,010,011,012,021,101,110,201		
	000,010,011,012,021,101,110,210		
	000,010,011,012,021,101,120,201		
	000,010,011,012,021,101,120,210		
	000,010,011,012,021,101,201,210		
	000,010,011,012,021,102,110,120		
	000,010,011,012,021,102,110,201		
	000,010,011,012,021,102,110,210		
	000,010,011,012,021,102,120,201		
	000,010,011,012,021,102,120,210		
	000,010,011,012,021,102,201,210		
	000,010,011,012,021,110,120,201		
	000,010,011,012,021,110,120,210		
	000,010,011,012,021,110,201,210		
	000,010,011,012,021,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
5	000,001,012,021,100,101,102,120		
	000,001,012,021,100,101,102,201		
	000,001,012,021,100,101,102,210		
	000,001,012,021,100,101,120,201		
	000,001,012,021,100,101,120,210		
	000,001,012,021,100,101,201,210		
	000,001,012,021,100,102,120,201		
	000,001,012,021,100,102,120,210		
	000,001,012,021,100,102,201,210		
	000,001,012,021,100,120,201,210		
	000,001,012,021,101,102,120,201		
	000,001,012,021,101,102,120,210		
	000,001,012,021,101,102,201,210		
	000,001,012,021,101,120,201,210		
	000,001,012,021,102,120,201,210		
	000,001,012,100,101,102,120,201		
	000,001,012,100,101,102,120,210		
	000,001,012,100,101,102,201,210		
	000,001,012,100,101,120,201,210		
	000,001,012,100,102,120,201,210		
	000,001,012,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;\ 011 \rightsquigarrow 00$	
	000,010,011,012,100,101,102,110		
	000,010,011,012,100,101,102,120		
	000,010,011,012,100,101,102,201		
	000,010,011,012,100,101,102,210		
	000,010,011,012,100,101,110,120		
	000,010,011,012,100,101,110,201		
	000,010,011,012,100,101,110,210		
	000,010,011,012,100,101,120,201		
	000,010,011,012,100,101,120,210		
	000,010,011,012,100,101,201,210		
	000,010,011,012,100,102,110,120		
	000,010,011,012,100,102,110,201		
	000,010,011,012,100,102,110,210		
	000,010,011,012,100,102,120,201		
	000,010,011,012,100,102,120,210		
	000,010,011,012,100,102,201,210		
	000,010,011,012,100,110,120,201		
	000,010,011,012,100,110,120,210		
	000,010,011,012,100,110,201,210		
	000,010,011,012,100,120,201,210		
	000,010,011,012,101,102,110,120		
	000,010,011,012,101,102,110,201		
	000,010,011,012,101,102,110,210		
	000,010,011,012,101,102,120,201		
	000,010,011,012,101,102,120,210		
	000,010,011,012,101,102,201,210		
	000,010,011,012,101,110,120,201		
	000,010,011,012,101,110,120,210		
	000,010,011,012,101,110,201,210		
	000,010,011,012,101,120,201,210		
	000,010,011,012,102,110,120,201		
	000,010,011,012,102,110,120,210		
	000,010,011,012,102,110,201,210		
	000,010,011,012,102,120,201,210		
	000,010,011,012,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;\ 002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
6	000,001,011,021,100,101,102,120		
	000,001,011,021,100,101,110,120		
	000,001,011,021,100,101,120,201		
	000,001,011,021,100,101,120,210		
	000,001,011,021,100,102,110,120		
	000,001,011,021,100,102,120,201		
	000,001,011,021,100,102,120,210		
	000,001,011,021,100,110,120,201		
	000,001,011,021,100,110,120,210		
	000,001,011,021,100,120,201,210		
	000,001,011,021,101,102,110,120		
	000,001,011,021,101,102,120,201		
	000,001,011,021,101,102,120,210		
	000,001,011,021,101,110,120,201		
	000,001,011,021,101,110,120,210		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,011,021,101,120,201,210		
	000,001,011,021,102,110,120,201		
	000,001,011,021,102,110,120,210		
	000,001,011,021,102,120,201,210		
	000,001,011,021,110,120,201,210		
	000,001,011,100,101,102,110,120		
	000,001,011,100,101,102,120,201		
	000,001,011,100,101,102,120,210		
	000,001,011,100,101,110,120,201		
	000,001,011,100,101,110,120,210		
	000,001,011,100,101,120,201,210		
	000,001,011,100,102,110,120,201		
	000,001,011,100,102,110,120,210		
	000,001,011,100,102,120,201,210		
	000,001,011,100,110,120,201,210		
	000,001,011,101,102,110,120,201		
	000,001,011,101,102,110,120,210		
	000,001,011,101,102,120,201,210		
	000,001,011,101,110,120,201,210		
	000,001,011,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 012;\ 012 \rightsquigarrow 012$	
	001,011,012,021,100,101,102,110		
	001,011,012,021,100,101,102,120		
	001,011,012,021,100,101,102,201		
	001,011,012,021,100,101,102,210		
	001,011,012,021,100,101,110,120		
	001,011,012,021,100,101,110,201		
	001,011,012,021,100,101,110,210		
	001,011,012,021,100,101,120,201		
	001,011,012,021,100,101,120,210		
	001,011,012,021,100,101,201,210		
	001,011,012,021,100,102,110,120		
	001,011,012,021,100,102,110,201		
	001,011,012,021,100,102,110,210		
	001,011,012,021,100,102,120,201		
	001,011,012,021,100,102,120,210		
	001,011,012,021,100,102,201,210		
	001,011,012,021,100,110,120,201		
	001,011,012,021,100,110,120,210		
	001,011,012,021,100,110,201,210		
	001,011,012,021,100,120,201,210		
	001,011,012,100,101,102,110,120		
	001,011,012,100,101,102,110,201		
	001,011,012,100,101,102,110,210		
	001,011,012,100,101,102,120,201		
	001,011,012,100,101,102,120,210		
	001,011,012,100,101,102,201,210		
	001,011,012,100,101,110,120,201		
	001,011,012,100,101,110,120,210		
	001,011,012,100,101,110,201,210		
	001,011,012,100,101,120,201,210		
	001,011,012,100,102,110,120,201		
	001,011,012,100,102,110,120,210		
	001,011,012,100,102,110,201,210		
	001,011,012,100,102,120,201,210		
	001,011,012,100,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010$	$x^2 + x^3 + \frac{x}{1-x}$
7	000,001,010,021,100,101,102,110		
	000,001,010,021,100,101,102,120		
	000,001,010,021,100,101,102,201		
	000,001,010,021,100,101,102,210		
	000,001,010,021,100,101,110,120		
	000,001,010,021,100,101,110,201		
	000,001,010,021,100,101,110,210		
	000,001,010,021,100,101,120,201		
	000,001,010,021,100,101,120,210		
	000,001,010,021,100,101,201,210		
	000,001,010,021,100,102,110,120		
	000,001,010,021,100,102,110,201		
	000,001,010,021,100,102,110,210		
	000,001,010,021,100,102,120,201		
	000,001,010,021,100,102,120,210		
	000,001,010,021,100,102,201,210		
	000,001,010,021,100,110,120,201		
	000,001,010,021,100,110,120,210		
	000,001,010,021,100,110,201,210		
	000,001,010,021,100,120,201,210		
	000,001,010,021,101,102,110,120		
	000,001,010,021,101,102,110,201		
	000,001,010,021,101,102,110,210		
	000,001,010,021,101,102,120,201		
	000,001,010,021,101,102,120,210		
	000,001,010,021,101,102,201,210		
	000,001,010,021,101,110,120,201		
	000,001,010,021,101,110,120,210		
	000,001,010,021,101,110,201,210		
	000,001,010,021,101,120,201,210		
	000,001,010,021,102,110,120,201		
	000,001,010,021,102,110,120,210		
	000,001,010,021,102,110,201,210		
	000,001,010,021,102,120,201,210		
	000,001,010,021,110,120,201,210		
	000,001,010,100,101,102,110,120		
	000,001,010,100,101,102,110,201		
	000,001,010,100,101,102,110,210		
	000,001,010,100,101,102,120,201		
	000,001,010,100,101,102,120,210		
	000,001,010,100,101,102,201,210		
	000,001,010,100,101,110,120,201		
	000,001,010,100,101,110,120,210		
	000,001,010,100,101,110,201,210		
	000,001,010,100,101,120,201,210		
	000,001,010,100,102,110,120,201		
	000,001,010,100,102,110,120,210		
	000,001,010,100,102,110,201,210		
	000,001,010,100,102,120,201,210		
	000,001,010,100,110,120,201,210		
	000,001,010,101,102,110,120,201		
	000,001,010,101,102,110,120,210		
	000,001,010,101,102,110,201,210		
	000,001,010,101,102,120,201,210		
	000,001,010,101,110,120,201,210		
	000,001,010,102,110,120,201,210		
	000,001,011,021,100,101,102,110		
	000,001,011,021,100,101,102,201		
	000,001,011,021,100,101,102,210		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$	
	000,001,011,021,100,101,110,201			
	000,001,011,021,100,101,110,210			
	000,001,011,021,100,101,201,210			
	000,001,011,021,100,102,110,201			
	000,001,011,021,100,102,110,210			
	000,001,011,021,100,102,201,210			
	000,001,011,021,100,110,201,210			
	000,001,011,021,101,102,110,201			
	000,001,011,021,101,102,110,210			
	000,001,011,021,101,102,201,210			
	000,001,011,021,101,110,201,210			
	000,001,011,021,102,110,201,210			
	000,001,011,100,101,102,110,201			
	000,001,011,100,101,102,110,210			
	000,001,011,100,101,102,201,210			
	000,001,011,100,101,110,201,210			
	000,001,011,100,102,110,201,210			
	000,001,011,101,102,110,201,210	$0 \rightsquigarrow 00, 0$		
	001,010,011,021,100,101,102,110			
	001,010,011,021,100,101,102,120			
	001,010,011,021,100,101,102,201			
	001,010,011,021,100,101,102,210			
	001,010,011,021,100,101,110,120			
	001,010,011,021,100,101,110,201			
	001,010,011,021,100,101,110,210			
	001,010,011,021,100,101,120,201			
	001,010,011,021,100,101,120,210			
	001,010,011,021,100,101,201,210			
	001,010,011,021,100,102,110,120			
	001,010,011,021,100,102,110,201			
	001,010,011,021,100,102,110,210			
	001,010,011,021,100,102,120,201			
	001,010,011,021,100,102,120,210			
	001,010,011,021,100,102,201,210			
	001,010,011,021,100,110,120,201			
	001,010,011,021,100,110,120,210			
	001,010,011,021,100,110,201,210			
	001,010,011,021,100,120,201,210			
	001,010,011,021,101,102,110,120			
	001,010,011,021,101,102,110,201			
	001,010,011,021,101,102,110,210			
	001,010,011,021,101,102,120,201			
	001,010,011,021,101,102,120,210			
	001,010,011,021,101,102,201,210			
	001,010,011,021,101,110,120,201			
	001,010,011,021,101,110,120,210			
	001,010,011,021,101,110,201,210			
	001,010,011,021,101,120,201,210			
	001,010,011,021,102,110,120,201			
	001,010,011,021,102,110,120,210			
	001,010,011,021,102,110,201,210			
	001,010,011,021,102,120,201,210			
	001,010,011,021,110,120,201,210			
	001,010,011,100,101,102,110,120			
	001,010,011,100,101,102,110,201			
	001,010,011,100,101,102,110,210			
	001,010,011,100,101,102,120,201			
	001,010,011,100,101,102,120,210			
	001,010,011,100,101,102,201,210			
	001,010,011,100,101,110,120,201			
	001,010,011,100,101,110,120,210			
	001,010,011,100,101,110,201,210			
	001,010,011,100,101,120,201,210			
	001,010,011,100,102,110,120,201			
	001,010,011,100,102,110,120,210			
	001,010,011,100,102,110,201,210			
	001,010,011,100,102,120,201,210			
	001,010,011,100,110,120,201,210			
	001,010,011,101,102,110,120,201			
	001,010,011,101,102,110,120,210			
	001,010,011,101,102,110,201,210			
	001,010,011,101,102,120,201,210			
	001,010,011,101,110,120,201,210			
	001,010,011,102,110,120,201,210			
	001,010,012,021,100,101,102,110			
	001,010,012,021,100,101,102,120			
	001,010,012,021,100,101,102,201			
	001,010,012,021,100,101,102,210			
	001,010,012,021,100,101,110,120			
	001,010,012,021,100,101,110,201			
	001,010,012,021,100,101,110,210			
	001,010,012,021,100,101,120,201			
	001,010,012,021,100,101,120,210			
	001,010,012,021,100,101,201,210			
	001,010,012,021,100,102,110,120			
	001,010,012,021,100,102,110,201			
	001,010,012,021,100,102,110,210			
	001,010,012,021,100,102,120,201			
	001,010,012,021,100,102,120,210			
	001,010,012,021,100,102,201,210			
	001,010,012,021,100,110,120,201			
	001,010,012,021,100,110,120,210			
	001,010,012,021,100,110,201,210			
	001,010,012,021,100,120,201,210			
	001,010,012,021,101,102,110,120			
	001,010,012,021,101,102,110,201			
	001,010,012,021,101,102,110,210			
	001,010,012,021,101,102,120,201			
	001,010,012,021,101,102,120,210			
	001,010,012,021,101,102,201,210			
	001,010,012,021,101,110,120,201			
	001,010,012,021,101,110,120,210			
	001,010,012,021,101,110,201,210			
	001,010,012,021,101,120,201,210			
	001,010,012,021,102,110,120,201			
	001,010,012,021,102,110,120,210			
	001,010,012,021,102,110,201,210			
	001,010,012,021,102,120,201,210			
	001,010,012,021,110,120,201,210			
	001,010,012,100,101,102,110,120			
	001,010,012,100,101,102,110,201			
	001,010,012,100,101,102,110,210			
	001,010,012,100,101,102,120,201			
	001,010,012,100,101,102,120,210			
	001,010,012,100,101,102,201,210			

Continuation of Table 4			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,010,012,100,101,110,120,201 001,010,012,100,101,110,120,210 001,010,012,100,101,110,201,210 001,010,012,100,101,120,201,210 001,010,012,100,102,110,120,201 001,010,012,100,102,110,120,210 001,010,012,100,102,110,201,210 001,010,012,100,102,120,201,210 001,010,012,100,110,120,201,210 001,010,012,101,102,110,120,201 001,010,012,101,102,110,120,210 001,010,012,101,102,110,201,210 001,010,012,101,102,120,201,210 001,010,012,101,110,120,201,210 001,010,012,102,110,120,201,210 001,011,012,021,101,102,110,120 001,011,012,021,101,102,110,201 001,011,012,021,101,102,110,210 001,011,012,021,101,102,120,201 001,011,012,021,101,102,120,210 001,011,012,021,101,102,201,210 001,011,012,021,101,110,120,201 001,011,012,021,101,110,120,210 001,011,012,021,101,110,201,210 001,011,012,021,101,120,201,210 001,011,012,021,102,110,120,201 001,011,012,021,102,110,120,210 001,011,012,021,102,110,201,210 001,011,012,021,102,120,201,210 001,011,012,021,110,120,201,210 001,011,012,101,102,110,120,201 001,011,012,101,102,110,120,210 001,011,012,101,102,110,201,210 001,011,012,101,102,120,201,210 001,011,012,101,110,120,201,210 001,011,012,102,110,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
8	000,011,012,021,100,101,102,110 000,011,012,021,100,101,102,120 000,011,012,021,100,101,102,201 000,011,012,021,100,101,102,210 000,011,012,021,100,101,110,120 000,011,012,021,100,101,110,201 000,011,012,021,100,101,110,210 000,011,012,021,100,101,120,201 000,011,012,021,100,101,120,210 000,011,012,021,100,101,201,210 000,011,012,021,100,102,110,120 000,011,012,021,100,102,110,201 000,011,012,021,100,102,110,210 000,011,012,021,100,102,120,201 000,011,012,021,100,102,120,210 000,011,012,021,100,102,201,210 000,011,012,021,100,110,120,201 000,011,012,021,100,110,120,210 000,011,012,021,100,110,201,210 000,011,012,021,100,120,201,210 000,011,012,021,101,102,110,120 000,011,012,021,101,102,110,201 000,011,012,021,101,102,110,210 000,011,012,021,101,102,120,201 000,011,012,021,101,102,120,210 000,011,012,021,101,102,201,210 000,011,012,021,101,110,120,201 000,011,012,021,101,110,120,210 000,011,012,021,101,110,201,210 000,011,012,021,101,120,201,210 000,011,012,021,102,110,120,201 000,011,012,021,102,110,120,210 000,011,012,021,102,110,201,210 000,011,012,021,102,120,201,210 000,011,012,021,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
9	000,011,012,100,101,102,110,120 000,011,012,100,101,102,110,201 000,011,012,100,101,102,110,210 000,011,012,100,101,102,120,201 000,011,012,100,101,102,120,210 000,011,012,100,101,102,201,210 000,011,012,100,101,110,120,201 000,011,012,100,101,110,120,210 000,011,012,100,101,110,201,210 000,011,012,100,101,120,201,210 000,011,012,100,102,110,120,201 000,011,012,100,102,110,120,210 000,011,012,100,102,110,201,210 000,011,012,100,102,120,201,210 000,011,012,100,110,120,201,210 000,011,012,101,102,110,120,201 000,011,012,101,102,110,120,210 000,011,012,101,102,110,201,210 000,011,012,101,102,120,201,210 000,011,012,101,110,120,201,210 000,011,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
10	000,010,012,021,100,101,102,110 000,010,012,021,100,101,102,120 000,010,012,021,100,101,102,201 000,010,012,021,100,101,102,210 000,010,012,021,100,101,110,120 000,010,012,021,100,101,110,201 000,010,012,021,100,101,110,210 000,010,012,021,100,101,120,201 000,010,012,021,100,101,120,210 000,010,012,021,100,101,201,210 000,010,012,021,100,102,110,120 000,010,012,021,100,102,110,201 000,010,012,021,100,102,110,210 000,010,012,021,100,102,120,201 000,010,012,021,100,102,120,210 000,010,012,021,100,102,201,210 000,010,012,021,100,110,120,201 000,010,012,021,100,110,120,210 000,010,012,021,100,110,201,210 000,010,012,021,100,120,201,210 000,010,012,021,101,102,110,120		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,012,021,101,102,110,201 000,010,012,021,101,102,110,210 000,010,012,021,101,102,120,201 000,010,012,021,101,102,120,210 000,010,012,021,101,102,201,210 000,010,012,021,101,110,120,201 000,010,012,021,101,110,120,210 000,010,012,021,101,110,201,210 000,010,012,021,101,120,201,210 000,010,012,021,102,110,120,201 000,010,012,021,102,110,120,210 000,010,012,021,102,110,201,210 000,010,012,021,102,120,201,210 000,010,012,021,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 01$; $01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
11	000,001,021,100,101,102,110,120 000,001,021,100,101,110,120,201 000,001,021,100,101,110,120,210 000,001,021,100,102,110,120,201 000,001,021,100,102,110,120,210 000,001,021,100,110,120,201,210 000,001,021,101,102,110,120,201 000,001,021,101,102,110,120,210 000,001,021,101,110,120,201,210 000,001,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 00, 012$; $012 \rightsquigarrow 00, 012$	
	001,011,021,100,101,102,110,120 001,011,021,100,101,102,120,201 001,011,021,100,101,102,120,210 001,011,021,100,101,110,120,201 001,011,021,100,101,110,120,210 001,011,021,100,101,120,201,210 001,011,021,100,102,110,120,201 001,011,021,100,102,110,120,210 001,011,021,100,102,120,201,210 001,011,021,100,110,120,201,210 001,011,100,101,102,110,120,201 001,011,100,101,102,110,120,210 001,011,100,101,102,120,201,210 001,011,100,101,110,120,201,210 001,011,100,102,110,120,201,210 001,012,021,100,101,102,110,120 001,012,021,100,101,102,110,201 001,012,021,100,101,102,110,210 001,012,021,100,101,110,120,201 001,012,021,100,101,110,120,210 001,012,021,100,101,110,201,210 001,012,021,100,102,110,120,201 001,012,021,100,102,110,120,210 001,012,021,100,102,110,201,210 001,012,021,100,110,120,201,210 001,012,100,101,102,110,120,201 001,012,100,101,102,110,120,210 001,012,100,101,102,110,201,210 001,012,100,101,110,120,201,210 001,012,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00$	$x + x^3 + \frac{2x^2}{1-x}$
12	000,010,012,100,101,102,110,120 000,010,012,100,101,102,110,201 000,010,012,100,101,102,110,210 000,010,012,100,101,110,120,201 000,010,012,100,101,110,120,210 000,010,012,100,101,110,201,210 000,010,012,100,102,110,120,201 000,010,012,100,102,110,120,210 000,010,012,100,102,110,201,210 000,010,012,100,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 3x^3 + 3x^4$
13	000,010,012,100,101,102,120,201 000,010,012,100,101,102,120,210 000,010,012,100,101,102,201,210 000,010,012,100,101,120,201,210 000,010,012,100,102,120,201,210 000,010,012,101,102,110,120,201 000,010,012,101,102,110,120,210 000,010,012,101,102,110,201,210 000,010,012,101,110,120,201,210 000,010,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 011$	$x + 2x^2 + 3x^3 + 3x^4 + x^5$
14	000,010,012,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 0022$; $0022 \rightsquigarrow 01$	$x+2x^2+3x^3+3x^4+2x^5+x^6$
15	000,001,021,100,101,102,120,201 000,001,021,100,101,102,120,210 000,001,021,100,101,120,201,210 000,001,021,100,102,120,201,210 000,001,021,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 012$; $011 \rightsquigarrow 00$; $012 \rightsquigarrow 00, 012$	$x + x^3 + x^4 + \frac{2x^2}{1-x}$

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
16	000,001,021,100,101,102,110,201 000,001,021,100,101,102,110,210 000,001,021,100,101,110,201,210 000,001,021,100,102,110,201,210 000,001,021,101,102,110,201,210 000,001,100,101,102,110,120,201 000,001,100,101,102,110,120,210 000,001,100,101,110,120,201,210 000,001,100,102,110,120,201,210 000,001,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 00, 01$	
	001,011,021,100,101,102,110,201 001,011,021,100,101,102,110,210 001,011,021,100,101,102,201,210 001,011,021,100,101,110,201,210 001,011,021,100,102,110,201,210 001,011,100,101,102,110,201,210 001,012,021,100,101,102,120,201 001,012,021,100,101,102,120,210 001,012,021,100,101,102,201,210 001,012,021,100,101,120,201,210 001,012,021,100,102,120,201,210 001,012,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010, 01$	
	001,011,021,101,102,110,120,201 001,011,021,101,102,110,120,210 001,011,021,101,102,120,201,210 001,011,021,101,110,120,201,210 001,011,021,102,110,120,201,210 001,011,101,102,110,120,201,210 001,012,021,101,102,110,120,201 001,012,021,101,102,110,120,210 001,012,021,101,102,110,201,210 001,012,021,101,110,120,201,210 001,012,021,102,110,120,201,210 001,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow (00)^2$	
17	000,001,021,100,101,102,201,210 000,001,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011, 01;$ $011 \rightsquigarrow 00$	$x + 2x^2 + 3x^3 + \frac{4x^4}{1-x}$
18	000,001,100,101,102,110,201,210	$a_m \rightsquigarrow (00)^{m+1}, a_{m+1},$ $a_m = 01 \cdots m$	
	000,010,011,021,100,101,102,110 000,010,011,021,100,101,102,120 000,010,011,021,100,101,102,201 000,010,011,021,100,101,102,210 000,010,011,021,100,101,110,120 000,010,011,021,100,101,110,201 000,010,011,021,100,101,110,210 000,010,011,021,100,101,120,201 000,010,011,021,100,101,120,210 000,010,011,021,100,101,201,210 000,010,011,021,100,102,110,120 000,010,011,021,100,102,110,201 000,010,011,021,100,102,110,210 000,010,011,021,100,102,120,201 000,010,011,021,100,102,120,210 000,010,011,021,100,102,201,210 000,010,011,021,100,110,120,201 000,010,011,021,100,110,120,210 000,010,011,021,100,110,201,210 000,010,011,021,100,120,201,210 000,010,011,021,101,102,110,120 000,010,011,021,101,102,110,201 000,010,011,021,101,102,110,210 000,010,011,021,101,102,120,201 000,010,011,021,101,102,120,210 000,010,011,021,101,102,201,210 000,010,011,021,101,110,120,201 000,010,011,021,101,110,120,210 000,010,011,021,101,110,201,210 000,010,011,021,101,120,201,210 000,010,011,021,102,110,120,201 000,010,011,021,102,110,120,210 000,010,011,021,102,110,201,210 000,010,011,021,102,120,201,210 000,010,011,021,110,120,201,210	$0 \rightsquigarrow 0, 01;\ 01 \rightsquigarrow 01$	
	001,010,021,100,101,102,110,120 001,010,021,100,101,102,110,201 001,010,021,100,101,102,110,210 001,010,021,100,101,102,120,201 001,010,021,100,101,102,120,210 001,010,021,100,101,102,201,210 001,010,021,100,101,110,120,201 001,010,021,100,101,110,120,210 001,010,021,100,101,110,201,210 001,010,021,100,101,120,201,210 001,010,021,100,102,110,120,201 001,010,021,100,102,110,120,210 001,010,021,100,102,110,201,210 001,010,021,100,102,120,201,210 001,010,021,100,110,120,201,210 001,010,021,101,102,110,120,201 001,010,021,101,102,110,120,210 001,010,021,101,102,110,201,210 001,010,021,101,102,120,201,210 001,010,021,101,110,120,201,210 001,010,021,102,110,120,201,210 001,010,100,101,102,110,120,201 001,010,100,101,102,110,120,210 001,010,100,101,102,110,201,210 001,010,100,101,102,120,201,210 001,010,100,101,110,120,201,210 001,010,100,102,110,120,201,210 001,010,101,102,110,120,201,210 001,011,021,101,102,110,201,210 001,012,021,101,102,120,201,210	$0 \rightsquigarrow 00, 0;\ 00 \rightsquigarrow 00$	
	010,011,012,021,100,101,102,110 010,011,012,021,100,101,102,120 010,011,012,021,100,101,102,201 010,011,012,021,100,101,102,210 010,011,012,021,100,101,110,120 010,011,012,021,100,101,110,201 010,011,012,021,100,101,110,210 010,011,012,021,100,101,120,201 010,011,012,021,100,101,120,210		

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,011,012,021,100,101,201,210 010,011,012,021,100,102,110,120 010,011,012,021,100,102,110,201 010,011,012,021,100,102,110,210 010,011,012,021,100,102,120,201 010,011,012,021,100,102,120,210 010,011,012,021,100,102,201,210 010,011,012,021,100,110,120,201 010,011,012,021,100,110,120,210 010,011,012,021,100,110,201,210 010,011,012,021,100,120,201,210 010,011,012,021,101,102,110,120 010,011,012,021,101,102,110,201 010,011,012,021,101,102,110,210 010,011,012,021,101,102,120,201 010,011,012,021,101,102,120,210 010,011,012,021,101,102,201,210 010,011,012,021,101,110,120,201 010,011,012,021,101,110,120,210 010,011,012,021,101,110,201,210 010,011,012,021,101,120,201,210 010,011,012,021,102,110,120,201 010,011,012,021,102,110,120,210 010,011,012,021,102,110,201,210 010,011,012,021,102,120,201,210 010,011,012,021,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $a_m = 0^m$	$\frac{x}{(1-x)^2}$
19	000,010,011,100,101,102,110,120 000,010,011,100,101,102,120,201 000,010,011,100,101,102,120,210 000,010,011,100,102,110,120,201 000,010,011,100,102,110,120,210 000,010,011,100,102,120,201,210 000,010,011,101,102,110,120,201 000,010,011,101,102,110,120,210 000,010,011,101,102,120,201,210 000,010,011,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
20	000,010,011,100,101,102,110,201 000,010,011,100,101,102,110,210 000,010,011,100,101,102,201,210 000,010,011,100,102,110,201,210 000,010,011,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 002$	$\frac{x(1+x^3)}{(1-x)^2}$
21	010,011,012,100,101,102,110,210 010,011,012,100,101,102,120,210 010,011,012,100,101,102,201,210 010,011,012,100,101,110,120,210 010,011,012,100,101,110,201,210 010,011,012,100,101,120,201,210 010,011,012,100,102,110,120,210 010,011,012,100,102,110,201,210 010,011,012,100,102,120,201,210 010,011,012,100,110,120,201,210 010,011,012,101,102,110,120,210 010,011,012,101,102,110,201,210 010,011,012,101,102,120,201,210 010,011,012,101,110,120,201,210 010,011,012,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow b_1^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
22	000,010,011,100,101,110,120,201 000,010,011,100,101,110,120,210 000,010,011,100,101,120,201,210 000,010,011,100,110,120,201,210 000,010,011,101,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 0$; $01 \rightsquigarrow 01$	
	010,011,012,100,101,102,110,120 010,011,012,100,101,102,110,201 010,011,012,100,101,102,120,201 010,011,012,100,101,110,120,201 010,011,012,100,102,110,120,201 010,011,012,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1+x)}{1-x-x^2}$
23	000,010,011,100,101,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 00$; $01 \rightsquigarrow 01$	$\frac{x(1-x-x^2)}{(1-x)(1-2x)}$
24	000,010,100,101,102,110,120,201•	$a_{m,j} \rightsquigarrow \{b_i\}_{i=1}^{j-m}, c_{2m-j}$, $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = 0^1 \cdots (m-1)^2 (2m)(2m-1)$, $c_m = 0^2 \cdots (m-1)^2$	
	000,010,100,101,102,110,120,210•	$a_{m,j} \rightsquigarrow \{b_i\}_{i=1}^{j-m}, c_{2m-j}$, $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}$; $c_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2}$, $a_{m,j} = 0^2 \cdots (m-1)^2 j$, $b_m = 0^1 \cdots (m-1)^2 (2m)m$, $c_m = 0^2 \cdots (m-1)^2$	Theorem 17

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
25	000,010,100,101,102,120,201,210•	$a_{m,j} \rightsquigarrow (0021)^{j-m}, b_{m,j},$ $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j};$ $b_{m,j} \rightsquigarrow (0021)^{j-m},$ $\{a_{2m+1-j,i}\}_{i=2m+1-j}^{4m+2-2j},$ $a_{m,j} = 0^2 \cdots (m-1)^2 j,$ $b_{m,j} = 0^2 \cdots (m-1)^2 jj$	
	000,010,100,102,110,120,201,210•	$a_{m,j} \rightsquigarrow (0021)^{j-m}, b_{2m-j},$ $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j};$ $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2};$ $0021 \rightsquigarrow 00212,$ $a_{m,j} = 0^2 \cdots (m-1)^2 j,$ $b_m = 0^2 \cdots m^2$	
	000,010,101,102,110,120,201,210•	$a_{m,j} \rightsquigarrow (0021)^{j-m}, b_{2m-j},$ $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j};$ $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2};$ $0021 \rightsquigarrow 00211,$ $a_{m,j} = 0^2 \cdots (m-1)^2 j,$ $b_m = 0^2 \cdots m^2$	Theorem 18
26	000,010,100,101,102,110,201,210•	$a_{m,j} \rightsquigarrow$ $(0021)^{j-m}, b_{2m-j}, \{a_{m,i}\}_{i=j}^{2m};$ $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2},$ $a_{m,j} = 0^2 \cdots (m-1)^2 j,$ $b_m = 0^2 \cdots m^2$	Theorem 19
27	000,010,100,101,110,120,201,210•	$a_{m,j} \rightsquigarrow (b_{2m-j})^{j+1-m},$ $\{a_{2m-j,i}\}_{i=2m-j}^{4m-2j};$ $b_m \rightsquigarrow \{a_{m+1,i}\}_{i=m+1}^{2m+2},$ $a_{m,j} = 0^2 \cdots (m-1)^2 j,$ $b_m = 0^2 \cdots m^2$	Theorem 20
28	000,012,021,100,101,102,110,120 000,012,021,100,101,102,110,201 000,012,021,100,101,102,110,210 000,012,021,100,101,110,120,201 000,012,021,100,101,110,120,210 000,012,021,100,101,110,201,210 000,012,021,101,102,110,120,201 000,012,021,101,102,110,120,210 000,012,021,101,102,110,201,210 000,012,021,101,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 001;$ $01 \rightsquigarrow 010, 010;\ 001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^4$
29	000,012,021,100,101,102,120,201 000,012,021,100,101,102,120,210 000,012,021,100,101,102,201,210 000,012,021,100,101,120,201,210 000,012,021,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (001)^2;$ $01 \rightsquigarrow 010, 001;\ 001 \rightsquigarrow 010$	
	000,012,021,100,102,110,120,201 000,012,021,100,102,110,120,210 000,012,021,100,102,110,201,210 000,012,021,100,110,120,201,210 000,012,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 001;$ $01 \rightsquigarrow 001, 011;\ 001 \rightsquigarrow 011$	
	000,012,100,101,102,110,120,201 000,012,100,101,102,110,120,210 000,012,100,101,102,110,201,210 000,012,100,101,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 01;$ $01 \rightsquigarrow 010, 010;\ 001 \rightsquigarrow 010$	$x + 2x62 + 4x^3 + 3x^4$

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
30	000,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 010, 010$; $001 \rightsquigarrow 010$; $002 \rightsquigarrow 001, 010$	$x + 2x^2 + 4x^3 + 3x^4 + x^5$
31	000,012,021,100,102,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow (001)^2$; $001 \rightsquigarrow 0011$	
	000,012,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 002$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 011$; $002 \rightsquigarrow 011, 011$	$x + 2x^2 + 4x^3 + 4x^4$
32	000,012,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 4x^4 + x^5$
33	000,011,021,100,101,102,110,120 000,011,021,100,101,102,120,201 000,011,021,100,101,102,120,210 000,011,021,100,102,110,120,201 000,011,021,100,102,110,120,210 000,011,021,100,102,120,201,210 000,011,021,101,102,110,120,201 000,011,021,101,102,110,120,210 000,011,021,101,102,120,201,210 000,011,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	
	001,021,100,101,102,110,120,201 001,021,100,101,102,110,120,210 001,021,100,101,110,120,201,210 001,021,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 012$; $012 \rightsquigarrow 00, 012$	$\frac{x(1+x^2-2x^3+x^4)}{(1-x)^2}$
34	000,011,021,100,101,102,110,201 000,011,021,100,101,102,110,210 000,011,021,100,101,102,201,210 000,011,021,100,102,110,201,210 000,011,021,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 01$; $002 \rightsquigarrow 002$	
	000,011,100,101,102,110,120,201 000,011,100,101,102,110,120,210 000,011,100,101,102,120,201,210 000,011,100,102,110,120,201,210 000,011,101,102,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 012$; $012 \rightsquigarrow 012$	
	001,021,100,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 01$	
	001,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 011, 012$; $011 \rightsquigarrow 010, 011$; $012 \rightsquigarrow 00, 012$	
	001,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	
	001,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 01$	$\frac{x(1+x^2-x^3)}{(1-x)^2}$
35	000,011,021,100,101,110,120,201 000,011,021,100,101,110,120,210 000,011,021,100,101,120,201,210 000,011,021,100,110,120,201,210 000,011,021,101,110,120,201,210	$0 \rightsquigarrow 00, 00$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	000,011,100,101,102,110,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 01$	
	011,012,021,100,101,102,110,120 011,012,021,100,101,102,110,201 011,012,021,100,101,102,110,210 011,012,021,100,101,102,120,201 011,012,021,100,101,102,120,210 011,012,021,100,101,102,201,210 011,012,021,100,101,110,120,201 011,012,021,100,101,110,120,210 011,012,021,100,101,110,201,210 011,012,021,100,101,120,201,210 011,012,021,100,102,110,120,201 011,012,021,100,102,110,120,210 011,012,021,100,102,110,201,210 011,012,021,100,102,120,201,210 011,012,021,100,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010$	$\frac{x(1+x^2)}{(1-x)^2}$
36	000,011,021,100,101,110,201,210	$0 \rightsquigarrow 00, 0$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	010,012,021,100,101,102,110,120 010,012,021,100,101,102,110,201 010,012,021,100,101,102,110,210 010,012,021,100,101,102,120,201 010,012,021,100,101,102,120,210 010,012,021,100,101,102,201,210 010,012,021,100,101,110,120,201 010,012,021,100,101,110,120,210 010,012,021,100,101,110,201,210 010,012,021,100,101,120,201,210 010,012,021,100,102,110,120,201 010,012,021,100,102,110,120,210 010,012,021,100,102,110,201,210 010,012,021,100,102,120,201,210 010,012,021,100,110,120,201,210 010,012,021,101,102,110,120,201 010,012,021,101,102,110,120,210 010,012,021,101,102,110,201,210 010,012,021,101,102,120,201,210 010,012,021,101,110,120,201,210 010,012,021,102,110,120,201,210 011,012,021,101,102,110,120,201 011,012,021,101,102,110,120,210 011,012,021,101,102,110,201,210 011,012,021,101,102,120,201,210 011,012,021,101,110,120,201,210 011,012,021,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01$, $a_m = 0^m$	
	011,012,100,101,102,110,201,210 011,012,100,101,102,120,201,210 011,012,100,101,110,120,201,210 011,012,100,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^2)}{(1-x)^3}$

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
37	011,012,100,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_1 \rightsquigarrow 010$; $b_2 \rightsquigarrow b_1, 010$; $b_m \rightsquigarrow c_m, (010)^{m-1}$; $c_m \rightsquigarrow (010)^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$, $c_m = 0^m m0$	$\frac{x(1-x+x^2+x^4)}{(1-x)^3}$
38	000,011,100,101,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 0, 012$; $012 \rightsquigarrow 012$	
	011,012,100,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x}{(1-x)(1-x-x^2)}$
39	010,012,100,101,102,110,120,210 010,012,100,101,102,110,201,210 010,012,100,101,110,120,201,210 010,012,100,102,110,120,201,210 011,012,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (0021)^{m-1}, b_1$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^2+x^3)}{(1-x)^3}$
40	010,012,100,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{c_i\}_{i=2}^m, b_1$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$, $c_m = 0^m m(m-1)$	$\frac{x(1-3x+4x^2-2x^3)}{(1-x)^5} + \frac{x^5(-1+2x-x^2+x^3)}{(1-x)^5}$
41	010,012,100,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (0021)^{m-1}, b_m$, $a_m = 0^m$, $b_m = 0^m m$	
	010,012,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow (b_1)^m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-2x+2x^2)}{(1-x)^4}$
42	010,011,021,100,101,102,110,120 010,011,021,100,101,102,110,201 010,011,021,100,101,102,110,210 010,011,021,100,101,102,120,201 010,011,021,100,101,102,120,210 010,011,021,100,101,102,201,210 010,011,021,100,101,110,120,201 010,011,021,100,101,110,120,210 010,011,021,100,101,110,201,210 010,011,021,100,101,120,201,210 010,011,021,100,102,110,120,201 010,011,021,100,102,110,120,210 010,011,021,100,102,110,201,210 010,011,021,100,102,120,201,210 010,011,021,100,110,120,201,210 010,011,021,101,102,110,120,201 010,011,021,101,102,110,120,210 010,011,021,101,102,110,201,210 010,011,021,101,102,120,201,210 010,011,021,101,110,120,201,210 010,011,021,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b_1, \ldots, b_m$; $b_m \rightsquigarrow b_1, \ldots, b_m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x}{1-2x}$
43	010,011,100,101,102,120,201,210 010,011,100,102,110,120,201,210 010,011,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = 0^m j$	$\frac{x(x^3+x^2-2x+1)}{(1-2x)(1-x)^2}$
44	010,011,100,101,102,110,120,201 010,011,100,101,102,110,120,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow c_2, \ldots, c_j, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$; $c_m \rightsquigarrow c_2, \ldots, c_{m-1}$, $a_m = 0^m$, $b_{m,j} = 0^m j$, $c_m = 0^m m(m-1)$	$\frac{x(1-2x+2x^3)}{(1-x)(1-2x)(1-x-x^2)}$
45	010,011,100,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, \{b_{m,i}\}_{i=j}^m$, $a_m = 0^m$, $b_{m,j} = 0^m j$	$\frac{x(x^3+2x^2-3x+1)}{(1-x)(1-2x)^2}$
46	000,010,021,100,101,102,110,120 000,010,021,100,101,102,110,201 000,010,021,100,101,102,110,210 000,010,021,100,101,102,120,201 000,010,021,100,101,102,120,210 000,010,021,100,101,102,201,210 000,010,021,100,101,110,120,201 000,010,021,100,101,110,120,210 000,010,021,100,101,110,201,210 000,010,021,100,101,120,201,210 000,010,021,100,102,110,120,201 000,010,021,100,102,110,120,210 000,010,021,100,102,110,201,210 000,010,021,100,102,120,201,210 000,010,021,100,110,120,201,210 000,010,021,101,102,110,120,201 000,010,021,101,102,110,120,210 000,010,021,101,102,110,201,210 000,010,021,101,102,120,201,210 000,010,021,101,110,120,201,210 000,010,021,102,110,120,201,210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^m$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-2x^2-\sqrt{1-2x-3x^2}}{2x^2}$

Continuation of Table 4

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
47	010,011,100,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (b_{m+2-j})^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = 0^m j$	$\frac{x(1-2x)}{(1-3x+x^2)(1-x))}$
48	012,021,100,101,102,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 01$, $a_m = 0^m$	
	012,021,100,102,110,120,201,210 012,021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow (010)^2$; $010 \rightsquigarrow 010$, $a_m = 0^m$	
	012,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow (010)^m, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$	$\frac{x(1-x+2x^2)}{(1-x)^3}$
49	011,021,100,101,102,110,120,201 011,021,100,101,102,110,120,210 011,021,100,101,102,120,201,210 011,021,100,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^{m}$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m}$	$\frac{x(1-2x+2x^2-2x^3)}{(1-x)^2(1-2x)}$
50	000,021,100,101,102,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $01 \rightsquigarrow 010, 011, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$; $011 \rightsquigarrow 010, a_1, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	
	000,021,100,102,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $01 \rightsquigarrow 010, b_0, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$; $010 \rightsquigarrow 0101$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3 + x^4$
51	000,021,100,101,102,110,201,210	$a_0 \rightsquigarrow b_0, 01$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $01 \rightsquigarrow 010, b_0, 01$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + \frac{x^3}{1-x}$
52	011,021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{c_i\}_{i=2}^{m}, 010$; $c_m \rightsquigarrow \{c_i\}_{i=2}^{m}, 010$; $010 \rightsquigarrow 010$, $a_m = 0^m$, $b_m = 0^m 1$, $c_m = 0^m 12$	$\frac{x(1-3x+4x^2-3x^3)}{(1-x)^3(1-2x)}$
53	011,021,100,101,102,110,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow 010, \{b_i\}_{i=1}^{m}$, $a_m = 0^m$, $b_m = 0^m 1$	
	011,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^{m}$; $b_{m,j} \rightsquigarrow (010)^j, c_{m+1-j}, \{b_{m-j,i}\}_{i=1}^{m-j}$; $c_m \rightsquigarrow c_m, \{b_{m-1,i}\}_{i=1}^{m-1}$, $a_m = 0^m$, $b_{m,j} = 0^m j$, $c_m = 0^m 12$	$\frac{x(x^2-x+1)}{(1-x)(1-2x)}$
54	000,021,100,101,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, 002$; $01 \rightsquigarrow (b_0)^2, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-4x^2-x^3}{2x^2} - \frac{(1+x-x^2)\sqrt{1-2x-3x^2}}{2x^2}$
55	000,100,101,102,110,120,201,210•		Theorem 21
56	011,021,100,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^{m}$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m}, 012$; $012 \rightsquigarrow 012$, $a_m = 0^m$, $b_m = 0^m 1$	$\frac{x(1-3x+3x^2)}{(1-2x)(1-x)^2}$
57	010,021,100,101,102,110,120,201 010,021,100,101,102,110,120,210 010,021,100,101,102,110,201,210 010,021,100,101,102,120,201,210 010,021,100,101,110,120,201,210 010,021,100,102,110,120,201,210 010,021,101,102,110,120,201,210	$a_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}$, $a_m = 0^m$	$\frac{1-2x-\sqrt{1-4x}}{2x}$
58	010,100,101,102,110,120,201,210•	$a_m \rightsquigarrow a_{m+1}, a_m, \{b_{m,i}\}_{i=2}^{m}$; $b_{m,j} \rightsquigarrow (0021)^{j-1}, a_{m+2-j}, a_{m+1-j}, \{b_{m+1-j,i}\}_{i=2}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = 0^m j$	$x(1 + x^3 C^3)C^2$, $C = \frac{1-\sqrt{1-4x}}{2x}$
59	012,021,100,101,102,110,120,201 012,021,100,101,102,110,120,210 012,021,100,101,102,110,201,210 012,021,100,101,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $01 \rightsquigarrow 010, 011$; $011 \rightsquigarrow 011$	$\frac{x(x^3-2x^2+x-1)}{(1-x)^3}$
60	000,021,100,101,102,110,120,201 000,021,100,101,102,110,120,210		

Continuation of Table 4			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,021,101,102,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^m, 002$; $01 \rightsquigarrow 010, b_0, 002$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, 002$; $002 \rightsquigarrow b_0, 002$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-\sqrt{1-2x-3x^2}}{2x^2} - 1 + x^3$
61	021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^m, 012$; $c_m \rightsquigarrow \{c_i\}_{i=1}^{m+1}, 012$; $012 \rightsquigarrow c_1, 012$, $a_m = 0^m$, $b_m = 0^m 1$, $c_m = 0^m 11$	$\frac{1-\sqrt{1-4x}}{2x} - 1 + \frac{x^3}{(1-x)^2}$
End of Table 4			

Theorem 17. *Let* $B_\tau = \{000, 010, 100, 101, 102, 110, 120, \tau\}$. *Then* $B_{201} \overset{\mathbf{I}}{\sim} B_{210}$. *Moreover, the generating function* $F_{B_{201}}(x)$ *is given by*

$$\frac{1 - x^2 - 4x^3 - 7x^4 - 5x^5 - 2x^6 - (1 + x + 2x^2 + 2x^3 + x^4)\sqrt{1 - 2x - 3x^2}}{2x^3(1 + x)(x^2 + 2x + 2)}.$$

Proof. From the fact that the generating trees are isomorphic, it remains to find the generating function $F_{B_{201}}(x)$. According to Class 24 in Table 4, define $A_{m,j}(x)$ (respectively, $B_m(x), C_m(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B_{201}; a_{m,j})$ (respectively, $\mathcal{T}(B_{201}; b_m), \mathcal{T}(B_{201}; c_m)$), where its root stays at level 0. Thus,

$$\begin{aligned}
A_{m,j}(x) &= x + x \sum_{i=1}^{j-m} B_i(x) + xC_{2m-j} + x \sum_{i=2m-j}^{4m-2j} A_{2m-j,s}(x), \\
B_m(x) &= x + x \sum_{i=1}^{m-1} B_i(x), \\
C_m(x) &= x + x \sum_{i=m+1}^{2m+2} A_{m+1,s}(x).
\end{aligned}$$

Clearly, $B_m(x) = x(1 + x)^{m-1}$ for all $m \geq 1$. Let $A_m(x) = \sum_{i=m}^{2m} A_{m,j}(x)$. Then

$$\begin{aligned}
A_m(x) &= (m + 1)x + x^2 \sum_{i=1}^{m} (m + 1 - i)(1 + x)^{i-1} + x \sum_{i=0}^{m} (A_i(x) + C_i(x)), \\
C_m(x) &= x + xA_{m-1}(x).
\end{aligned}$$

Define $A(v) = \sum_{m\geq 0} A_m(x)v^m$ and $C(v) = \sum_{m\geq 0} C_m(x)v^m$. Then, the above recurrence can be written as

$$\begin{aligned}
A(v) &= \frac{x}{(1 - v)(1 - v - vx)} + \frac{x}{1 - v} \left(C(v) + A(v)\right), \\
C(v) &= \frac{x}{1 - v} + \frac{x}{v}(A(v) - A(0)).
\end{aligned}$$

Thus,

$$A(v) = \frac{x}{(1-v)(1-v-vx)} + \frac{x}{1-v}\left(\frac{x}{1-v} + \frac{x}{v}(A(v) - A(0)) + A(v)\right).$$

By taking $v = \frac{1-x-\sqrt{1-2x-3x^2}}{2}$ and solving for $A(0)$, we complete the proof. □

Theorem 18. *Let*

$$\begin{aligned} B_1 &= \{000, 010, 100, 101, 102, 120, 201, 210\}, \\ B_2 &= \{000, 010, 100, 102, 110, 120, 201, 210\}, \\ B_3 &= \{000, 010, 101, 102, 110, 120, 201, 210\}. \end{aligned}$$

Then, for $j = 1, 2, 3$*,*

$$F_{B_j}(x) = \frac{1 - 2x - x^2 + x^3 - (1-x)\sqrt{1-2x-3x^2}}{x^2}.$$

Proof. Since the proof of the cases are similar, we only present the details of the case B_1. According to Class 25 in Table 4, define $A_{m,j}(x)$ (respectively, $B_{m,j}(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B_1; a_{m,j})$ (respectively, $\mathcal{T}(B_1; b_{m,j})$), where its root stays at level 0. Thus,

$$\begin{aligned} A_{m,j}(x) &= x + (j-m)x^2 + xB_{m,j}(x) + xA_{2m-j}(x), \\ B_{m,j}(x) &= x + (j-m)x^2 + xA_{2m+1-j}(x), \end{aligned}$$

where $A_m(x) = \sum_{i=m}^{2m} A_{m,j}(x)$ and $B_m(x) = \sum_{i=m}^{2m} B_{m,j}(x)$. Then

$$\begin{aligned} A_m(x) &= (m+1)x + \binom{m+1}{2}x^2 + xB_m + x\sum_{i=0}^{m} A_i(x), \\ B_m(x) &= (m+1)x + \binom{m+1}{2}x^2 + xB_m + x\sum_{i=1}^{m+1} A_i(x). \end{aligned}$$

Define $A(v) = \sum_{m\geq 0} A_m(x)v^m$ and $B(v) = \sum_{m\geq 0} B_m(x)v^m$. Then, the above recurrences lead to

$$A(v) = \frac{x(1+x)(vx-v+1)}{(1-v)^3} + \frac{x(1+x)}{1-v}A(v) + \frac{x^2}{v}(A(v) - A(0)) - \frac{x^2}{1-v}A(0).$$

By taking $v = \frac{1-x-\sqrt{1-2x-3x^2}}{2}$ and solving for $A(0)$, we complete the proof. □

Theorem 19. *Let* $B = \{000, 010, 100, 101, 102, 110, 201, 210\}$*. Then,*

$$F_B(x) = \frac{1 - 2x - x^2 + x^3 - x^4 - (1 - x - x^3)\sqrt{1-2x-3x^2}}{2x^2(1+x)}.$$

Proof. According to Class 26 in Table 4, define $A_{m,j}(x)$ (respectively, $B_m(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B; a_{m,j})$ (respectively, $\mathcal{T}(B; b_m)$), where its root stays at level 0. Thus,

$$A_{m,j}(x) = x + (j-m)x^2 + xB_{2m-j}(x) + x\sum_{i=j}^{2m} A_{m,i}(x),$$
$$B_{m,j}(x) = x + x\sum_{i=m+1}^{2m+2} A_{m+1,i}(x).$$

Define $A(v,u) = \sum_{m\geq 0}\sum_{j=m}^{2m} A_{m,j}(x)u^{m-j}v^m$ and $B(v) = \sum_{m\geq 0} B_m(x)v^m$. The above recurrences give

$$A(v) = \frac{x(1-uv+uvx)}{(1-v)(1-uv)^2} + \frac{x}{1-uv}B(v) + \frac{x}{1-u}(A(v,1) - uA(v,u)),$$
$$B(v) = \frac{x}{1-v} + \frac{x}{v}(A(v,1) - A(0,1)).$$

By taking $u = \frac{1}{1-x}$ and then $v = \frac{1-x-\sqrt{1-2x-3x^2}}{2}$ and solving for $A(0,1)$, we complete the proof. □

Theorem 20. *Let* $B = \{000, 010, 100, 101, 110, 120, 201, 210\}$. *Then,*

$$F_B(x) = \frac{v_0(1+x-v_0)}{x(1-v_0)},$$

where $v_0 = x^2 + x^3 + 3x^4 + \cdots$ *is the root of the polynomial* $v(1-v)^2 - vx(1-v) - x$.

Proof. According to Class 26 in Table 4, define $A_{m,j}(x)$ (respectively, $B_m(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B; a_{m,j})$ (respectively, $\mathcal{T}(B; b_m)$), where its root stays at level 0. Thus,

$$A_{m,j}(x) = x + (j+1-m)xB_{2m-j}(x) + x\sum_{i=2m-j}^{4m-2j} A_{2m-j,i}(x),$$
$$B_m(x) = x + x\sum_{i=m+1}^{2m+2} A_{m+1,i}(x).$$

Define $A(v) = \sum_{m\geq 0}\sum_{j=m}^{2m} A_{m,j}(x)v^m$ and $B(v) = \sum_{m\geq 0} B_m(x)v^m$. Then, the above recurrences lead to

$$A(v) = \frac{x}{(1-v)^2}\left(1 + \frac{x}{1-v} + \frac{x}{v}(A(v) - A(0))\right) + \frac{x}{1-v}A(v).$$

Let $v_0 = x^2 + x^3 + 3x^4 + \cdots$ be the root of the polynomial $v(1-v)^2 - vx(1-v) - x$. Then, by taking $v = v_0$, we complete the proof. □

Theorem 21. *Let* $B = \{000, 100, 101, 102, 110, 120, 201, 210\}$. *Then*

$$F_B(x) = \frac{2 - 2x - 5x^2 - x^3 - (2 - x^2)\sqrt{1 - 2x - 3x^2}}{2x^2(1 + x)}.$$

Proof. The rules of the generating tree $\mathcal{T}(B)$ are given by

$$\begin{aligned}
a_m &\rightsquigarrow b_m, c_{m,m+1}, \ldots, c_{m,2m+1},\\
b_m &\rightsquigarrow a_{m+1}, c_{m,m+1}, \ldots, c_{m,2m+1},\\
c_{m,j} &\rightsquigarrow (010)^{j-m}, b_{2m+1-j}, c_{2m+1-j,2m+2-j}, \ldots, c_{2m+1-j,4m+3-2j}.
\end{aligned}$$

Define $A_m(x)$ (respectively, $B_m(x), C_{m,j}(x)$) to be the generating function for the number of nodes at level $n \geq 0$ for the subtree of $\mathcal{T}(B; a_m)$ (respectively, $\mathcal{T}(B; b_m)$, $\mathcal{T}(B; c_{m,j})$), where its root stays at level 0. Thus,

$$\begin{aligned}
A_m(x) &= x + xB_m(x) + x \sum_{i=m+1}^{2m+1} C_{m,i}(x),\\
B_m(x) &= x + xA_{m+1}(x) + x \sum_{i=m+1}^{2m+1} C_{m,i}(x),\\
C_{m,j}(x) &= x + (j - m)x^2 + xB_{2m+1-j}(x) + x \sum_{i=2m+2-j}^{4m+3-2j} C_{2m+1-j,i}(x).
\end{aligned}$$

Define $C_m(x) = \sum_{i=m+1}^{2m+1} C_{m,i}(x)$. Then $C_{m,j}(x) = x + (j - m)x^2 + xB_{2m+1-j}(x) + xC_{2m+1-j}(x)$, which leads to $C_m(x) = (m+1)x + \binom{m+2}{2}x^2 + x\sum_{i=0}^{m}(B_i(x) + C_i(x))$. Hence,

$$\begin{aligned}
A_m(x) &= x + xB_m(x) + xC_m(x),\\
B_m(x) &= x + xA_{m+1}(x) + xC_m(x),\\
C_m(x) &= (m + 1)x + \binom{m+2}{2}x^2 + x\sum_{i=0}^{m}(B_i(x) + C_i(x)).
\end{aligned}$$

Define $f(v) = \sum_{m\geq 0} f_m(x)v^m$ for $f \in \{A, B, C\}$. Then, the above recurrences lead to

$$\begin{aligned}
A(v) &= \frac{x}{1 - v} + xB(v) + xC(v),\\
B(v) &= \frac{x}{1 - v} + \frac{x}{v}(A(v) - A(0)) + xC(v),\\
C(v) &= \frac{x}{(1 - v)^2} + \frac{x^2}{(1 - v)^3} + \frac{x}{1 - v}(B(v) + C(v)).
\end{aligned}$$

By taking $v = \frac{1-x-\sqrt{1-2x-3x^2}}{2}$ and solving for $A(0)$, we complete the proof. □

7. d-Tables, Where $d = 9, 10, 11, 12$

In this section, we show that $w_9 = 35$, $w_{10} = 21$, $w_{11} = 10$, and $w_{12} = 4$ (of course, $w_{13} = 1$). By our procedure, we present the generating functions for all of the w_d I-Wilf-equivalences. Actually, we see that there is only 1 set among $\sum_{d=9}^{13} \binom{13}{d}$ sets of d length-3 patterns with $d = 9, 10, 11, 12, 13$ that cannot be reduced to a smaller set of length-3 patterns. In Tables 5-8, we present all the distinct I-Wilf-equivalences of sets of $d = 9, 10, 11, 12$, length-3 patterns, respectively. Due to the similarity with the work in previous sections, we omit all the proofs.

Table 5: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 9$.

Beginning of Table 5			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100,101,102 000,001,010,011,012,021,100,101,110 000,001,010,011,012,021,100,101,120 000,001,010,011,012,021,100,101,201 000,001,010,011,012,021,100,101,210 000,001,010,011,012,021,100,102,110 000,001,010,011,012,021,100,102,120 000,001,010,011,012,021,100,102,201 000,001,010,011,012,021,100,102,210 000,001,010,011,012,021,100,110,120 000,001,010,011,012,021,100,110,201 000,001,010,011,012,021,100,110,210 000,001,010,011,012,021,100,120,201 000,001,010,011,012,021,100,120,210 000,001,010,011,012,021,100,201,210 000,001,010,011,012,021,101,102,110 000,001,010,011,012,021,101,102,120 000,001,010,011,012,021,101,102,201 000,001,010,011,012,021,101,102,210 000,001,010,011,012,021,101,110,120 000,001,010,011,012,021,101,110,201 000,001,010,011,012,021,101,110,210 000,001,010,011,012,021,101,120,201 000,001,010,011,012,021,101,120,210 000,001,010,011,012,021,101,201,210 000,001,010,011,012,021,102,110,120 000,001,010,011,012,021,102,110,201 000,001,010,011,012,021,102,110,210 000,001,010,011,012,021,102,120,201 000,001,010,011,012,021,102,120,210 000,001,010,011,012,021,102,201,210 000,001,010,011,012,021,110,120,201 000,001,010,011,012,021,110,120,210 000,001,010,011,012,021,110,201,210 000,001,010,011,012,021,120,201,210 000,001,010,011,012,100,101,102,110 000,001,010,011,012,100,101,102,120 000,001,010,011,012,100,101,102,201 000,001,010,011,012,100,101,102,210 000,001,010,011,012,100,101,110,120 000,001,010,011,012,100,101,110,201 000,001,010,011,012,100,101,110,210 000,001,010,011,012,100,101,120,201 000,001,010,011,012,100,101,120,210 000,001,010,011,012,100,101,201,210 000,001,010,011,012,100,102,110,120 000,001,010,011,012,100,102,110,201 000,001,010,011,012,100,102,110,210 000,001,010,011,012,100,102,120,201 000,001,010,011,012,100,102,120,210 000,001,010,011,012,100,102,201,210 000,001,010,011,012,100,110,120,201 000,001,010,011,012,100,110,120,210 000,001,010,011,012,100,110,201,210 000,001,010,011,012,100,120,201,210 000,001,010,011,012,101,102,110,120 000,001,010,011,012,101,102,110,201 000,001,010,011,012,101,102,110,210 000,001,010,011,012,101,102,120,201 000,001,010,011,012,101,102,120,210 000,001,010,011,012,101,102,201,210 000,001,010,011,012,101,110,120,201 000,001,010,011,012,101,110,120,210 000,001,010,011,012,101,110,201,210 000,001,010,011,012,101,120,201,210 000,001,010,011,012,102,110,120,201 000,001,010,011,012,102,110,120,210 000,001,010,011,012,102,110,201,210 000,001,010,011,012,102,120,201,210 000,001,010,011,012,110,120,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101,102,110 000,001,010,012,021,100,101,102,120 000,001,010,012,021,100,101,102,201 000,001,010,012,021,100,101,102,210		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,012,021,100,101,110,120		
	000,001,010,012,021,100,101,110,201		
	000,001,010,012,021,100,101,110,210		
	000,001,010,012,021,100,101,120,201		
	000,001,010,012,021,100,101,120,210		
	000,001,010,012,021,100,101,201,210		
	000,001,010,012,021,100,102,110,120		
	000,001,010,012,021,100,102,110,201		
	000,001,010,012,021,100,102,110,210		
	000,001,010,012,021,100,102,120,201		
	000,001,010,012,021,100,102,120,210		
	000,001,010,012,021,100,102,201,210		
	000,001,010,012,021,100,110,120,201		
	000,001,010,012,021,100,110,120,210		
	000,001,010,012,021,100,110,201,210		
	000,001,010,012,021,100,120,201,210		
	000,001,010,012,021,101,102,110,120		
	000,001,010,012,021,101,102,110,201		
	000,001,010,012,021,101,102,110,210		
	000,001,010,012,021,101,102,120,201		
	000,001,010,012,021,101,102,120,210		
	000,001,010,012,021,101,102,201,210		
	000,001,010,012,021,101,110,120,201		
	000,001,010,012,021,101,110,120,210		
	000,001,010,012,021,101,110,201,210		
	000,001,010,012,021,101,120,201,210		
	000,001,010,012,021,102,110,120,201		
	000,001,010,012,021,102,110,120,210		
	000,001,010,012,021,102,110,201,210		
	000,001,010,012,021,102,120,201,210		
	000,001,010,012,021,110,120,201,210		
	000,001,010,012,100,101,102,110,120		
	000,001,010,012,100,101,102,110,201		
	000,001,010,012,100,101,102,110,210		
	000,001,010,012,100,101,102,120,201		
	000,001,010,012,100,101,102,120,210		
	000,001,010,012,100,101,102,201,210		
	000,001,010,012,100,101,110,120,201		
	000,001,010,012,100,101,110,120,210		
	000,001,010,012,100,101,110,201,210		
	000,001,010,012,100,101,120,201,210		
	000,001,010,012,100,102,110,120,201		
	000,001,010,012,100,102,110,120,210		
	000,001,010,012,100,102,110,201,210		
	000,001,010,012,100,102,120,201,210		
	000,001,010,012,100,110,120,201,210		
	000,001,010,012,101,102,110,120,201		
	000,001,010,012,101,102,110,120,210		
	000,001,010,012,101,102,110,201,210		
	000,001,010,012,101,102,120,201,210		
	000,001,010,012,101,110,120,201,210		
	000,001,010,012,102,110,120,201,210		
	000,001,011,012,021,100,101,102,110		
	000,001,011,012,021,100,101,102,120		
	000,001,011,012,021,100,101,102,201		
	000,001,011,012,021,100,101,102,210		
	000,001,011,012,021,100,101,110,120		
	000,001,011,012,021,100,101,110,201		
	000,001,011,012,021,100,101,110,210		
	000,001,011,012,021,100,101,120,201		
	000,001,011,012,021,100,101,120,210		
	000,001,011,012,021,100,101,201,210		
	000,001,011,012,021,100,102,110,120		
	000,001,011,012,021,100,102,110,201		
	000,001,011,012,021,100,102,110,210		
	000,001,011,012,021,100,102,120,201		
	000,001,011,012,021,100,102,120,210		
	000,001,011,012,021,100,102,201,210		
	000,001,011,012,021,100,110,120,201		
	000,001,011,012,021,100,110,120,210		
	000,001,011,012,021,100,110,201,210		
	000,001,011,012,021,100,120,201,210		
	000,001,011,012,021,101,102,110,120		
	000,001,011,012,021,101,102,110,201		
	000,001,011,012,021,101,102,110,210		
	000,001,011,012,021,101,102,120,201		
	000,001,011,012,021,101,102,120,210		
	000,001,011,012,021,101,102,201,210		
	000,001,011,012,021,101,110,120,201		
	000,001,011,012,021,101,110,120,210		
	000,001,011,012,021,101,110,201,210		
	000,001,011,012,021,101,120,201,210		
	000,001,011,012,021,102,110,120,201		
	000,001,011,012,021,102,110,120,210		
	000,001,011,012,021,102,110,201,210		
	000,001,011,012,021,102,120,201,210		
	000,001,011,012,021,110,120,201,210		
	000,001,011,012,100,101,102,110,120		
	000,001,011,012,100,101,102,110,201		
	000,001,011,012,100,101,102,110,210		
	000,001,011,012,100,101,102,120,201		
	000,001,011,012,100,101,102,120,210		
	000,001,011,012,100,101,102,201,210		
	000,001,011,012,100,101,110,120,201		
	000,001,011,012,100,101,110,120,210		
	000,001,011,012,100,101,110,201,210		
	000,001,011,012,100,101,120,201,210		
	000,001,011,012,100,102,110,120,201		
	000,001,011,012,100,102,110,120,210		
	000,001,011,012,100,102,110,201,210		
	000,001,011,012,100,102,120,201,210		
	000,001,011,012,100,110,120,201,210		
	000,001,011,012,101,102,110,120,201		
	000,001,011,012,101,102,110,120,210		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,011,012,101,102,110,201,210		
	000,001,011,012,101,102,120,201,210		
	000,001,011,012,101,110,120,201,210		
	000,001,011,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x(1+x)^{2}$2
3	000,001,012,021,100,101,102,110,120		
	000,001,012,021,100,101,102,110,201		
	000,001,012,021,100,101,102,110,210		
	000,001,012,021,100,101,110,120,201		
	000,001,012,021,100,101,110,120,210		
	000,001,012,021,100,101,110,201,210		
	000,001,012,021,100,102,110,120,201		
	000,001,012,021,100,102,110,120,210		
	000,001,012,021,100,102,110,201,210		
	000,001,012,021,100,110,120,201,210		
	000,001,012,021,101,102,110,120,201		
	000,001,012,021,101,102,110,120,210		
	000,001,012,021,101,102,110,201,210		
	000,001,012,021,101,110,120,201,210		
	000,001,012,021,102,110,120,201,210		
	000,001,012,100,101,102,110,120,201		
	000,001,012,100,101,102,110,120,210		
	000,001,012,100,101,102,110,201,210		
	000,001,012,100,101,110,120,201,210		
	000,001,012,100,102,110,120,201,210		
	000,001,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000,010,011,012,021,100,101,102,110		
	000,010,011,012,021,100,101,102,120		
	000,010,011,012,021,100,101,102,201		
	000,010,011,012,021,100,101,102,210		
	000,010,011,012,021,100,101,110,120		
	000,010,011,012,021,100,101,110,201		
	000,010,011,012,021,100,101,110,210		
	000,010,011,012,021,100,101,120,201		
	000,010,011,012,021,100,101,120,210		
	000,010,011,012,021,100,101,201,210		
	000,010,011,012,021,100,102,110,120		
	000,010,011,012,021,100,102,110,201		
	000,010,011,012,021,100,102,110,210		
	000,010,011,012,021,100,102,120,201		
	000,010,011,012,021,100,102,120,210		
	000,010,011,012,021,100,102,201,210		
	000,010,011,012,021,100,110,120,201		
	000,010,011,012,021,100,110,120,210		
	000,010,011,012,021,100,110,201,210		
	000,010,011,012,021,100,120,201,210		
	000,010,011,012,021,101,102,110,120		
	000,010,011,012,021,101,102,110,201		
	000,010,011,012,021,101,102,110,210		
	000,010,011,012,021,101,102,120,201		
	000,010,011,012,021,101,102,120,210		
	000,010,011,012,021,101,102,201,210		
	000,010,011,012,021,101,110,120,201		
	000,010,011,012,021,101,110,120,210		
	000,010,011,012,021,101,110,201,210		
	000,010,011,012,021,101,120,201,210		
	000,010,011,012,021,102,110,120,201		
	000,010,011,012,021,102,110,120,210		
	000,010,011,012,021,102,110,201,210		
	000,010,011,012,021,102,120,201,210		
	000,010,011,012,021,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
4	000,001,012,021,100,101,102,120,201		
	000,001,012,021,100,101,102,120,210		
	000,001,012,021,100,101,102,201,210		
	000,001,012,021,100,101,120,201,210		
	000,001,012,021,100,102,120,201,210		
	000,001,012,021,101,102,120,201,210		
	000,001,012,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;$ $011 \rightsquigarrow 00$	
	000,010,011,012,100,101,102,110,120		
	000,010,011,012,100,101,102,110,201		
	000,010,011,012,100,101,102,110,210		
	000,010,011,012,100,101,102,120,201		
	000,010,011,012,100,101,102,120,210		
	000,010,011,012,100,101,102,201,210		
	000,010,011,012,100,101,110,120,201		
	000,010,011,012,100,101,110,120,210		
	000,010,011,012,100,101,110,201,210		
	000,010,011,012,100,101,120,201,210		
	000,010,011,012,100,102,110,120,201		
	000,010,011,012,100,102,110,120,210		
	000,010,011,012,100,102,110,201,210		
	000,010,011,012,100,102,120,201,210		
	000,010,011,012,100,110,120,201,210		
	000,010,011,012,101,102,110,120,201		
	000,010,011,012,101,102,110,120,210		
	000,010,011,012,101,102,110,201,210		
	000,010,011,012,101,102,120,201,210		
	000,010,011,012,101,110,120,201,210		
	000,010,011,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;$ $002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
5	000,011,012,021,100,101,102,110,120		
	000,011,012,021,100,101,102,110,201		
	000,011,012,021,100,101,102,110,210		
	000,011,012,021,100,101,102,120,201		
	000,011,012,021,100,101,102,120,210		
	000,011,012,021,100,101,102,201,210		
	000,011,012,021,100,101,110,120,201		
	000,011,012,021,100,101,110,120,210		
	000,011,012,021,100,101,110,201,210		
	000,011,012,021,100,101,120,201,210		
	000,011,012,021,100,102,110,120,201		
	000,011,012,021,100,102,110,120,210		
	000,011,012,021,100,102,110,201,210		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,011,012,021,100,102,120,201,210		
	000,011,012,021,100,110,120,201,210		
	000,011,012,021,101,102,110,120,201		
	000,011,012,021,101,102,110,120,210		
	000,011,012,021,101,102,110,201,210		
	000,011,012,021,101,102,120,201,210		
	000,011,012,021,101,110,120,201,210		
	000,011,012,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
6	000,011,012,100,101,102,110,120,201		
	000,011,012,100,101,102,110,120,210		
	000,011,012,100,101,102,110,201,210		
	000,011,012,100,101,102,120,201,210		
	000,011,012,100,101,110,120,201,210		
	000,011,012,100,102,110,120,201,210		
	000,011,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 001$; $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
7	000,010,012,021,100,101,102,110,120		
	000,010,012,021,100,101,102,110,201		
	000,010,012,021,100,101,102,110,210		
	000,010,012,021,100,101,102,120,201		
	000,010,012,021,100,101,102,120,210		
	000,010,012,021,100,101,102,201,210		
	000,010,012,021,100,101,110,120,201		
	000,010,012,021,100,101,110,120,210		
	000,010,012,021,100,101,110,201,210		
	000,010,012,021,100,101,120,201,210		
	000,010,012,021,100,102,110,120,201		
	000,010,012,021,100,102,110,120,210		
	000,010,012,021,100,102,110,201,210		
	000,010,012,021,100,102,120,201,210		
	000,010,012,021,100,110,120,201,210		
	000,010,012,021,101,102,110,120,201		
	000,010,012,021,101,102,110,120,210		
	000,010,012,021,101,102,110,201,210		
	000,010,012,021,101,102,120,201,210		
	000,010,012,021,101,110,120,201,210		
	000,010,012,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (01)^2$; $01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
8	000,010,012,100,101,102,110,120,201		
	000,010,012,100,101,102,110,120,210		
	000,010,012,100,101,102,110,201,210		
	000,010,012,100,101,110,120,201,210		
	000,010,012,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 3x^3 + 3x^4$
9	000,010,012,100,101,102,120,201,210		
	000,010,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow 01, 011$	$x + 2x^2 + 3x^3 + 3x^4 + x^5$
10	000,012,021,100,101,102,110,120,201		
	000,012,021,100,101,102,110,120,210		
	000,012,021,100,101,102,110,201,210		
	000,012,021,100,101,110,120,201,210		
	000,012,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^4$
11	000,012,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 010, 001$; $001 \rightsquigarrow 010$	
	000,012,021,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow 001, 011$; $001 \rightsquigarrow 11$	
	000,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 001, 01$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 3x^4$
12	000,001,010,011,021,100,101,102,110		
	000,001,010,011,021,100,101,102,120		
	000,001,010,011,021,100,101,102,201		
	000,001,010,011,021,100,101,102,210		
	000,001,010,011,021,100,101,110,120		
	000,001,010,011,021,100,101,110,201		
	000,001,010,011,021,100,101,110,210		
	000,001,010,011,021,100,101,120,201		
	000,001,010,011,021,100,101,120,210		
	000,001,010,011,021,100,101,201,210		
	000,001,010,011,021,100,102,110,120		
	000,001,010,011,021,100,102,110,201		
	000,001,010,011,021,100,102,110,210		
	000,001,010,011,021,100,102,120,201		
	000,001,010,011,021,100,102,120,210		
	000,001,010,011,021,100,102,201,210		
	000,001,010,011,021,100,110,120,201		
	000,001,010,011,021,100,110,120,210		
	000,001,010,011,021,100,110,201,210		
	000,001,010,011,021,100,120,201,210		
	000,001,010,011,021,101,102,110,120		
	000,001,010,011,021,101,102,110,201		
	000,001,010,011,021,101,102,110,210		
	000,001,010,011,021,101,102,120,201		
	000,001,010,011,021,101,102,120,210		
	000,001,010,011,021,101,102,201,210		
	000,001,010,011,021,101,110,120,201		
	000,001,010,011,021,101,110,120,210		
	000,001,010,011,021,101,110,201,210		
	000,001,010,011,021,101,120,201,210		
	000,001,010,011,021,102,110,120,201		
	000,001,010,011,021,102,110,120,210		
	000,001,010,011,021,102,110,201,210		
	000,001,010,011,021,102,120,201,210		
	000,001,010,011,021,110,120,201,210		
	000,001,010,011,100,101,102,110,120		
	000,001,010,011,100,101,102,110,201		
	000,001,010,011,100,101,102,110,210		
	000,001,010,011,100,101,102,120,201		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,011,100,101,102,120,210		
	000,001,010,011,100,101,102,201,210		
	000,001,010,011,100,101,110,120,201		
	000,001,010,011,100,101,110,120,210		
	000,001,010,011,100,101,110,201,210		
	000,001,010,011,100,101,120,201,210		
	000,001,010,011,100,102,110,120,201		
	000,001,010,011,100,102,110,120,210		
	000,001,010,011,100,102,110,201,210		
	000,001,010,011,100,102,120,201,210		
	000,001,010,011,100,110,120,201,210		
	000,001,010,011,101,102,110,120,201		
	000,001,010,011,101,102,110,120,210		
	000,001,010,011,101,102,110,201,210		
	000,001,010,011,101,102,120,201,210		
	000,001,010,011,101,110,120,201,210		
	000,001,010,011,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101,102,110		
	001,010,011,012,021,100,101,102,120		
	001,010,011,012,021,100,101,102,201		
	001,010,011,012,021,100,101,102,210		
	001,010,011,012,021,100,101,110,120		
	001,010,011,012,021,100,101,110,201		
	001,010,011,012,021,100,101,110,210		
	001,010,011,012,021,100,101,120,201		
	001,010,011,012,021,100,101,120,210		
	001,010,011,012,021,100,101,201,210		
	001,010,011,012,021,100,102,110,120		
	001,010,011,012,021,100,102,110,201		
	001,010,011,012,021,100,102,110,210		
	001,010,011,012,021,100,102,120,201		
	001,010,011,012,021,100,102,120,210		
	001,010,011,012,021,100,102,201,210		
	001,010,011,012,021,100,110,120,201		
	001,010,011,012,021,100,110,120,210		
	001,010,011,012,021,100,110,201,210		
	001,010,011,012,021,100,120,201,210		
	001,010,011,012,021,101,102,110,120		
	001,010,011,012,021,101,102,110,201		
	001,010,011,012,021,101,102,110,210		
	001,010,011,012,021,101,102,120,201		
	001,010,011,012,021,101,102,120,210		
	001,010,011,012,021,101,102,201,210		
	001,010,011,012,021,101,110,120,201		
	001,010,011,012,021,101,110,120,210		
	001,010,011,012,021,101,110,201,210		
	001,010,011,012,021,101,120,201,210		
	001,010,011,012,021,102,110,120,201		
	001,010,011,012,021,102,110,120,210		
	001,010,011,012,021,102,110,201,210		
	001,010,011,012,021,102,120,201,210		
	001,010,011,012,021,110,120,201,210		
	001,010,011,012,100,101,102,110,120		
	001,010,011,012,100,101,102,110,201		
	001,010,011,012,100,101,102,110,210		
	001,010,011,012,100,101,102,120,201		
	001,010,011,012,100,101,102,120,210		
	001,010,011,012,100,101,102,201,210		
	001,010,011,012,100,101,110,120,201		
	001,010,011,012,100,101,110,120,210		
	001,010,011,012,100,101,110,201,210		
	001,010,011,012,100,101,120,201,210		
	001,010,011,012,100,102,110,120,201		
	001,010,011,012,100,102,110,120,210		
	001,010,011,012,100,102,110,201,210		
	001,010,011,012,100,102,120,201,210		
	001,010,011,012,100,110,120,201,210		
	001,010,011,012,101,102,110,120,201		
	001,010,011,012,101,102,110,120,210		
	001,010,011,012,101,102,110,201,210		
	001,010,011,012,101,102,120,201,210		
	001,010,011,012,101,110,120,201,210		
	001,010,011,012,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$x^2 + \frac{x}{1-x}$
13	001,011,012,021,100,101,102,110,120		
	001,011,012,021,100,101,102,110,201		
	001,011,012,021,100,101,102,110,210		
	001,011,012,021,100,101,102,120,201		
	001,011,012,021,100,101,102,120,210		
	001,011,012,021,100,101,102,201,210		
	001,011,012,021,100,101,110,120,201		
	001,011,012,021,100,101,110,120,210		
	001,011,012,021,100,101,110,201,210		
	001,011,012,021,100,101,120,201,210		
	001,011,012,021,100,102,110,120,201		
	001,011,012,021,100,102,110,120,210		
	001,011,012,021,100,102,110,201,210		
	001,011,012,021,100,102,120,201,210		
	001,011,012,021,100,110,120,201,210		
	001,011,012,100,101,102,110,120,201		
	001,011,012,100,101,102,110,120,210		
	001,011,012,100,101,102,110,201,210		
	001,011,012,100,101,102,120,201,210		
	001,011,012,100,101,110,120,201,210		
	001,011,012,100,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;\ 01 \rightsquigarrow 010$	
	000,001,011,021,100,101,102,110,120		
	000,001,011,021,100,101,102,120,201		
	000,001,011,021,100,101,102,120,210		
	000,001,011,021,100,101,110,120,201		
	000,001,011,021,100,101,110,120,210		
	000,001,011,021,100,101,120,201,210		
	000,001,011,021,100,102,110,120,201		
	000,001,011,021,100,102,110,120,210		
	000,001,011,021,100,102,120,201,210		
	000,001,011,021,100,110,120,201,210		

Continuation of Table 5			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,011,021,101,102,110,120,201 000,001,011,021,101,102,110,120,210 000,001,011,021,101,102,120,201,210 000,001,011,021,101,110,120,201,210 000,001,011,021,102,110,120,201,210 000,001,011,100,101,102,110,120,201 000,001,011,100,101,102,110,120,210 000,001,011,100,101,102,120,201,210 000,001,011,100,101,110,120,201,210 000,001,011,100,102,110,120,201,210 000,001,011,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 012$; $012 \rightsquigarrow 012$	$x^2 + x^3 + \frac{x}{1-x}$
14	000,001,010,021,100,101,102,110,120 000,001,010,021,100,101,102,110,201 000,001,010,021,100,101,102,110,210 000,001,010,021,100,101,102,120,201 000,001,010,021,100,101,102,120,210 000,001,010,021,100,101,102,201,210 000,001,010,021,100,101,110,120,201 000,001,010,021,100,101,110,120,210 000,001,010,021,100,101,110,201,210 000,001,010,021,100,101,120,201,210 000,001,010,021,100,102,110,120,201 000,001,010,021,100,102,110,120,210 000,001,010,021,100,102,110,201,210 000,001,010,021,100,102,120,201,210 000,001,010,021,100,110,120,201,210 000,001,010,021,101,102,110,120,201 000,001,010,021,101,102,110,120,210 000,001,010,021,101,102,110,201,210 000,001,010,021,101,102,120,201,210 000,001,010,021,101,110,120,201,210 000,001,010,021,102,110,120,201,210 000,001,010,100,101,102,110,120,201 000,001,010,100,101,102,110,120,210 000,001,010,100,101,102,110,201,210 000,001,010,100,101,102,120,201,210 000,001,010,100,101,110,120,201,210 000,001,010,100,102,110,120,201,210 000,001,010,101,102,110,120,201,210 000,001,011,021,100,101,102,110,201 000,001,011,021,100,101,102,110,210 000,001,011,021,100,101,102,201,210 000,001,011,021,100,101,110,201,210 000,001,011,021,100,102,110,201,210 000,001,011,021,101,102,110,201,210 000,001,011,100,101,102,110,201,210	$0 \rightsquigarrow 00, 0$	
	001,010,011,021,100,101,102,110,120 001,010,011,021,100,101,102,110,201 001,010,011,021,100,101,102,110,210 001,010,011,021,100,101,102,120,201 001,010,011,021,100,101,102,120,210 001,010,011,021,100,101,102,201,210 001,010,011,021,100,101,110,120,201 001,010,011,021,100,101,110,120,210 001,010,011,021,100,101,110,201,210 001,010,011,021,100,101,120,201,210 001,010,011,021,100,102,110,120,201 001,010,011,021,100,102,110,120,210 001,010,011,021,100,102,110,201,210 001,010,011,021,100,102,120,201,210 001,010,011,021,100,110,120,201,210 001,010,011,021,101,102,110,120,201 001,010,011,021,101,102,110,120,210 001,010,011,021,101,102,110,201,210 001,010,011,021,101,102,120,201,210 001,010,011,021,101,110,120,201,210 001,010,011,021,102,110,120,201,210 001,010,011,100,101,102,110,120,201 001,010,011,100,101,102,110,120,210 001,010,011,100,101,102,110,201,210 001,010,011,100,101,102,120,201,210 001,010,011,100,101,110,120,201,210 001,010,011,100,102,110,120,201,210 001,010,011,101,102,110,120,201,210 001,010,012,021,100,101,102,110,120 001,010,012,021,100,101,102,110,201 001,010,012,021,100,101,102,110,210 001,010,012,021,100,101,102,120,201 001,010,012,021,100,101,102,120,210 001,010,012,021,100,101,102,201,210 001,010,012,021,100,101,110,120,201 001,010,012,021,100,101,110,120,210 001,010,012,021,100,101,110,201,210 001,010,012,021,100,101,120,201,210 001,010,012,021,100,102,110,120,201 001,010,012,021,100,102,110,120,210 001,010,012,021,100,102,110,201,210 001,010,012,021,100,102,120,201,210		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,010,012,021,100,110,120,201,210		
	001,010,012,021,101,102,110,120,201		
	001,010,012,021,101,102,110,120,210		
	001,010,012,021,101,102,110,201,210		
	001,010,012,021,101,102,120,201,210		
	001,010,012,021,101,110,120,201,210		
	001,010,012,021,102,110,120,201,210		
	001,010,012,100,101,102,110,120,201		
	001,010,012,100,101,102,110,120,210		
	001,010,012,100,101,102,110,201,210		
	001,010,012,100,101,102,120,201,210		
	001,010,012,100,101,110,120,201,210		
	001,010,012,100,102,110,120,201,210		
	001,010,012,101,102,110,120,201,210		
	001,011,012,021,101,102,110,120,201		
	001,011,012,021,101,102,110,120,210		
	001,011,012,021,101,102,110,201,210		
	001,011,012,021,101,102,120,201,210		
	001,011,012,021,101,110,120,201,210		
	001,011,012,021,102,110,120,201,210		
	001,011,012,101,102,110,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
15	000,001,021,100,101,102,110,120,201		
	000,001,021,100,101,102,110,120,210		
	000,001,021,100,101,110,120,201,210		
	000,001,021,100,102,110,120,201,210		
	000,001,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	
	001,011,021,100,101,102,110,120,201		
	001,011,021,100,101,102,110,120,210		
	001,011,021,100,101,102,120,201,210		
	001,011,021,100,101,110,120,201,210		
	001,011,021,100,102,110,120,201,210		
	001,011,100,101,102,110,120,201,210		
	001,012,021,100,101,102,110,120,201		
	001,012,021,100,101,102,110,120,210		
	001,012,021,100,101,102,110,201,210		
	001,012,021,100,101,110,120,201,210		
	001,012,021,100,102,110,120,201,210		
	001,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00$	$x + x^3 + \frac{2x^2}{1-x}$
16	000,001,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 011, 012$; $011 \rightsquigarrow 00$; $012 \rightsquigarrow 00, 012$	$x + x^3 + x^4 + \frac{2x^2}{1-x}$
17	000,001,021,100,101,102,110,201,210		
	000,001,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow (00)^2, 01$	
	001,011,021,100,101,102,110,201,210		
	001,012,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 01$	
	001,011,021,101,102,110,120,201,210		
	001,012,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow (00)^2$	$x + 2x^2 + \frac{3x^3}{1-x}$
18	000,010,011,021,100,101,102,110,120		
	000,010,011,021,100,101,102,110,201		
	000,010,011,021,100,101,102,110,210		
	000,010,011,021,100,101,102,120,201		
	000,010,011,021,100,101,102,120,210		
	000,010,011,021,100,101,102,201,210		
	000,010,011,021,100,101,110,120,201		
	000,010,011,021,100,101,110,120,210		
	000,010,011,021,100,101,110,201,210		
	000,010,011,021,100,101,120,201,210		
	000,010,011,021,100,102,110,120,201		
	000,010,011,021,100,102,110,120,210		
	000,010,011,021,100,102,110,201,210		
	000,010,011,021,100,102,120,201,210		
	000,010,011,021,100,110,120,201,210		
	000,010,011,021,101,102,110,120,201		
	000,010,011,021,101,102,110,120,210		
	000,010,011,021,101,102,110,201,210		
	000,010,011,021,101,102,120,201,210		
	000,010,011,021,101,110,120,201,210		
	000,010,011,021,102,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 01$	
	001,010,021,100,101,102,110,120,201		
	001,010,021,100,101,102,110,120,210		
	001,010,021,100,101,102,110,201,210		
	001,010,021,100,101,102,120,201,210		
	001,010,021,100,101,110,120,201,210		
	001,010,021,100,102,110,120,201,210		
	001,010,021,101,102,110,120,201,210		
	001,010,100,101,102,110,120,201,210	$0 \rightsquigarrow 0, 00$; $00 \rightsquigarrow 00$	
	010,011,012,021,100,101,102,110,120		
	010,011,012,021,100,101,102,110,201		
	010,011,012,021,100,101,102,110,210		
	010,011,012,021,100,101,102,120,201		
	010,011,012,021,100,101,102,120,210		
	010,011,012,021,100,101,102,201,210		
	010,011,012,021,100,101,110,120,201		
	010,011,012,021,100,101,110,120,210		
	010,011,012,021,100,101,110,201,210		
	010,011,012,021,100,101,120,201,210		
	010,011,012,021,100,102,110,120,201		
	010,011,012,021,100,102,110,120,210		
	010,011,012,021,100,102,110,201,210		
	010,011,012,021,100,102,120,201,210		
	010,011,012,021,100,110,120,201,210		
	010,011,012,021,101,102,110,120,201		
	010,011,012,021,101,102,110,120,210		
	010,011,012,021,101,102,110,201,210		
	010,011,012,021,101,102,120,201,210		
	010,011,012,021,101,110,120,201,210		

Continuation of Table 5

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	010,011,012,021,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $a_m = 0^m$	$\frac{x}{(1-x)^2}$
19	001,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00, 012$; $012 \rightsquigarrow 00, 012$	
	000,011,021,100,102,110,120,201,210 000,011,021,101,102,110,120,201,210 000,011,021,100,101,102,110,120,201 000,011,021,100,101,102,110,120,210 000,011,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	$\frac{x(1+x^2(1-x)^2)}{(1-x)^2}$
20	000,010,011,100,101,102,110,120,201 000,010,011,100,101,102,110,120,210 000,010,011,100,101,102,120,201,210 000,010,011,100,102,110,120,201,210 000,010,011,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
21	000,011,021,100,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 01$; $002 \rightsquigarrow 002$	
	000,011,100,101,102,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 010, 012$; $012 \rightsquigarrow 012$	$\frac{x(1+x^2-x^3)}{(1-x)^2}$
22	000,010,011,100,101,102,110,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 002$	$\frac{x(1+x^3)}{(1-x)^2}$
23	000,011,021,100,101,110,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00, 002$; $002 \rightsquigarrow 002$	
	011,012,021,100,101,102,110,120,201 011,012,021,100,101,102,110,120,210 011,012,021,100,101,102,110,201,210 011,012,021,100,101,102,120,201,210 011,012,021,100,101,110,120,201,210 011,012,021,100,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010$, $a_m = 0^m$	$\frac{x(1+x^2)}{(1-x)^2}$
24	010,011,012,100,101,102,110,120,210 010,011,012,100,101,102,110,201,210 010,011,012,100,101,102,120,201,210 010,011,012,100,101,110,120,201,210 010,011,012,100,102,110,120,201,210 010,011,012,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (01)^{m-1}$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
25	010,012,021,100,101,102,110,120,201 010,012,021,100,101,102,110,120,210 010,012,021,100,101,102,110,201,210 010,012,021,100,101,102,120,201,210 010,012,021,100,101,110,120,201,210 010,012,021,100,102,110,120,201,210 010,012,021,101,102,110,120,201,210 011,012,021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 01$, $a_m = 0^m$	
	011,012,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (010)^m$, $a_m = 0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^2)}{(1-x)^3}$
26	012,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$; $01 \rightsquigarrow 010, 011$; $011 \rightsquigarrow 011$, $a_m = 0^m$	$\frac{x(1-x(1-x)^2)}{(1-x)^3}$
27	000,010,011,100,101,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 0$; $01 \rightsquigarrow 01$	
	010,011,012,100,101,102,110,120,201	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^{m-1}$, $a{=}0^m$, $b_m = 0^m m$	$\frac{x(1+x)}{1-x-x^2}$
28	010,012,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow (0021)^{m-1}, b_1$, $a{=}0^m$, $b_m = 0^m m$	$\frac{x(1-x+x^2+x^3)}{(1-x)^3}$
29	010,011,021,100,101,102,110,120,201 010,011,021,100,101,102,110,120,210 010,011,021,100,101,102,110,201,210 010,011,021,100,101,102,120,201,210 010,011,021,100,101,110,120,201,210 010,011,021,100,102,110,120,201,210 010,011,021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow \{b_i\}_{i=1}^m$, $a_m = 0^m$, $b_m = 0^m 1$	$\frac{x}{1-2x}$
30	011,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow 010, \{c_i\}_{i=1}^m$; $c_m \rightsquigarrow \{c_i\}_{i=1}^m$, $a_m = 0^m$, $b_m = 0^m 1$, $c_m = 0^m 12$	$\frac{x(1-2x+2x^2-2x^3)}{(1-x)^2(1-2x)}$
31	010,011,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_{m,i}\}_{i=1}^m$; $b_{m,j} \rightsquigarrow$ $(0021)^{j-1}, \{b_{m+1-j,i}\}_{i=1}^{m+1-j}$, $a_m = 0^m$, $b_{m,j} = a_m j$	$\frac{x((1-x)^2+x^3)}{(1-x)^2(1-2x)}$
32	000,010,021,100,101,102,110,120,201 000,010,021,100,101,102,110,120,210 000,010,021,100,101,102,110,201,210		

Continuation of Table 5			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,010,021,100,101,102,120,201,210 000,010,021,100,101,110,120,201,210 000,010,021,100,102,110,120,201,210 000,010,021,101,102,110,120,201,210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^{m}$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-2x^2-\sqrt{1-2x-3x^2}}{2x^2}$
	000,021,100,101,102,110,120,201,210	$a_0 \rightsquigarrow b_0, 01$; $01 \rightsquigarrow 010, b_0, c$; $c \rightsquigarrow b_0, c$; $a_m \rightsquigarrow b_m, \{a_i\}_{i=1}^{m}, c$; $b_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}, c$, $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$, $c = 002$	
33	000,010,100,101,102,110,120,201,210●		Theorem 22
34	010,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow \{a_i\}_{i=1}^{m+1}$, $a_m = 0^m$	$\frac{1-2x-\sqrt{1-4x}}{2x}$
End of Table 5			

Theorem 22. *We have*

$$F_{\{000,010,100,101,102,110,120,201,210\}}(x) = \frac{2-3x-3x^2+(x-2)\sqrt{1-2x-3x^2}}{2x^2(1+x)}.$$

Proof. Let $a_{mj} = 0^2 1^2 \cdots (m-1)^2 j$ with $m \le j \le 2m$ and $b_m = 0^2 1^2 \cdots m^2$. Then the succession rules of the generating tree $\mathcal{T}(B)$ are given by

$$\begin{aligned} a_{mj} &\rightsquigarrow c^{j-m}, b_{2m-j}, a_{2m-j,2m-j}, \ldots, a_{2m-j,4m-j}, \\ b_m &\rightsquigarrow a_{m+1,m+1}, \ldots, a_{m+1,2m+2}. \end{aligned}$$

Note that there are no children for vertex c. Define $A_{m,j}(x)$ (respectively, $B_m(x)$) to be the generating function for the number of nodes at level $n \ge 0$ for the subtree of $\mathcal{T}(B; a_{m,j})$ (respectively, $\mathcal{T}(B; b_m)$), where its root stays at level 0. Thus,

$$\begin{aligned} A_{m,j}(x) &= x + (j-m)x^2 + xB_{2m-j}(x) + x\sum_{i=2m-j}^{4m-2j} A_{2m-j,s}(x), \\ B_m(x) &= x + x\sum_{i=m+1}^{2m+2} A_{m+1,i}(x). \end{aligned}$$

Let $A_m(x) = \sum_{i=m}^{2m} A_{m,j}(x)$. Then

$$\begin{aligned} A_m(x) &= (m+1)x + \binom{m+1}{2}x^2 + x\sum_{j=0}^{m}(B_j(x) + A_j(x)), \\ B_m(x) &= x + xA_{m+1}(x). \end{aligned}$$

Define $A(v) = \sum_{m\ge1} A_m(x)v^m$ and $B(v) = \sum_{m\ge1} B_m(x)v^m$. Then, the above recurrence can be written as

$$\begin{aligned} A(v) &= \frac{x(vx-v+1)}{(1-v)^3} + \frac{x}{1-v}(A(v)+B(v)), \\ B(v) &= \frac{x}{1-v} + \frac{x}{v}(A(v)-A(0)). \end{aligned}$$

Thus,

$$A(v) = \frac{x(vx - v + 1)}{(1-v)^3} + \frac{x}{1-v}(A(v) + \frac{x}{1-v} + \frac{x}{v}(A(v) - A(0))).$$

By taking $v = \frac{1-x-\sqrt{1-2x-3x^2}}{2}$, we obtain

$$A(0) = \frac{2 - 3x - 3x^2 + (x-2)\sqrt{1-2x-3x^2}}{2x^2(1+x)},$$

as required. □

Table 6: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 10$.

Beginning of Table 6			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100,101,102,110 000,001,010,011,012,021,100,101,102,120 000,001,010,011,012,021,100,101,102,201 000,001,010,011,012,021,100,101,102,210 000,001,010,011,012,021,100,101,110,120 000,001,010,011,012,021,100,101,110,201 000,001,010,011,012,021,100,101,110,210 000,001,010,011,012,021,100,101,120,201 000,001,010,011,012,021,100,101,120,210 000,001,010,011,012,021,100,101,201,210 000,001,010,011,012,021,100,102,110,120 000,001,010,011,012,021,100,102,110,201 000,001,010,011,012,021,100,102,110,210 000,001,010,011,012,021,100,102,120,201 000,001,010,011,012,021,100,102,120,210 000,001,010,011,012,021,100,102,201,210 000,001,010,011,012,021,100,110,120,201 000,001,010,011,012,021,100,110,120,210 000,001,010,011,012,021,100,110,201,210 000,001,010,011,012,021,100,120,201,210 000,001,010,011,012,021,101,102,110,120 000,001,010,011,012,021,101,102,110,201 000,001,010,011,012,021,101,102,110,210 000,001,010,011,012,021,101,102,120,201 000,001,010,011,012,021,101,102,120,210 000,001,010,011,012,021,101,102,201,210 000,001,010,011,012,021,101,110,120,201 000,001,010,011,012,021,101,110,120,210 000,001,010,011,012,021,101,110,201,210 000,001,010,011,012,021,101,120,201,210 000,001,010,011,012,021,102,110,120,201 000,001,010,011,012,021,102,110,120,210 000,001,010,011,012,021,102,110,201,210 000,001,010,011,012,021,102,120,201,210 000,001,010,011,012,021,110,120,201,210 000,001,010,011,012,100,101,102,110,120 000,001,010,011,012,100,101,102,110,201 000,001,010,011,012,100,101,102,110,210 000,001,010,011,012,100,101,102,120,201 000,001,010,011,012,100,101,102,120,210 000,001,010,011,012,100,101,102,201,210 000,001,010,011,012,100,101,110,120,201 000,001,010,011,012,100,101,110,120,210 000,001,010,011,012,100,101,110,201,210 000,001,010,011,012,100,101,120,201,210 000,001,010,011,012,100,102,110,120,201 000,001,010,011,012,100,102,110,120,210 000,001,010,011,012,100,102,110,201,210 000,001,010,011,012,100,102,120,201,210 000,001,010,011,012,100,110,120,201,210 000,001,010,011,012,101,102,110,120,201 000,001,010,011,012,101,102,110,120,210 000,001,010,011,012,101,102,110,201,210 000,001,010,011,012,101,102,120,201,210 000,001,010,011,012,101,110,120,201,210 000,001,010,011,012,102,110,120,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101,102,110,120 000,001,010,012,021,100,101,102,110,201 000,001,010,012,021,100,101,102,110,210 000,001,010,012,021,100,101,102,120,201 000,001,010,012,021,100,101,102,120,210 000,001,010,012,021,100,101,102,201,210 000,001,010,012,021,100,101,110,120,201 000,001,010,012,021,100,101,110,120,210 000,001,010,012,021,100,101,110,201,210 000,001,010,012,021,100,101,120,201,210 000,001,010,012,021,100,102,110,120,201 000,001,010,012,021,100,102,110,120,210 000,001,010,012,021,100,102,110,201,210		

Continuation of Table 6

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,012,021,100,102,120,201,210		
	000,001,010,012,021,100,110,120,201,210		
	000,001,010,012,021,101,102,110,120,201		
	000,001,010,012,021,101,102,110,120,210		
	000,001,010,012,021,101,102,110,201,210		
	000,001,010,012,021,101,102,120,201,210		
	000,001,010,012,021,101,110,120,201,210		
	000,001,010,012,021,102,110,120,201,210		
	000,001,010,012,100,101,102,110,120,201		
	000,001,010,012,100,101,102,110,120,210		
	000,001,010,012,100,101,102,110,201,210		
	000,001,010,012,100,101,102,120,201,210		
	000,001,010,012,100,101,110,120,201,210		
	000,001,010,012,100,102,110,120,201,210		
	000,001,010,012,101,102,110,120,201,210		
	000,001,011,012,021,100,101,102,110,120		
	000,001,011,012,021,100,101,102,110,201		
	000,001,011,012,021,100,101,102,110,210		
	000,001,011,012,021,100,101,102,120,201		
	000,001,011,012,021,100,101,102,120,210		
	000,001,011,012,021,100,101,102,201,210		
	000,001,011,012,021,100,101,110,120,201		
	000,001,011,012,021,100,101,110,120,210		
	000,001,011,012,021,100,101,110,201,210		
	000,001,011,012,021,100,101,120,201,210		
	000,001,011,012,021,100,102,110,120,201		
	000,001,011,012,021,100,102,110,120,210		
	000,001,011,012,021,100,102,110,201,210		
	000,001,011,012,021,100,102,120,201,210		
	000,001,011,012,021,100,110,120,201,210		
	000,001,011,012,021,101,102,110,120,201		
	000,001,011,012,021,101,102,110,120,210		
	000,001,011,012,021,101,102,110,201,210		
	000,001,011,012,021,101,102,120,201,210		
	000,001,011,012,021,101,110,120,201,210		
	000,001,011,012,021,102,110,120,201,210		
	000,001,011,012,100,101,102,110,120,201		
	000,001,011,012,100,101,102,110,120,210		
	000,001,011,012,100,101,102,110,201,210		
	000,001,011,012,100,101,102,120,201,210		
	000,001,011,012,100,101,110,120,201,210		
	000,001,011,012,100,102,110,120,201,210		
	000,001,011,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x + 2x^2 + x^3$
3	000,001,012,021,100,101,102,110,120,201		
	000,001,012,021,100,101,102,110,120,210		
	000,001,012,021,100,101,102,110,201,210		
	000,001,012,021,100,101,110,120,201,210		
	000,001,012,021,100,102,110,120,201,210		
	000,001,012,021,101,102,110,120,201,210		
	000,001,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000,010,011,012,021,100,101,102,110,120		
	000,010,011,012,021,100,101,102,110,201		
	000,010,011,012,021,100,101,102,110,210		
	000,010,011,012,021,100,101,102,120,201		
	000,010,011,012,021,100,101,102,120,210		
	000,010,011,012,021,100,101,102,201,210		
	000,010,011,012,021,100,101,110,120,201		
	000,010,011,012,021,100,101,110,120,210		
	000,010,011,012,021,100,101,110,201,210		
	000,010,011,012,021,100,101,120,201,210		
	000,010,011,012,021,100,102,110,120,201		
	000,010,011,012,021,100,102,110,120,210		
	000,010,011,012,021,100,102,110,201,210		
	000,010,011,012,021,100,102,120,201,210		
	000,010,011,012,021,100,110,120,201,210		
	000,010,011,012,021,101,102,110,120,201		
	000,010,011,012,021,101,102,110,120,210		
	000,010,011,012,021,101,102,110,201,210		
	000,010,011,012,021,101,102,120,201,210		
	000,010,011,012,021,101,110,120,201,210		
	000,010,011,012,021,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
4	000,001,012,021,100,101,102,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 011;$ $011 \rightsquigarrow 00$	
	000,010,011,012,100,101,102,110,120,201		
	000,010,011,012,100,101,102,110,120,210		
	000,010,011,012,100,101,102,110,201,210		
	000,010,011,012,100,101,102,120,201,210		
	000,010,011,012,100,101,110,120,201,210		
	000,010,011,012,100,102,110,120,201,210		
	000,010,011,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 002;$ $002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
5	000,011,012,021,100,101,102,110,120,201		
	000,011,012,021,100,101,102,110,120,210		
	000,011,012,021,100,101,102,110,201,210		
	000,011,012,021,100,101,102,120,201,210		
	000,011,012,021,100,101,110,120,201,210		
	000,011,012,021,100,102,110,120,201,210		
	000,011,012,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (001)^2;$ $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
6	000,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 01;$ $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3 + x^4$
7	000,010,012,021,100,101,102,110,120,201		
	000,010,012,021,100,101,102,110,120,210		
	000,010,012,021,100,101,102,110,201,210		
	000,010,012,021,100,101,102,120,201,210		
	000,010,012,021,100,101,110,120,201,210		
	000,010,012,021,100,102,110,120,201,210		
	000,010,012,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2;$ $01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$

Continuation of Table 6

No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
8	000,010,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 01, 002$; $01 \rightsquigarrow 011$; $002 \rightsquigarrow (011)^2$	$x + 2x^2 + 3x^3 + 3x^4$
9	000,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow (001)^2$; $01 \rightsquigarrow (010)^2$; $001 \rightsquigarrow 010$	$x + 2x^2 + 4x^3 + 2x^4$
10	000,001,010,011,021,100,101,102,110,120		
	000,001,010,011,021,100,101,102,110,201		
	000,001,010,011,021,100,101,102,110,210		
	000,001,010,011,021,100,101,102,120,201		
	000,001,010,011,021,100,101,102,120,210		
	000,001,010,011,021,100,101,102,201,210		
	000,001,010,011,021,100,101,110,120,201		
	000,001,010,011,021,100,101,110,120,210		
	000,001,010,011,021,100,101,110,201,210		
	000,001,010,011,021,100,101,120,201,210		
	000,001,010,011,021,100,102,110,120,201		
	000,001,010,011,021,100,102,110,120,210		
	000,001,010,011,021,100,102,110,201,210		
	000,001,010,011,021,100,102,120,201,210		
	000,001,010,011,021,100,110,120,201,210		
	000,001,010,011,021,101,102,110,120,201		
	000,001,010,011,021,101,102,110,120,210		
	000,001,010,011,021,101,102,110,201,210		
	000,001,010,011,021,101,102,120,201,210		
	000,001,010,011,021,101,110,120,201,210		
	000,001,010,011,021,102,110,120,201,210		
	000,001,010,011,100,101,102,110,120,201		
	000,001,010,011,100,101,102,110,120,210		
	000,001,010,011,100,101,102,110,201,210		
	000,001,010,011,100,101,102,120,201,210		
	000,001,010,011,100,101,110,120,201,210		
	000,001,010,011,100,102,110,120,201,210		
	000,001,010,011,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101,102,110,120		
	001,010,011,012,021,100,101,102,110,201		
	001,010,011,012,021,100,101,102,110,210		
	001,010,011,012,021,100,101,102,120,201		
	001,010,011,012,021,100,101,102,120,210		
	001,010,011,012,021,100,101,102,201,210		
	001,010,011,012,021,100,101,110,120,201		
	001,010,011,012,021,100,101,110,120,210		
	001,010,011,012,021,100,101,110,201,210		
	001,010,011,012,021,100,101,120,201,210		
	001,010,011,012,021,100,102,110,120,201		
	001,010,011,012,021,100,102,110,120,210		
	001,010,011,012,021,100,102,110,201,210		
	001,010,011,012,021,100,102,120,201,210		
	001,010,011,012,021,100,110,120,201,210		
	001,010,011,012,021,101,102,110,120,201		
	001,010,011,012,021,101,102,110,120,210		
	001,010,011,012,021,101,102,110,201,210		
	001,010,011,012,021,101,102,120,201,210		
	001,010,011,012,021,101,110,120,201,210		
	001,010,011,012,021,102,110,120,201,210		
	001,010,011,012,100,101,102,110,120,201		
	001,010,011,012,100,101,102,110,120,210		
	001,010,011,012,100,101,102,110,201,210		
	001,010,011,012,100,101,102,120,201,210		
	001,010,011,012,100,101,110,120,201,210		
	001,010,011,012,100,102,110,120,201,210		
	001,010,011,012,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$	$x + 2x^2 + \frac{x^3}{1-x}$
11	000,001,011,021,100,101,102,110,120,201		
	000,001,011,021,100,101,102,110,120,210		
	000,001,011,021,100,101,102,120,201,210		
	000,001,011,021,100,101,110,120,201,210		
	000,001,011,021,100,102,110,120,201,210		
	000,001,011,021,101,102,110,120,201,210		
	000,001,011,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow 00, 012$; $012 \rightsquigarrow 012$	
	001,011,012,021,100,101,102,110,120,201		
	001,011,012,021,100,101,102,110,120,210		
	001,011,012,021,100,101,102,110,201,210		
	001,011,012,021,100,101,102,120,201,210		
	001,011,012,021,100,101,110,120,201,210		
	001,011,012,021,100,102,110,120,201,210		
	001,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010$	$x + 2x^2 + 2x^3 + \frac{x^4}{1-x}$
12	000,001,011,021,100,101,102,110,201,210		
	000,001,010,021,100,101,102,110,120,201		
	000,001,010,021,100,101,102,110,120,210		
	000,001,010,021,100,101,102,110,201,210		
	000,001,010,021,100,101,102,120,201,210		
	000,001,010,021,100,101,110,120,201,210		
	000,001,010,021,100,102,110,120,201,210		
	000,001,010,021,101,102,110,120,201,210		
	000,001,010,100,101,102,110,120,201,210	$0 \rightsquigarrow 0, 00$	
	001,010,011,021,100,101,102,110,120,201		
	001,010,011,021,100,101,102,110,120,210		
	001,010,011,021,100,101,102,110,201,210		
	001,010,011,021,100,101,102,120,201,210		
	001,010,011,021,100,101,110,120,201,210		
	001,010,011,021,100,102,110,120,201,210		
	001,010,011,021,101,102,110,120,201,210		
	001,010,011,100,101,102,110,120,201,210		
	001,010,012,021,100,101,102,110,120,201		
	001,010,012,021,100,101,102,110,120,210		
	001,010,012,021,100,101,102,110,201,210		
	001,010,012,021,100,101,102,120,201,210		
	001,010,012,021,100,101,110,120,201,210		
	001,010,012,021,100,102,110,120,201,210		
	001,010,012,021,101,102,110,120,201,210		
	001,010,012,100,101,102,110,120,201,210		

Continuation of Table 6			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	001,011,012,021,101,102,110,120,201,210	$0 \rightsquigarrow (00)^2$; $00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
13	000,001,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $01 \rightsquigarrow (00)^2, 012$; $012 \rightsquigarrow 00, 012$	
	001,011,021,100,101,102,110,120,201,210 001,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00$; $01 \rightsquigarrow 010, 00$	$x + 2x^2 + 3x^3 + \frac{2x^4}{1-x}$
14	000,010,011,021,100,101,102,110,120,201 000,010,011,021,100,101,102,110,120,210 000,010,011,021,100,101,102,110,201,210 000,010,011,021,100,101,102,120,201,210 000,010,011,021,100,101,110,120,201,210 000,010,011,021,100,102,110,120,201,210 000,010,011,021,101,102,110,120,201,210	$0 \rightsquigarrow 0, 01$; $01 \rightsquigarrow 01$	
	001,010,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 0$; $00 \rightsquigarrow 00$	
	010,011,012,021,100,101,102,110,120,201 010,011,012,021,100,101,102,110,120,210 010,011,012,021,100,101,102,110,201,210 010,011,012,021,100,101,102,120,201,210 010,011,012,021,100,101,110,120,201,210 010,011,012,021,100,102,110,120,201,210 010,011,012,021,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m$, $a_m = 0^m$	$\frac{x}{(1-x)^2}$
15	000,011,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 010, 002$; $002 \rightsquigarrow 002$	$\frac{x(1+x^2-2x^3+x^4)}{(1-x)^2}$
16	000,010,011,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$; $00 \rightsquigarrow 00, 002$; $01 \rightsquigarrow 01$; $002 \rightsquigarrow 0021, 01$	$\frac{x(1+x^3-x^4)}{(1-x)^2}$
17	011,012,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b^m$; $b \rightsquigarrow 010$; $a_m = 0^m$, $b = 01$	$\frac{x(1+x^2)}{(1-x)^2}$
18	010,011,012,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_1^{m-1}$; $a_m = 0^m$, $b_m = a_m m$	$\frac{x(1-x+x^3)}{(1-x)^3}$
19	010,012,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, b^m$; $b \rightsquigarrow b$; $a_m = 0^m$, $b = 01$	$\frac{x(1-x+x^2)}{(1-x)^3}$
20	010,011,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, \{b_i\}_{i=1}^m$; $b_m \rightsquigarrow b_m, \ldots, b_1$; $a_m = 0^m$, $b_m = 0^m 1$	$\frac{x}{1-2x}$
21	000,010,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow b_m, \{a_i\}_{i=0}^m$; $b_m \rightsquigarrow \{a_i\}_{i=0}^{m+1}$; $a_m = 0^2 \cdots (m-1)^2 m$, $b_m = a_m m$	$\frac{1-x-2x^2-\sqrt{1-2x-3x^2}}{2x^2}$
End of Table 6			

Table 7: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 11$.

Beginning of Table 7			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100,101,102,110,120 000,001,010,011,012,021,100,101,102,110,201 000,001,010,011,012,021,100,101,102,110,210 000,001,010,011,012,021,100,101,102,120,201 000,001,010,011,012,021,100,101,102,120,210 000,001,010,011,012,021,100,101,102,201,210 000,001,010,011,012,021,100,101,110,120,201 000,001,010,011,012,021,100,101,110,120,210 000,001,010,011,012,021,100,101,110,201,210 000,001,010,011,012,021,100,101,120,201,210 000,001,010,011,012,021,100,102,110,120,201 000,001,010,011,012,021,100,102,110,120,210 000,001,010,011,012,021,100,102,110,201,210 000,001,010,011,012,021,100,102,120,201,210 000,001,010,011,012,021,100,110,120,201,210 000,001,010,011,012,021,101,102,110,120,201 000,001,010,011,012,021,101,102,110,120,210 000,001,010,011,012,021,101,102,110,201,210 000,001,010,011,012,021,101,102,120,201,210 000,001,010,011,012,021,101,110,120,201,210 000,001,010,011,012,021,102,110,120,201,210 000,001,010,011,012,100,101,102,110,120,201 000,001,010,011,012,100,101,102,110,120,210 000,001,010,011,012,100,101,102,110,201,210 000,001,010,011,012,100,101,102,120,201,210 000,001,010,011,012,100,101,110,120,201,210 000,001,010,011,012,100,102,110,120,201,210 000,001,010,011,012,101,102,110,120,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101,102,110,120,201 000,001,010,012,021,100,101,102,110,120,210 000,001,010,012,021,100,101,102,110,201,210 000,001,010,012,021,100,101,102,120,201,210 000,001,010,012,021,100,101,110,120,201,210 000,001,010,012,021,100,102,110,120,201,210		

Continuation of Table 7			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
	000,001,010,012,021,101,102,110,120,201,210 000,001,010,012,100,101,102,110,120,201,210 000,001,011,012,021,100,101,102,110,120,201 000,001,011,012,021,100,101,102,110,120,210 000,001,011,012,021,100,101,102,110,201,210 000,001,011,012,021,100,101,102,120,201,210 000,001,011,012,021,100,101,110,120,201,210 000,001,011,012,021,100,102,110,120,201,210 000,001,011,012,021,101,102,110,120,201,210 000,001,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x + 2x^2 + x^3$
3	000,001,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow (00)^2$	
	000,010,011,012,021,100,101,102,110,120,201 000,010,011,012,021,100,101,102,110,120,210 000,010,011,012,021,100,101,102,110,201,210 000,010,011,012,021,100,101,102,120,201,210 000,010,011,012,021,100,101,110,120,201,210 000,010,011,012,021,100,102,110,120,201,210 000,010,011,012,021,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2$	$x + 2x^2 + 2x^3$
4	000,010,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 001, 002;$ $002 \rightsquigarrow 01$	$x + 2x^2 + 2x^3 + x^4$
5	000,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (001)^2;$ $01 \rightsquigarrow 001$	$x + 2x^2 + 3x^3$
6	000,010,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow (01)^2;$ $01 \rightsquigarrow 011$	$x + 2x^2 + 3x^3 + 2x^4$
7	000,001,010,011,021,100,101,102,110,120,201 000,001,010,011,021,100,101,102,110,120,210 000,001,010,011,021,100,101,102,110,201,210 000,001,010,011,021,100,101,102,120,201,210 000,001,010,011,021,100,101,110,120,201,210 000,001,010,011,021,100,102,110,120,201,210 000,001,010,011,021,101,102,110,120,201,210 000,001,010,011,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101,102,110,120,201 001,010,011,012,021,100,101,102,110,120,210 001,010,011,012,021,100,101,102,110,201,210 001,010,011,012,021,100,101,102,120,201,210 001,010,011,012,021,100,101,110,120,201,210 001,010,011,012,021,100,102,110,120,201,210 001,010,011,012,021,101,102,110,120,201,210 001,010,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$x + 2x^2 + \frac{x^3}{1-x}$
8	000,001,011,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00, 012;$ $012 \rightsquigarrow 012$	
	001,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00;$ $01 \rightsquigarrow 010$	$x + 2x^2 + 2x^3 + \frac{x^4}{1-x}$
9	000,001,010,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 0, 00$	
	001,010,011,021,100,101,102,110,120,201,210 001,010,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow (00)^2;\ 00 \rightsquigarrow 00$	$x + \frac{2x^2}{1-x}$
10	000,010,011,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 0, 01;\ 01 \rightsquigarrow 01$	
	010,011,012,021,100,101,102,110,120,201,210	$a_m \rightsquigarrow a_{m+1}, (01)^m;$ $a_m = 0^m$	$\frac{x}{(1-x)^2}$
End of Table 7			

Table 8: Succession rules for the generating trees $\mathcal{T}(B)$ and generating functions $F_B(x)$, where $B \subset \mathcal{P}_3$ and $|B| = 12, 13$.

Beginning of Table 8			
No.	B	Rules of $\mathcal{T}(B)$	$F_B(x)$
1	000,001,010,011,012,021,100,101,102,110,120,201 000,001,010,011,012,021,100,101,102,110,120,210 000,001,010,011,012,021,100,101,102,110,201,210 000,001,010,011,012,021,100,101,102,120,201,210 000,001,010,011,012,021,100,101,110,120,201,210 000,001,010,011,012,021,100,102,110,120,201,210 000,001,010,011,012,021,101,102,110,120,201,210 000,001,010,011,012,100,101,102,110,120,201,210	$0 \rightsquigarrow (00)^2$	$x + 2x^2$
2	000,001,010,012,021,100,101,102,110,120,201,210 000,001,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 00$	$x + 2x^2 + x^3$
3	000,010,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 01, 01$	$x + 2x^2 + 2x^3$
4	000,001,010,011,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 01 \rightsquigarrow 01$	
	001,010,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01;\ 00 \rightsquigarrow 00$	$x + 2x^2 + \frac{x^3}{1-x}$
1	000,001,010,011,012,021,100,101,102,110,120,201,210	$0 \rightsquigarrow 00, 01$	$x + 2x^2$
End of Table 8			

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