CORRIGENDUM TO NO NEW GOORMAGHTIGH PRIMES UP TO 10^{700}, #A98 OF VOLUME 24

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1. Introduction

The article as published has two types of errors. First, the computation did not search the appropriate parameters required to establish the result; it also contained an indexing error. Second, the statement of the Goormaghtigh Conjecture swapped the two exponents, which led to an incorrect proof of Theorem 2 and thus of Corollary 1. The second error was noted in the Math Review of the published article, and the first set of errors was discovered during a subsequent audit of the paper by GPT-5.6 Sol. This corrigendum repairs both defects.

The corrected statement of the Goormaghtigh Conjecture is as follows.

Conjecture 1 (Goormaghtigh Conjecture)**.** *The only solutions to*

$$N = \frac{x^m - 1}{x - 1} = \frac{y^n - 1}{y - 1} \tag{1}$$

with integers $y > x \geq 2$, $m > n > 2$ *are* $N = 31$ *and* $N = 8191$.

2. Corrected Computation

The corrected computation differs from the original in the following ways.

1. The original computation checked Mersenne primes, but not other Mersenne numbers with prime exponents. The latter was not required for the paper's titular claim, but it was necessary for the stronger statement that there are no new Goormaghtigh numbers below 10^{700} with both exponents prime. The relevant Mersenne numbers have now been checked.

2. The original computation used an unjustified bound on n dependent on x; the new computation does not.

3. The original computation checked all $x < 10^{700/m}$. All $x < 10^{700/(m-1)}$ need to be checked; that has now been done.

4. The corrected computation uses Theorem 3 of Bennett-Garbuz-Marten to restrict to exponents n where $5(n-1) < 700$.

5. The corrected computation uses Theorem 6 of Bennett-Garbuz-Marten to eliminate exponent pairs where $m - 1 = 3(n-1)$. The original computation transposed m and n and thus failed to eliminate any pairs.

6. The original computation used incorrect indices when performing the lookup to the precomputation. The corrected computation does not.

The corrected $x \geq 3$ computation took approximately 70 minutes using 112 processes on two 64-core AMD EPYC 7742 processors.

Lean was used to verify that the correct bounds were used in the search design. It did not verify the proofs from the cited papers nor the C code itself.

3. Corrected Proof of Theorem 2

Theorem 2. *Assuming the abc conjecture, there are only finitely many Goormaghtigh numbers where neither representation is of length three or four.*

Proof. Rewrite (1) as

$$(x^m - 1)(y - 1) = (y^n - 1)(x - 1).$$

Rearranging,

$$y^n(x-1) + (y-x) = x^m(y-1).$$

Let

$$g = \gcd(y^n(x-1), y-x, x^m(y-1)).$$

Let $a = y^n(x-1)/g$, $b = (y-x)/g$, and $c = x^m(y-1)/g$. Then $\gcd(a,b,c) = 1$ and we have $a + b = c$.

Further

$$\operatorname{rad}(abc) \leq x(x-1)y(y-1)(y-x) < x^2y^3,$$

and $c \geq x^m$.

For a Goormaghtigh number N, we have $x^{m-1} < N < 2x^{m-1}$ and $y^{n-1} < N < 2y^{n-1}$; therefore $y^{n-1} < 2x^{m-1}$. Combined with the fact that $3/(n-1) \leq 3/4$, we get

$$y^3 = (y^{n-1})^{3/(n-1)} < 2^{3/4}x^{3(m-1)/4},$$

and thus

$$\operatorname{rad}(abc) \leq 2^{3/4}x^{5/4+3m/4}.$$

Choose $\epsilon = 1/25$. We have $m > n \geq 5$, so $m \geq 6$, and

$$\operatorname{rad}(abc)^{1+\epsilon} < 2^{39/50}x^{(65+39m)/50}.$$

For $m \geq 10$, the right hand side is less than x^m and thus c for all $x \geq 2$. For $9 \geq m \geq 6$, it is true for sufficiently large x. Therefore, there are only finitely many solutions to the Goormaghtigh equation of length 5 or greater that do not give a triple satisfying $c > \mathrm{rad}(abc)^{1+\epsilon}$. By the abc conjecture, there are only finitely many such triples. It is easy to check that each triple corresponds to finitely many Goormaghtigh numbers.

Therefore there are only finitely many Goormaghtigh numbers with $n \geq 5$. $\square$

To provide readers with evidence that there are no defects in the corrected proof, the author has produced a Lean proof which verifies correctness of Theorem 2 subject to the abc conjecture.

Acknowledgement. The author thanks Christian Ballot for noting the error in the proof of Theorem 2 in his Math Review of the published article.

AI Disclosure. The author used Claude Opus 4.8 (Anthropic) and GPT-5.6 Sol (OpenAI) in fixing the proof and computations. The author has reviewed the corrected versions and takes full responsibility for the work. This corrigendum was entirely written by him.

Repository. The code used in the new computation, along with the Lean verifications mentioned above, is available at `https://github.com/31and8191/goor1/`. A revised version of the full paper is available on the arXiv at `https://arxiv.org/abs/2410.03677`.