RATIONAL POINTS OF SOME GENUS 3 CURVES FROM THE RANK 0 QUOTIENT STRATEGY

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Received: 12/9/23, Accepted: 5/24/25, Published: 6/27/25

Abstract

In 1922, Mordell conjectured that the set of rational points on a smooth curve C over $\mathbb{Q}$ with genus $g \geq 2$ is finite. This has been proved by Faltings in 1983. However, Coleman determined in 1985 an upper bound of $\#C(\mathbb{Q})$ by following Chabauty's approach which considers the special case when the Jacobian variety of C has Mordell-Weil rank $< g$. In 2006, Stoll improved Coleman's bound. The Chabauty-Coleman method has already been implemented to compute the rational points of genus 3 hyperelliptic curves and those of Picard curves. But it happens that this work has not yet been done for all genus 3 curves. In this paper, we describe an algorithm to compute the complete set of rational points $C(\mathbb{Q})$ for any genus 3 curve $C/\mathbb{Q}$ which is a degree-2 cover of a genus 1 curve whose Jacobian has rank 0. We implemented this algorithm in Magma, and we ran it on approximately 40,000 curves selected from databases of plane quartics and genus 3 hyperellitic curves. We discuss some interesting examples, and we exhibit curves for which the number of rational points meets Stoll's bound.

1. Introduction

Given a projective, smooth, absolutely integral curve C over $\mathbb{Q}$, we are interested in determining the set $C(\mathbb{Q})$ of rational points on C. The genus g of C, which is a

DOI: 10.5281/zenodo.15755985

nonnegative integer depending on C up to birational equivalence, is an important data. If $g = 0$, then either $C(\mathbb{Q}) = \emptyset$ or C is isomorphic to the projective line. If $g = 1$ and $C(\mathbb{Q}) \neq \emptyset$, then C is an elliptic curve. In the latter case, a famous theorem by Mordell in [13] certifies that $C(\mathbb{Q})$ is a finitely generated abelian group. This means that $C(\mathbb{Q})$ can be described only from a finite number of its points. Moreover, at the end of his paper Mordell conjectured that if g is greater than or equal to 2, then $C(\mathbb{Q})$ is finite. In 1929, Weil [19] generalized Mordell's theorem to all abelian varieties over number fields, and then Faltings [5] proved Mordell's conjecture in 1983. But Faltings' proof is not effective. Actually, the problem of constructing an algorithm which computes the rational points of a given curve with genus at least 2 is of topical interest. There are some methods adapted to special families of curves, but the problem is difficult in general. One of the main known methods is based on the work by Chabauty [3] and Coleman [4]. Assuming that p is a prime of good reduction for C, we denote by P a point in $C(\mathbb{Q}_p)$. The embedding

$$\iota : C \hookrightarrow J$$

$$Q \longmapsto [Q - P]$$

induces an isomorphism $\iota^* : H^0(J_{\mathbb{Q}_p}, \Omega^1) \longrightarrow H^0(C_{\mathbb{Q}_p}, \Omega^1)$ between spaces of holomorphic differential forms on C and its Jacobian variety J. Denote by $\overline{J(\mathbb{Q})}$ the p-adic closure of $J(\mathbb{Q})$ in $J(\mathbb{Q}_p)$. Chabauty [3] proved that if the rank of the Jacobian of C is less than g, then the intersection $\iota(C(\mathbb{Q}_p)) \cap \overline{J(\mathbb{Q})}$ is a finite set. Then, Coleman [4] proposed an effective version of this result by using his theory of p-adic integration on curves. Indeed, from properties of the pairing

$$\langle , \rangle : H^0(C_{\mathbb{Q}_p}, \Omega^1) \times J(\mathbb{Q}_p) \longrightarrow \mathbb{Q}_p$$

$$(\omega, [\sum P_i - Q_i]) \longmapsto \sum \int_{P_i}^{Q_i} \omega,$$

and the fact that $H^0(C_{\mathbb{Q}_p}, \Omega^1)$ is g-dimensional while J has Mordell-Weil rank $< g$, one deduces that there exist linearly independant differentials $\omega_1, \ldots, \omega_k \in H^0(C_{\mathbb{Q}_p}, \Omega^1)$ such that $\langle \omega_i, [D] \rangle = 0$ for all divisor classes $[D]$ in $J(\mathbb{Q})$. Coleman actually computed all the $\mathbb{Q}$-rational points of C by investigating integrals of the ω_i's on $J(\mathbb{Q}_p)$ which vanish on $J(\mathbb{Q})$. That is why these 1-forms are usually called the *annihilating differentials.* Furthermore, he obtained an upper bound of the number of rational points.

Theorem 1 ([4]). *Let C be a smooth curve over $\mathbb{Q}$ of genus $g \geq 2$ whose Jacobian has Mordell-Weil rank $r < g$, and let p be a prime of good reduction such that*

$p > 2g$. *Denote by* $\overline{C}$ *the reduction of* C *modulo* p. *Then*

$$\#C(\mathbb{Q}) \leq \#\overline{C}(\mathbb{F}_p) + 2g - 2.$$

Later, Stoll [17] improved this bound.

Theorem 2 ([17]). *Let* C *be a smooth curve over* $\mathbb{Q}$ *of genus* $g \geq 2$ *whose Jacobian has Mordell-Weil rank* $r < g - 1$, *and let* p *be a prime of good reduction such that* $p > 2r + 2$. *Denote by* $\overline{C}$ *the reduction of* C *modulo* p. *Then*

$$\#C(\mathbb{Q}) \leq \#\overline{C}(\mathbb{F}_p) + 2r.$$

Balakrishnan with her co-authors in [1] implemented the Chabauty-Coleman method to compute the rational points of genus 3 hyperelliptic curves selected from the database [2]. On the other hand, Hashimoto and Morrison [8] did the same work for Picard curves selected from the database [18]. But it happens that this work has not yet been done for all genus 3 curves. In this paper, we describe an algorithm to compute the complete set of rational points $C(\mathbb{Q})$ for any genus 3 curve $C/\mathbb{Q}$ which is a degree-2 cover of a genus 1 curve $D/\mathbb{Q}$ whose Jacobian has rank 0. The first step is to check whether D possesses a $\mathbb{Q}$-rational point or not. When $D(\mathbb{Q}) = \emptyset$, obviously $C(\mathbb{Q}) = \emptyset$. If we find a rational point P_0, then (D, P_0) is an elliptic curve. Mordell's theorem tells us that $D(\mathbb{Q})$ is a finite group. Furthermore, Mazur [10, 11] proved that this group is isomorphic to one of the following fifteen groups:

$$\mathbb{Z}/n\mathbb{Z} \quad \text{with} \quad 1 \leq n \leq 10 \quad \text{or} \quad n = 12,$$

and

$$\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2n\mathbb{Z} \quad \text{with } 1 \leq n \leq 4.$$

One may use the Magma function *MordellWeilGroup* to decide the exact nature of $D(\mathbb{Q})$. Since the quotient map $\psi : C \longrightarrow D$ is defined over $\mathbb{Q}$, we have $\psi(C(\mathbb{Q})) \subseteq D(\mathbb{Q})$ and one easily deduces $C(\mathbb{Q})$ from $\psi^{-1}(D(\mathbb{Q}))$. Note that the rank 0 quotient strategy is not specific to genus 3 curves. Indeed, it is easily seen that the algorithm described in Section 2.2 can be used to compute the rational points of any curve of genus at least 2 satisfying the requirements on the inputs. For instance, a specific genus 2 case is discussed by Siksek in [15]. In addition, the rank zero quotient strategy could be combined with the Chabauty-Coleman method in order to implement an efficient Point-Counting algorithm which takes advantage of strenghts of each of both procedures so that if the implemented function fails to find the annihilating differentials as required in the Chabauty-Coleman method, it could switch in searching symmetries and elliptic quotients. It seems that this trick has been used for the implementation of the Magma's Chabauty function concerning genus 2 curves.

Our paper is organized as follows. After this introduction, we quickly recall some properties of genus 3 curves in Section 2.1. Then, we describe the rank 0 quotient

Curves having a genus 1 rank 0 quotient		Numbers of curves having a genus 1 rank 1 quotient	Numbers of curves having a genus 1 quotient of rank at least 2
Numbers of curves whose quotient has no rational points: **7534**	Numbers of curves whose quotient is an elliptic curve: **31030**	**57347**	**4089**

Table 1: Statistics resulting from our selection procedure on $100,000$ plane quartics from a database constructed in [6].

strategy in Section 2.2. We discuss our implementation in Section 2.3. Section 3 is devoted to illustrations. We present some interesting examples, and we exhibit curves for which the number of rational points meets Stoll's bound.

2. The Rank 0 Quotient Strategy

As mentioned in the introduction, this algorithm can be used to compute the rational points of any curve of genus at least 2 satisfying the requirements on the inputs. In this section, we first specify the needed properties for a genus 3 curve to be eligible, and then we present the algortihm. We describe our implementation and experiments using curves selected from the database [2] and from another database constructed by the authors of [6].

2.1. Genus 3 Curves Are Eligible

While families of curves with genus $g \leq 2$ are homogenous in the sense that two curves with the same genus over an algebraically closed field are of the same nature (rational, elliptic, hyperelliptic), things are different when the genus is greater than 2. For instance, a (projective, smooth, absolutely integral) curve C of genus 3 may be either hyperelliptic or nonhyperelliptic, that is a plane quartic. Assume that this curve C is a degree-2 cover of a genus 1 curve D. It is known that the Jacobian variety of C is isogenous to the product of the Jacobian variety of D by an abelian surface; see for instance [14]. When C is also a hyperelliptic curve, it is defined by $C : y^2 = f(x^2)$, where f is a polynomial of degree 4 and D is the locus $y^2 = f(x)$. If C is a plane quartic, then it is the locus of a ternary quartic of the form $Y^4 - h(X,Z)Y^2 + r(X,Z)$ so that D is given by $D : Y^2 - h(X,Z)Y + r(X,Z) = 0$. The present paper actually describes how to compute the rational points of C from those of D in the case when the Jacobian of D has Mordell-Weil rank 0. This is quite suitable for genus 3 hyperelliptic curves endowed with an extra involution giving rise to a rank 0 quotient. Ciani quartics whose Jacobians have rank ≤ 2 are also eligible. Recall that a Ciani quartic is a smooth curve defined by an equation of the form

$$a_1X^4 + a_2Y^4 + a_3Z^4 + 2(b_1Y^2Z^2 + b_2X^2Z^2 + b_3X^2Y^2) = 0.$$

It is known that the Jacobian variety of such a curve is isogenous to the product of three quotient elliptic curves; see [9, Section 2.3]. Therefore, if the Jacobian has rank ≤ 2, then at least one of these elliptic curves has rank 0.

2.2. The Algorithm

A simple presentation of the rank 0 quotient strategy is as follows.

Algorithm 1: Rank 0 quotient strategy

Input : A curve $C/\mathbb{Q}$ and an involution σ on C defined over $\mathbb{Q}$ such that the quotient $D := C/\langle\sigma\rangle$ has genus 1 and its Jacobian has rank 0.

Output: The set S of $\mathbb{Q}$-rational points of C.

Set $\psi : C \longrightarrow D$ the quotient map, and compute $D(\mathbb{Q})$.
If $D(\mathbb{Q}) = \emptyset$, then $S = \emptyset$. Otherwise, compute $\psi^{-1}(P)$ for every $P \in D(\mathbb{Q})$.
Set $\Omega(P) := \psi^{-1}(P) \cap C(\mathbb{Q})$, then $S = \bigcup_{P\in D(\mathbb{Q})} \Omega(P)$.

return S

2.3. Implementation

We implemented this algorithm in Magma. The code file is available on the GitHub repositories of the second author[12].

We ran our code on $38,564$ plane quartics selected from a database constructed by Sutherland and his co-authors in [6]. Actually, our original dataset was made of $100,000$ plane quartics which had to pass identification and selection processes. We first tested whether their equations show certain symmetries by using the Magma function `AutomorphismGroupOfPlaneQuartic`.

For every involution $\sigma \in \mathrm{Aut}(C)$, we checked whether the associated quotient curve has genus 1. We got the confirmation that all the $100,000$ curves were symmetric under at least one of the involutions

$$(X : Y : Z) \longrightarrow (-X : Y : Z); \quad (X : Y : Z) \rightarrow (X : -Y : Z);$$

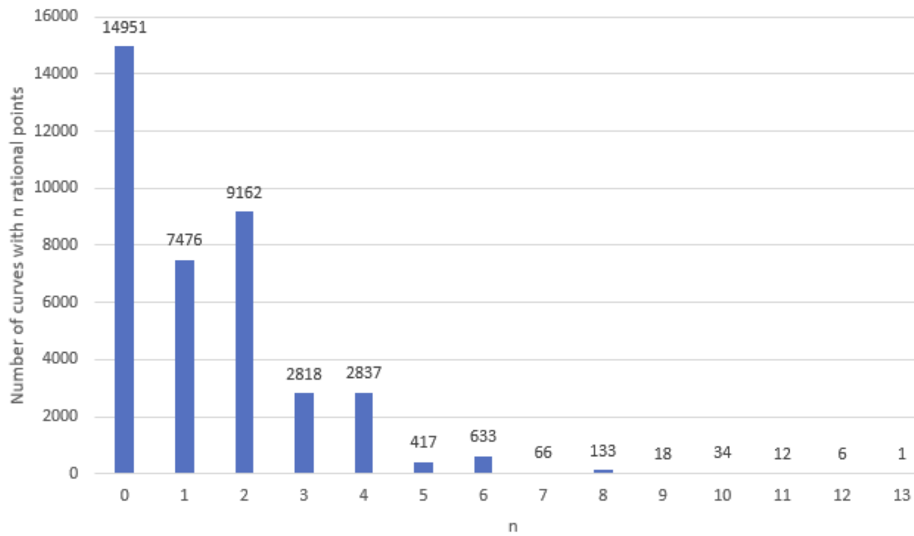

Figure 1: Statistics obtained when running Algorithm 1 on eligible **38,564** quartic covers selected from a database constructed in [6].

and

$$(X : Y : Z) \longrightarrow (X : Y : -Z).$$

Furthermore, all the corresponding quotient curves have genus 1. But only 38,564 have rank 0, among them there are 17,404 Ciani quartics. Running Algorithm 1 reveals that most of these curves have no rational points. The statistics resulting from the selection process are summarized in table 1. In addition, the statistics obtained when running Algorithm 1 on the eligible 38,564 quartic covers are presented in Figure 1. We also ran our code on hyperelliptic curves selected from the database [2]. Actually, we applied the identication and selection processes that we described previously to all the 67,879 hyperelliptic curves of this database. It turns out that only 130 curves have a genus 1 rank 0 quotient, and each of them possesses at least one rational point. The statistics obtained when running Algorithm 1 on the eligible 130 hyperelliptic curves are presented in 2. All the code files as well as the results obtains during our experiments about planes quartics and genus 3 hyperelliptic curves are available on the GitHub repositories of the second author [12].

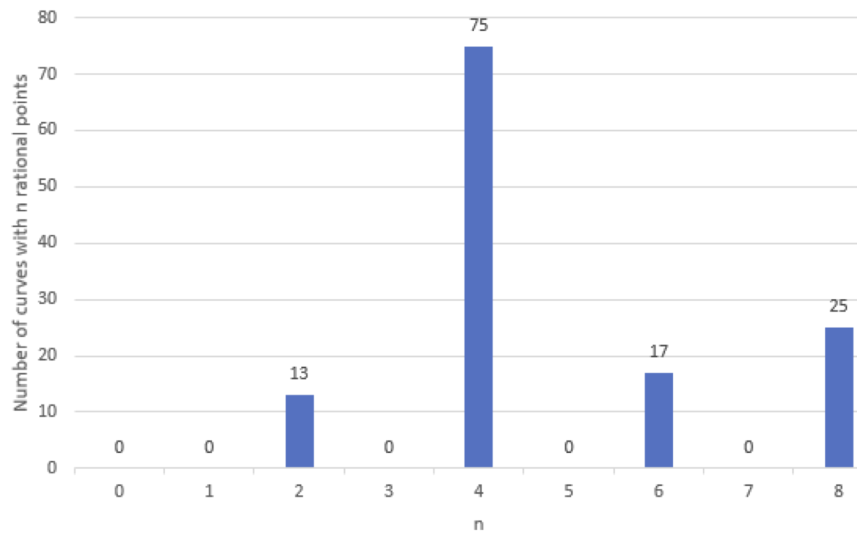

Figure 2: Statistics obtained when running Algorithm 1 on **130** hyperelliptic curves selected from the database [2].

3. Examples

Theorem 1 and Theorem 2 give upper bounds of the number of rational points on smooth curves of genus $g \geq 2$ under certain conditions. In practice, when computing the rational points of these curves one observes that some of them have a number of rational points which meets these bounds, but some others do not. Such observations have already been illustrated in [7]. The following examples also illustrate this point.

Example 1 (Rational points on Fermat curves)**.** Let k be a positive integer, and let a_1, a_2, a_3 nonzero rational numbers which are $4k$-powers. By Fermat's last theorem, there are no triple $(X, Y, Z) \in \mathbb{Q}^3$ such that $XYZ \neq 0$ and (X, Y, Z) is a root of the trinomial $a_1X^{4k} + a_2Y^{4k} - a_3Z^{4k}$. Hence, the only $\mathbb{Q}$-rational points on the curve

$$C_k : X^{4k} + Y^{4k} - Z^{4k} = 0$$

are the trivial ones: $(0, 1, 1)$, $(0, 1, -1)$, $(1, 0, 1)$, and $(1, 0, -1)$. We conclude that the number of rational points on C_1 meets Stoll's bound for all primes $p > 2$. But $\#C_k(\mathbb{Q})$ never meets Coleman's bound for any k.

Example 2 (Rational points of a Ciani quartic)**.** Consider the Ciani quartic

$$C : 15X^4 + 112Y^4 + 112Z^4 + 288Y^2Z^2 - 88X^2Z^2 - 88X^2Y^2 = 0$$

Let

$$\sigma_1 : (X, Y, Z) \mapsto (-X, Y, Z), \quad \sigma_2 : (X, Y, Z) \mapsto (X, -Y, Z),$$

and

$$\sigma_3 : (X, Y, Z) \mapsto (X, Y, -Z)$$

be the three canonical involutions of C. The associated quotient elliptic curves are

$$E_1 : 112X^4 + 112Y^4 + 15Y^2Z^2 - 88ZY^3 - 88YZX^2 + 288X^2Y^2 = 0,$$

$$E_2 : 112Y^4 + 112Z^4 + 15X^2Z^2 - 88XZ^3 - 88XZY^2 + 288Z^2Y^2 = 0,$$

$$E_3 : 112X^4 + 112Z^4 + 15Y^2X^2 - 88YX^3 - 88YXZ^2 + 288X^2Z^2 = 0.$$

Set $E : y^2 = x^3 + \frac{7}{2}x^2 - \frac{15}{16}x$. By using the following transformations

$$E_1 \longrightarrow E : (X, Y, Z) \longmapsto \left(\frac{28}{15}X^2Y + \frac{58}{15}Y^3 - Y^2Z, \frac{56}{15}X^3 + \frac{116}{15}XY^2 - 2XYZ, -\frac{8}{15}Y^3\right),$$

$$E_2 \longrightarrow E : (X, Y, Z) \longmapsto \left(\frac{28}{15}Y^2Z + \frac{58}{15}Z^3 - Z^2X, \frac{56}{15}Y^3 + \frac{116}{15}YZ^2 - 2XYZ, -\frac{8}{15}Z^3\right),$$

$$E_3 \longrightarrow E : (X, Y, Z) \longmapsto \left(\frac{28}{15}Z^2X + \frac{58}{15}X^3 - X^2Y, \frac{56}{15}Z^3 + \frac{116}{15}ZX^2 - 2XYZ, -\frac{8}{15}X^3\right),$$

we see that E_1, E_2 and E_3 are isomorphic to E. Magma tells us that E has rank 0. By running our code, we obtained

$$E(\mathbb{Q}) = \left\{\infty, (0, 0), \left(\frac{3}{4}, -\frac{3}{2}\right), \left(-\frac{3}{4}, \frac{3}{2}\right), \left(\frac{1}{4}, 0\right), \left(\frac{5}{4}, -\frac{5}{2}\right), \left(\frac{5}{4}, \frac{5}{2}\right), \left(-\frac{15}{4}, 0\right)\right\},$$

and

$$C(\mathbb{Q}) = \{(2 : 1 : 0), (-2 : 1 : 0), (2 : 0 : 1), (-2 : 0 : 1)\}.$$

Hence, Stoll's bound is sharp at 17, since C has rank 0 and its reduction modulo 17 satisfies

$$\overline{C}(\mathbf{F}_{17}) = \{(2 : 1 : 0), (15 : 1 : 0), (2 : 0 : 1), (15 : 0 : 1)\}.$$

Example 3 (Rational points of a genus 3 hyperellitic curve)**.** The hyperelliptic curve

$$C : Y^2 = X^8 + 2X^4Z^4 - 4X^2Z^6 + Z^8$$

is invariant under the symmetry $\sigma : (X, Y, Z) \mapsto (-X, Y, Z)$, and the quotient $D := C/\langle\sigma\rangle$ is a genus 1 rank 0 curve. By running our code, we found

$$D(\mathbb{Q}) = \left\{\infty, (0 : \frac{1}{2} : 1), (0 : -\frac{1}{2} : 1), (1 : \frac{1}{2} : 1), (1 : -\frac{1}{2} : 1)\right\},$$

and

$$C(\mathbb{Q}) = \{(1 : -1 : 0), (1 : 1 : 0), (-1 : 0 : 1), (0 : -1 : 1), (0 : 1 : 1), (1 : 0 : 1)\}.$$

Example 4 (The case of a genus 3 nonhyperelliptic curve which is not a Ciani quartic)**.** Consider the plane quartic

$$C : 3X^4 - 2X^2Y^2 - Y^4 + 4X^3Z - 4XY^2Z + 2X^2Z^2 - 2Y^2Z^2 + 2XZ^3 + Z^4 = 0.$$

From its equation, we see that this curve is invariant under the involution $\sigma_2 : (X, Y, Z) \mapsto (X, -Y, Z)$, but not under $\sigma_1 : (X, Y, Z) \mapsto (-X, Y, Z)$ nor $\sigma_3 : (X, Y, Z) \mapsto (X, Y, -Z)$. Actually, Magma confirms that the automorphism group of C is $\mathbb{Z}/2$. This means that the only nontrivial automorphism of C is σ_2. The quotient $E := C/\langle\sigma_2\rangle$ is an elliptic curve of rank 0. By running our code, we found

$$E(\mathbb{Q}) = \left\{\infty, (-1 : 0 : 1), \left(-\frac{1}{2} : \frac{3}{4} : 1\right), \left(\frac{1}{2} : \frac{1}{4} : 1\right)\right\},$$

and

$$C(\mathbb{Q}) = \left\{(-1 : 0 : 1), (-1 : 1 : 0), (1 : 1 : 0), \left(-\frac{1}{2} : -\frac{1}{2} : 1\right), \left(-\frac{1}{2} : \frac{1}{2} : 1\right)\right\}.$$

We conclude that $\#C(\mathbb{Q})$ cannot meet Coleman's bound, since the reductions of C satisfy $\#\overline{C}(\mathbf{F}_p) \geq 2$ for any prime p of good reduction.

Acknowledgements. This study has been carried out with financial support from the French State, managed by CNRS in the frame of the *Dispositif de Soutien aux Collaborations avec l'Afrique subsaharienne* (via the REDGATE Project and the IRN AFRIMath). Experiments presented in this paper were carried out by using the Magma software through the account of the second author at Boston University Library. The first two authors were supported by Simons Foundation in the frame work of the Africa Mathematics Project [16]. We are very grateful to Jennifer Balakrishnan for many helpful discussions and for having brought our attention to recent papers on Coleman integration on curves and Point-Counting algorithms. We thank Oana Padurariu for helpful suggestions. We are also very grateful to Steffen Müller for his comments on early versions of this work. We would like to thank Andrew Sutherland for his comments on this work and for sharing with us some of the data he used with his co-authors in [6]. The authors thank the anonymous referees for carefully reading this paper and their helpful feedback.

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