

3.5 Implicit Differentiation

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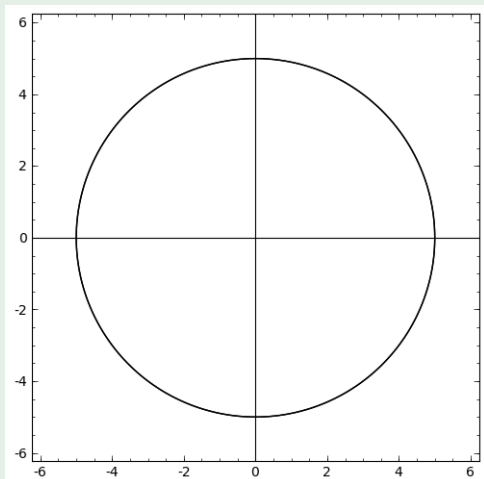
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Implicit functions

Example

Some functions are defined implicitly by a relation between x and y

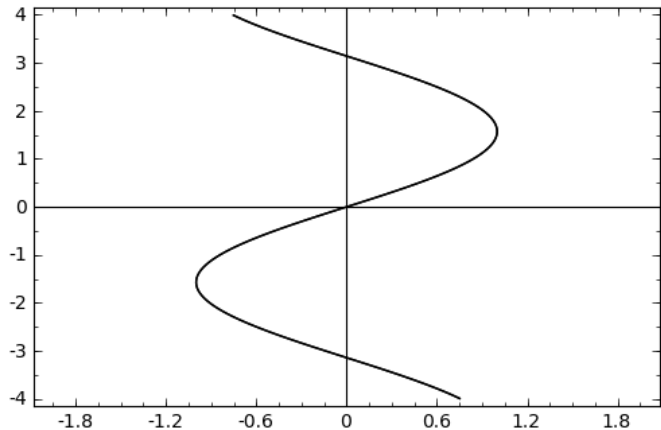
- $x^2 + y^2 = 25$



Other implicit functions

Example

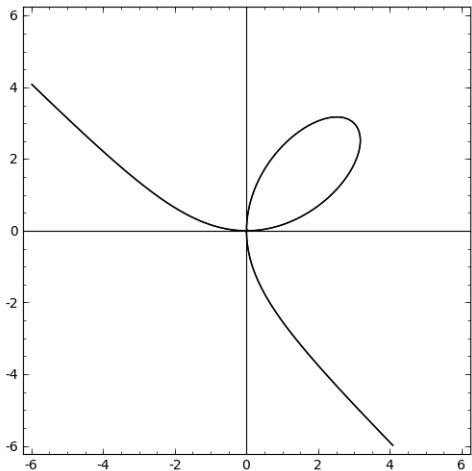
$$x = \sin(y)$$



The folium of Descartes

Example

$$x^3 + y^3 = 6xy$$



Definition

Method of implicit differentiation: differentiate both sides of the equation with respect to x and then solve the resulting equation for y' .

Example

Find $\frac{dy}{dx}$

- if $x^2 + y^2 = 25$
- if $x^3 + y^3 = 6xy$
- if $2\sqrt{x} + \sqrt{y} = 3$
- if $1 + x = \sin(xy^2)$

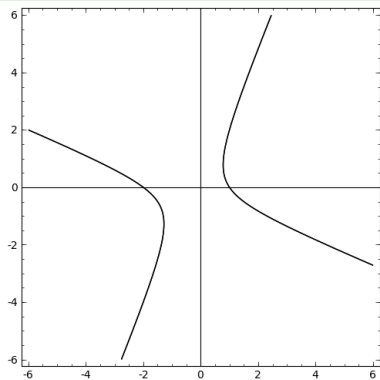
Example

Example

Find an equation of the tangent line to the curve

$$x^2 + 2xy - y^2 + x = 2$$

at $(1, 2)$.



Derivatives of Inverse Trigonometric Functions

Example

Find $\frac{dy}{dx}$ if $\sin y = x$, $-1 \leq x \leq 1$.

Derivatives of Inverse Trigonometric Functions

Fact (Derivatives)

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Derivatives of Inverse Trigonometric Functions

Fact (Derivatives, cont'd)

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2}.$$

Example

Find the derivative of the following functions

- $y = \sqrt{\tan^{-1} x}$
- $y = \sin^{-1}(x^2 + 1)$
- $y = \cos^{-1}(e^{x^2})$.