4.1 Maximum and Minimum Values

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- f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the minimum value of f on D.

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- f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the minimum value of f on D.
- The maximum and minimum values of f are called the extreme values of f.

Example

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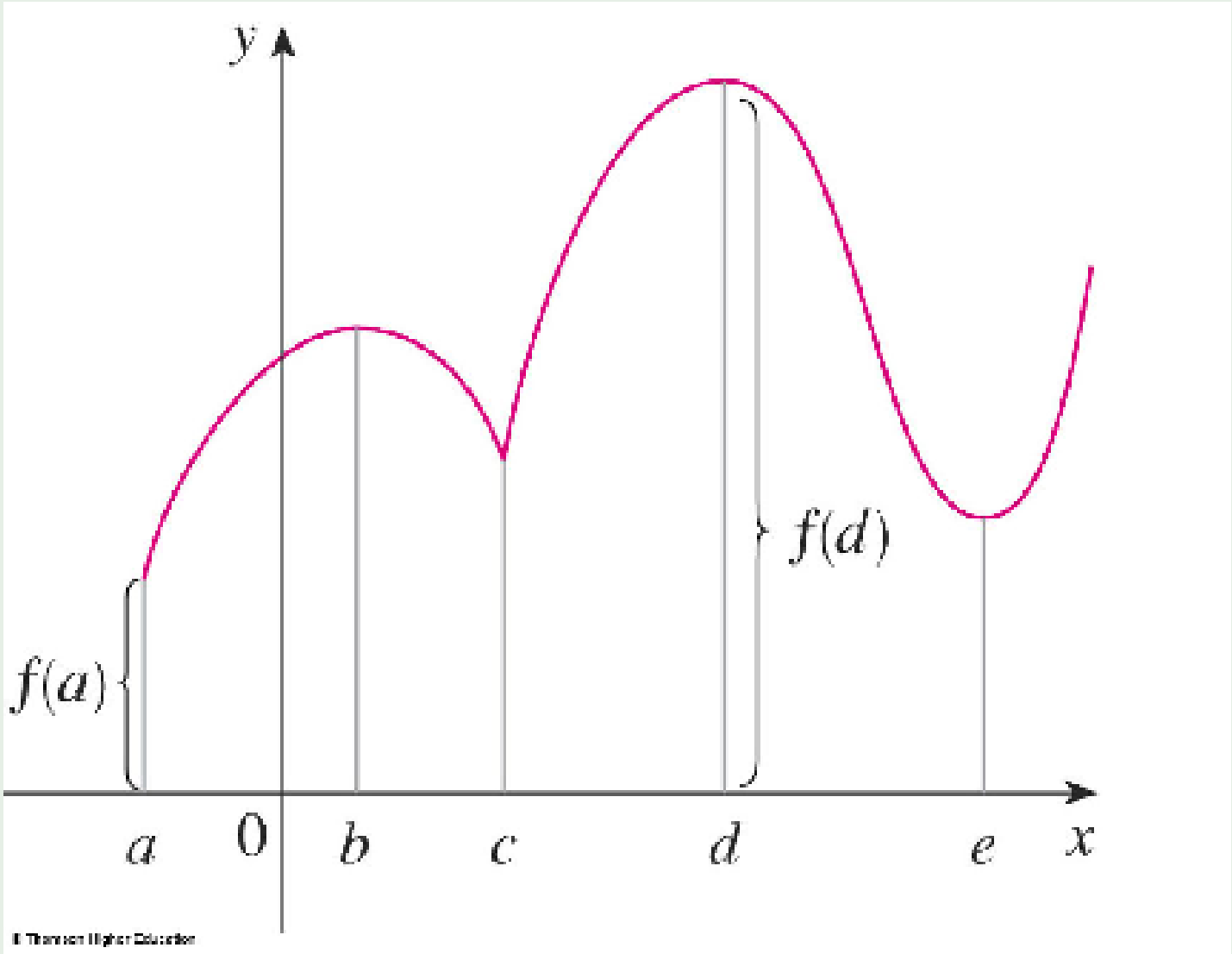

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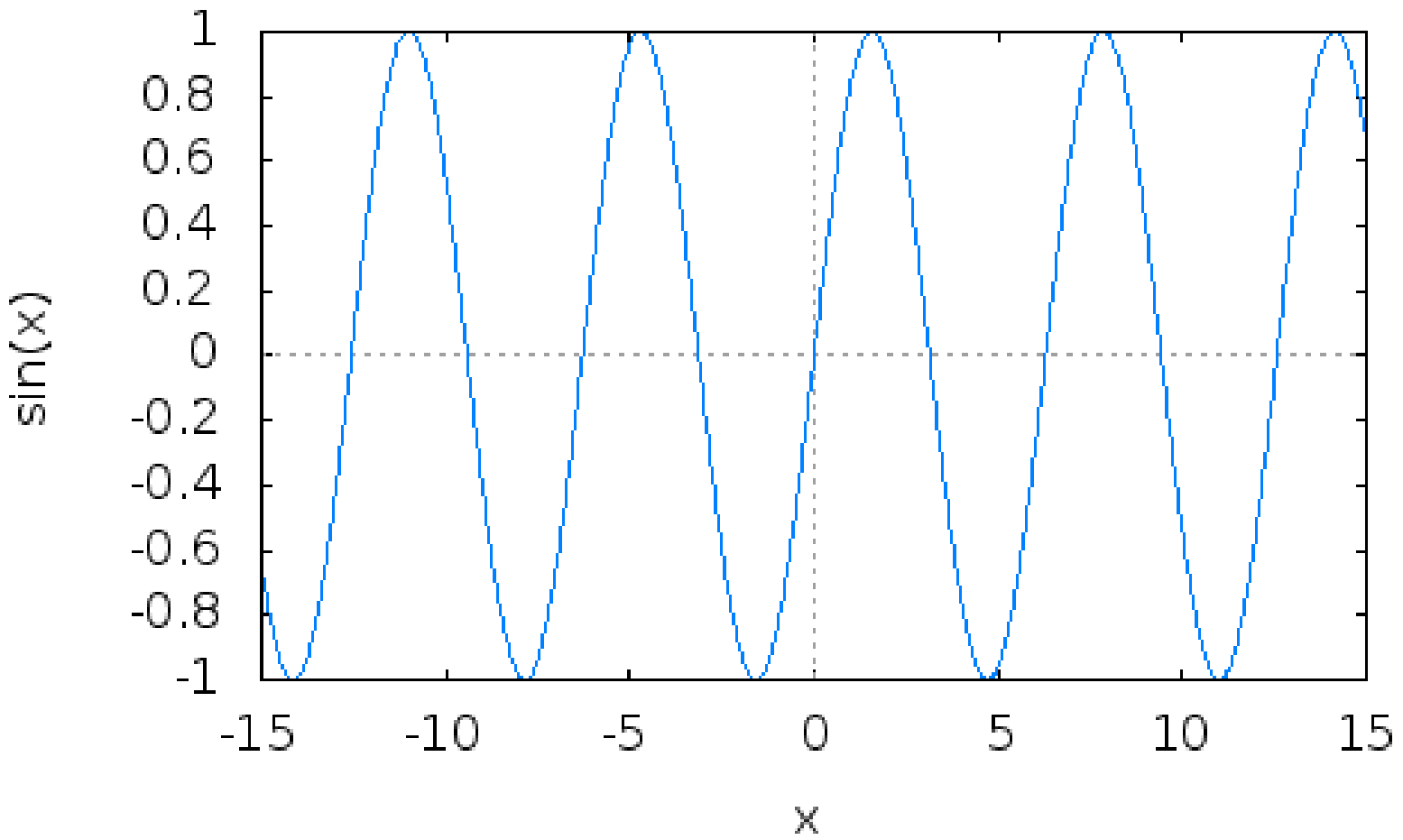
Let $f(x) = \sin x$



Example

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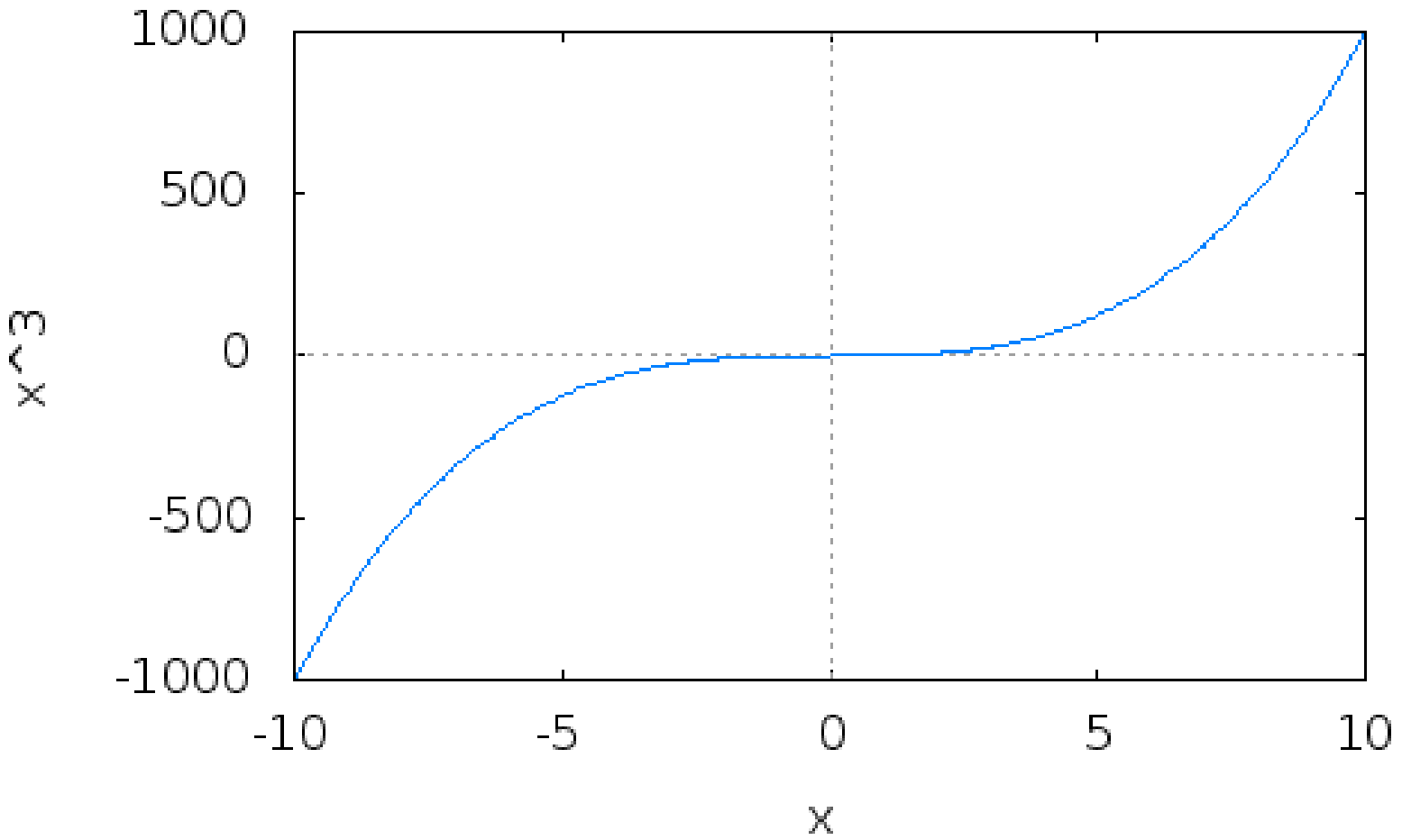
Let $f(x) = x^3$



The Extreme Value Theorem

Fact

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Example

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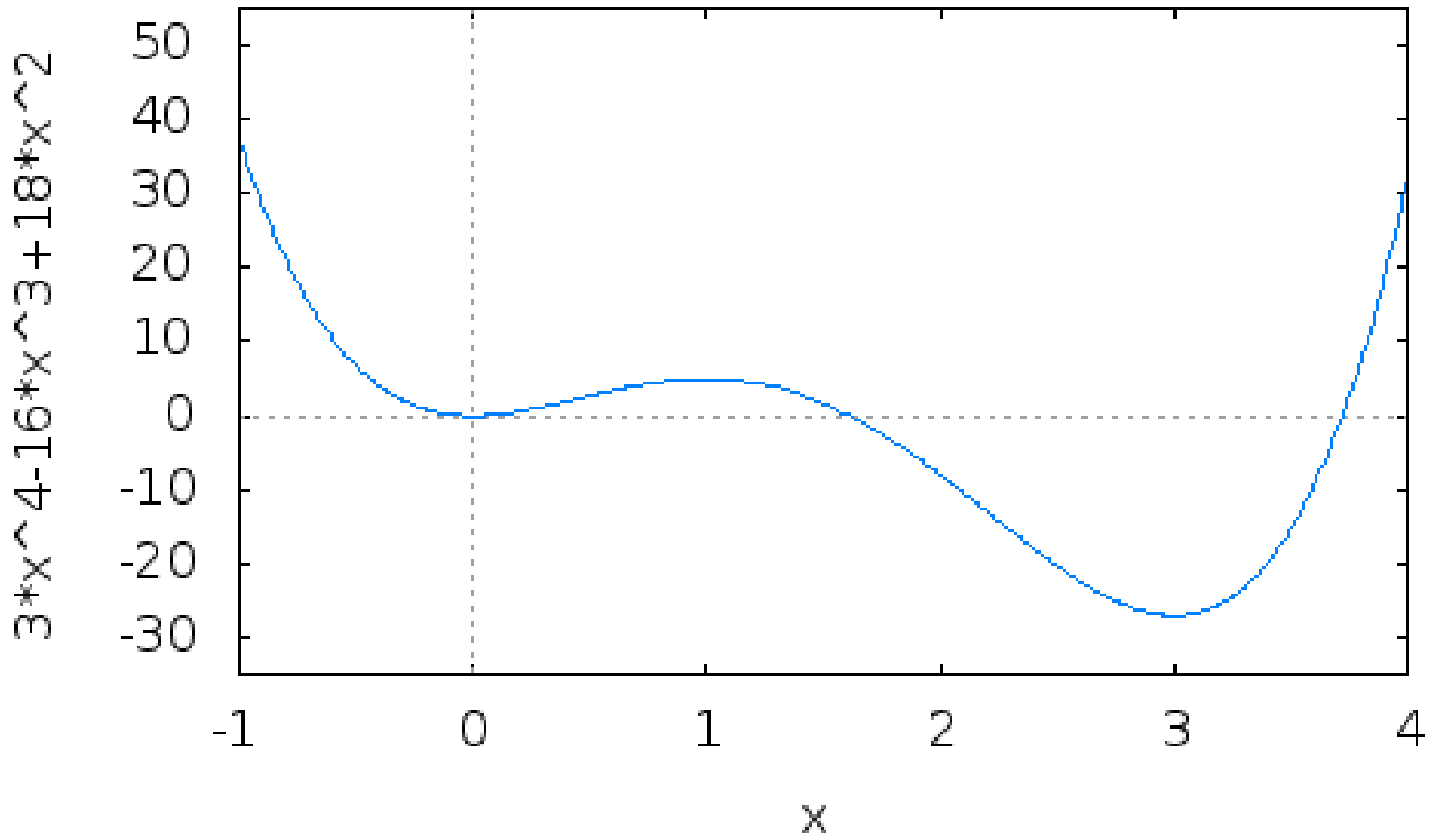
Let $f(x) = 3x^4 - 16x^3 + 18x^2$ with $-1 \leq x \leq 4$.



Fermat's Theorem

Fact

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

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- Let $f(x) = |x|$. What does the Fermat's theorem say for this function?
- What about $f(x) = x^3$?

Critical Numbers

Definition

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Examples

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- $f(x) = \frac{x-1}{x^2-x+1}$
- $f(x) = x^{-2} \ln x$

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To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

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To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. *Find the values of f at the critical numbers of f in (a, b).*

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2. *Find the values of f at the endpoints of the interval.*

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3. *The largest value from 1 and 2 is the absolute maximum value.*

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To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. *Find the values of f at the critical numbers of f in (a, b).*
2. *Find the values of f at the endpoints of the interval.*
3. *The largest value from 1 and 2 is the absolute maximum value.*
4. *The smallest is the absolute minimum value.*

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- $f(x) = (x^2 - 1)^3$, $[-1, 2]$.
- $f(x) = x - \ln x$, $[1/2, 2]$.
- $f(x) = e^{-x} - e^{-2x}$, $[0, 1]$.