The Dot Product
Lecture 2

Marius Ionescu

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The Dot Product

- If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number

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- It is also called the **scalar product** or **inner product**.

Examples

- $\langle 2, 1\rangle \cdot \langle -1, 3\rangle$.

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- $\langle 3, -2, 1\rangle \cdot \langle 0, 1, 1\rangle$.

Properties of the Dot Product

- If $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ are vectors in V_3 and c is a scalar, then
  1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
  2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
  3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
  4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
  5. $\mathbf{0} \cdot \mathbf{a} = 0$.

The angle between two vectors

- If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

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- If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Orthogonal vectors

- $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Direction Angles

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- The cosines of these direction angles are called the **direction cosines** of the the vector $\mathbf{a}$:

$$\cos\alpha = \frac{a_1}{|\mathbf{a}|}, \ \cos\beta = \frac{a_2}{|\mathbf{a}|}, \ \cos\gamma = \frac{a_3}{|\mathbf{a}|}.$$

Projections

- Scalar projection of **b** onto **a**:

$$\text{comp}_a \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$

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- Vector projection of **b** onto **a**

$$\text{proj}_a b = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$$

Work done by a constant force

- The work done by a constant force $\mathbf{F}$ is

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- Example: A constant force $\mathbf{F} = -2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ moves an object along a straight line from the point $(1, 0, 0)$ to $(-3, 2, 3)$. Find the work done.