HOMEWORK #10, REAL ANALYSIS I, FALL 2012

MARIUS IONESCU

Problem 1 (Exercise 6.2.3 on page 161)**.** Consider the sequence of functions

$$h_n(x) = \frac{x}{1+x^n}$$

over the domain $[0, \infty)$.

(a) Find the pointwise limit of (h_n) on $[0, \infty)$.
(b) Explain how we know that the convergence *cannot* be uniform on $[0, \infty)$.
(c) Choose a smaller set over which the convergence is uniform and supply an argument to show that this is indeed the case.

Problem 2 (Exercise 6.2.7 on page 161)**.** Assume that (f_n) converges uniformly to f on A and that each f_n is uniformly continuous. Prove that f is uniformly continuous on A.

Problem 3 (Exercise 6.3.1 on page 166)**.** (a) Let

$$h_n(x) = \frac{\sin(nx)}{n}.$$

Show that $h_n \to 0$ uniformly on $\mathbb{R}$. At what points does the sequence of derivatives h_n' converge?
(b) Modify this example to show that it is possible for a sequence (f_n) to converge uniformly but for (f_n') to be unbounded.

Problem 4 (Exercise 6.4.3 on page 168)**.** (a) Show that $g(x) = \sum_{n=1}^{\infty} \cos(2^n x)/2^n$ is continuous on all of $\mathbb{R}$.
(b) Prove that $h(x) = \sum_{n=1}^{\infty} x^n/n^2$ is continuous on $[-1, 1]$.

Problem 5 (Exercise 6.5.1 on page 174)**.** Consider the function g defined by the power series

$$g(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots.$$

Date: **Due Sunday, 12/09/2012; You do not have to type this assignment using Latex.**

(a) Is g define on $(-1, 1)$? Is it continuous on this set? Is g defined on $(-1, 1]$? Is it continuous on this set? What happens on $[-1, 1]$? Can the power series for $g(x)$ possible converge for any other points $|x| > 1$? Explain.
(b) For what values of x is $g'(x)$ defined? Find a formula for g'.

Problem 6 (Exercise 6.5.2 on page 174)**.** Find suitable coefficients (a_n) so that the resulting power series $\sum a_n x^n$

(a) converges absolutely for all $x \in [-1, 1]$ and diverges off of this set;
(b) converges conditionally at $x = -1$ and diverges at $x = 1$;
(c) converges conditionally at both $x = -1$ and $x = 1$.
(d) Is it possible to find an example of a power series that converges conditionally at $x = -1$ and converges absolutely at $x = 1$?