Changing the index of an infinite series:

Suppose you have

$$\sum_{n=1}^{\infty} (2n+1)\, x^{n-1}$$

and you would rather start with $n = 0$. You can shift your indexes as follows. Let $n - 1 = k$ so that when $n = 1$, $k = 0$. (Notice that we choose this since under the $\sum$ is $n = 1$ and we want $n - 1 = 0$.) Solve for $n = k + 1$, and we replace every n with $k + 1$ in our infinite series:

$$\sum_{k+1=1}^{\infty} (2\,(k+1)+1)\, x^{(k+1)-1} \qquad \Rightarrow \qquad \sum_{k=0}^{\infty} (2k+3)\, x^{k}$$

Since both k and n are just indexing variables, we can reuse n in the above series on the right to get

$$\sum_{n=0}^{\infty} (2n+3)\, x^{n}$$

Notice that

$$\begin{aligned} \sum_{n=1}^{\infty} (2n+1)\, x^{n-1} &= 3 + 5x + 7x^2 + 9x^3 + \cdots \\ \sum_{n=0}^{\infty} (2n+3)\, x^{n} &= 3 + 5x + 7x^2 + 9x^3 + \cdots \end{aligned}$$

Suppose you have

$$\sum_{n=0}^{\infty} (-1)^n\, x^{n+1},$$

but you need the exponent of x to be n instead of $n + 1$. Then you can let $k = n + 1$ and shift the index. Here, $n = k - 1$ and

$$\sum_{k-1=0}^{\infty} (-1)^{k-1}\, x^{(k-1)+1} \qquad \Rightarrow \qquad \sum_{k=1}^{\infty} (-1)^{k-1}\, x^{k}$$

Reusing the letter n, we have

$$\sum_{n=1}^{\infty} (-1)^{n-1}\, x^{n}$$

Checking our series,

$$\begin{aligned} \sum_{n=0}^{\infty} (-1)^n\, x^{n+1} &= x - x^2 + x^3 - x^4 + \cdots \\ \sum_{n=1}^{\infty} (-1)^{n-1}\, x^{n} &= x - x^2 + x^3 - x^4 + \cdots \end{aligned}$$