**Math 213 – Calculus III** **Exam 1 (Compressed Version)** **Spring 2002**

1. Match each function to one of the given graphs, and to one of the given contour diagrams. The plots are given in a separate handout.

| $f(x,y)$ | Graph | Contour Diagram |
|---|---|---|
| $x^2 + \left(\frac{3y}{2}\right)^2$ | | |
| $x^2 - y^2$ | | |
| $\frac{x}{2} + y$ | | |
| $-x + y$ | | |
| $\sin(\pi(x-y))$ | | |
| $xy$ | | |
| $\sqrt{x^2 + \left(\frac{3y}{2}\right)^2}$ | | |
| $x + 2y^2$ | | |

2. (a) Find the equation of the plane that contains the points $(1,0,0)$, $(0,-3,0)$, and $(0,0,4)$.

   (b) Find the equation of a sphere centered at $(1,2,3)$ that is tangent to the $yz$-plane.

3. (a) Consider the function

$$f(x,y) = y^2 - \sin(x-y) - 3.$$

   Find a function $g(x,y,z)$ such that the graph of $f$ is the level surface $g(x,y,z) = 2$.

   (b) For each of the following functions, determine if the level surface $g(x,y,z) = 1$ can be expressed as the graph of a function $f(x,y)$. If it is not possible, explain why not. If is it possible, find the function $f(x,y)$.

   i. $g(x,y,z) = x^2 + 2y^2 - z^2$

   ii. $g(x,y,z) = -x - y + 3z + 8$

   iii. $g(x,y,z) = e^{x+y+z}$

4. For each of the following, find the indicated partial derivative.

(a) $f(x,y) = xy - 3\sin x,$ $\qquad f_x(x,y) =$

(b) $g(x,y) = e^{x-y},$ $\qquad f_y(x,y) =$

(c) $h(s,t) = \sqrt{s^2 + st + t^2},$ $\qquad \dfrac{\partial h}{\partial s} =$

(d) $f(x,y) = \ln(x^2 + 2y),$ $\qquad f_y(x,y) =$

(e) $P = bL^{\alpha}K^{1-\alpha},$ $\qquad \dfrac{\partial P}{\partial K} =$

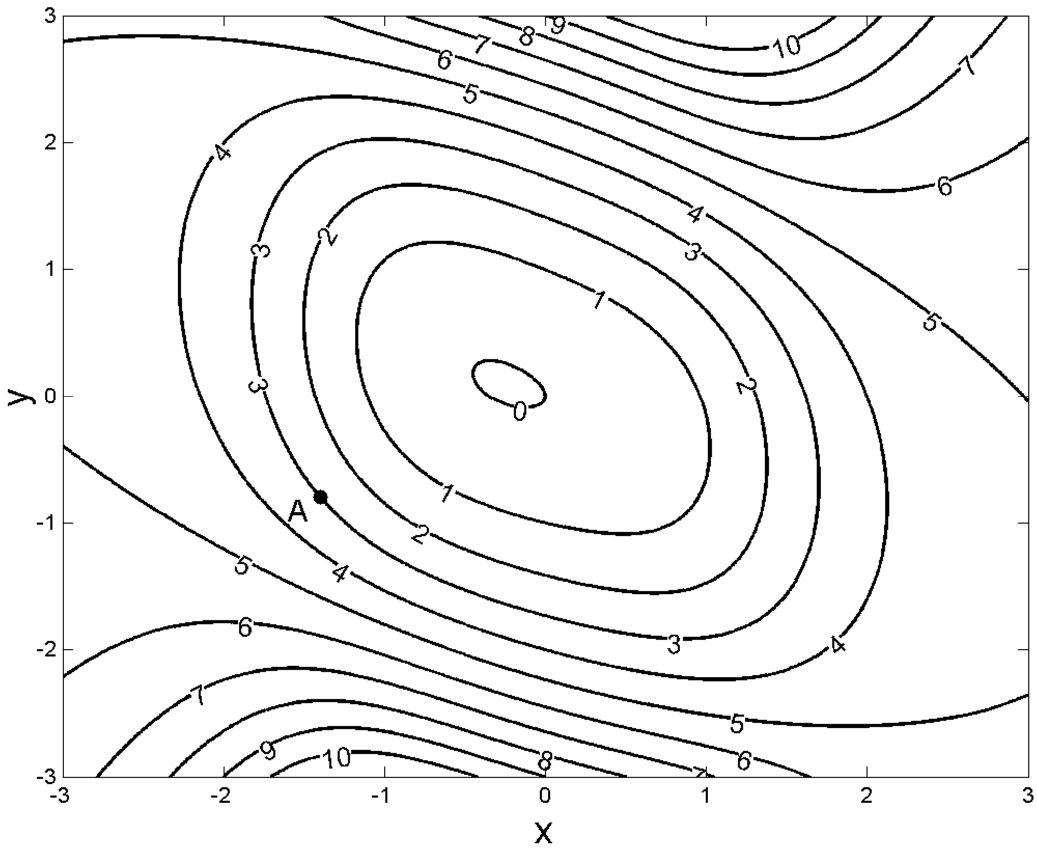
5. Let

$$f(x,y) = (y^2 + 2 + \sin y)x.$$

(a) Find the equation of the plane that is tangent to the graph of $f$ at the point $(3,0)$.

(b) Use the local linearization of $f$ near $(3,0)$ to estimate $f(3.2, 0.1)$

6. The following is the contour diagram for a function $f(x,y)$.




Consider the tangent plane to the graph of $f$ at the point labeled $A$.

(a) In the plot above, sketch the contour diagram of this tangent plane.

(b) Describe and explain your answer to part (a). That is, describe the contours lines, and explain how you know how the curves are shaped and oriented.