Getting Started with Differential Equations in Maple

In this Maple session, we see some of the basic tools for working with differential equations in Maple. We will only deal with first order equations here.

First, we need to load the "DEtools" library:

```
> with(DEtools):
```

Let's define a differential equation. As an example, we consider
y' = 1-y^2.
This is a first order, autonomous, ordinary differential equation.

The following defines a variable called "eq" that holds the differential equation:

```
> eq := diff(y(t),t) = 1-y(t)^2;
```

$$eq := \frac{\partial}{\partial t} y(t) = 1 - y(t)^2$$

A few points:

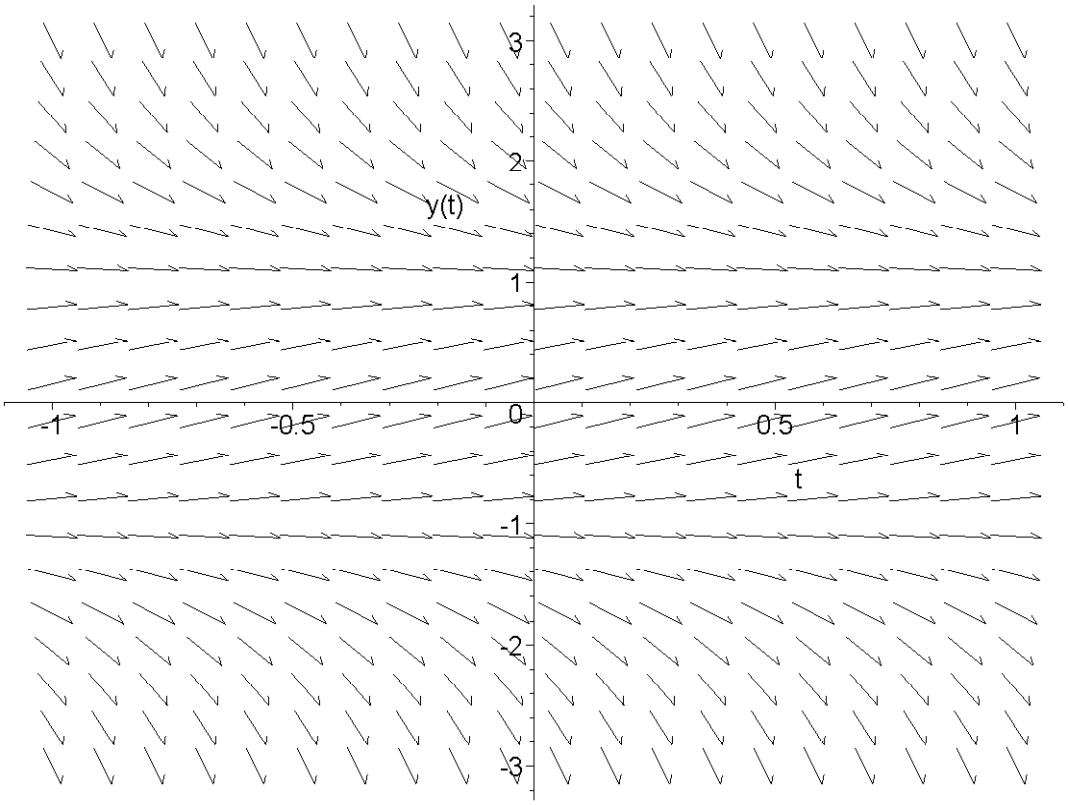
1. The derivative of y is specified with the "diff" command.
2. We **can not** drop the "(t)" from the dependent variable y. Maple treats "y" and "y(t)" differently, and our equation is for "y(t)".

Two useful commands are "DEplot" and "dsolve".

DEplot

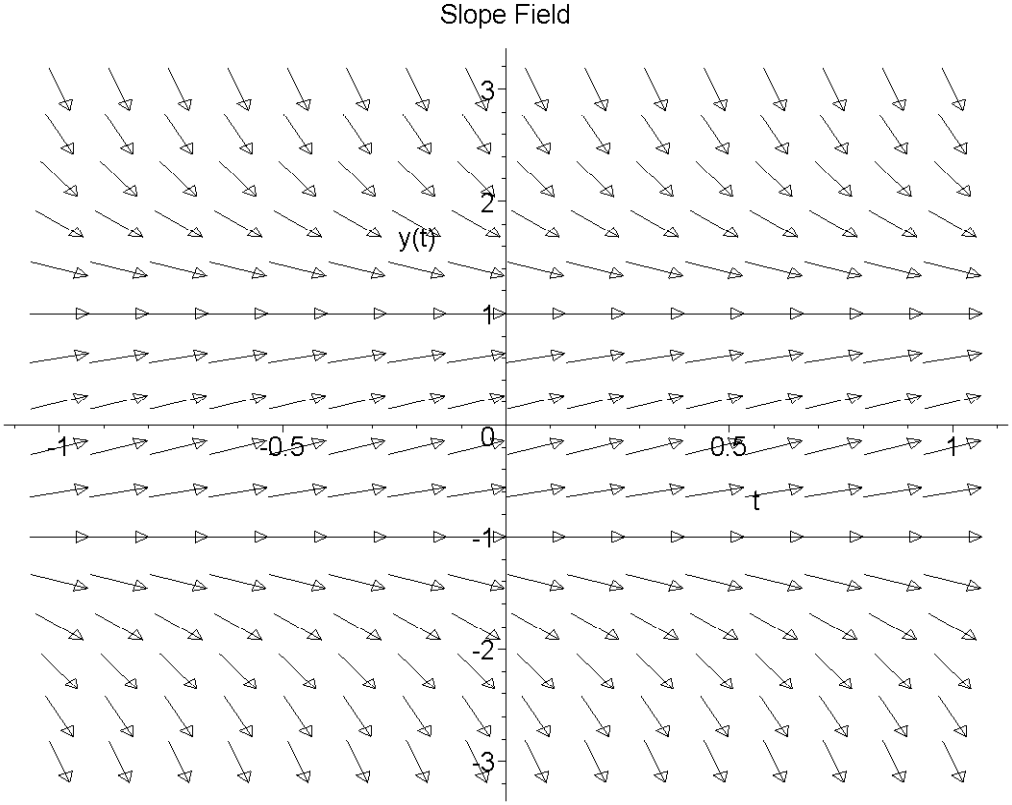
Let's create a slope field (also known as a "direction field") for this equation. One Maple command that does this is called "DEplot". In the first example given here, the first argument is the differential equation, the second is the function in the differential equation, the third is the range the independent variable, and the fourth is the range of the dependent variable. The option "arrows=THIN" tells the command to show the slope field with thin arrows, and "color=black" makes the arrows black.

```
> DEplot(eq,y(t),t=-1..1,y=-3..3,arrows=THIN,color=black);
```



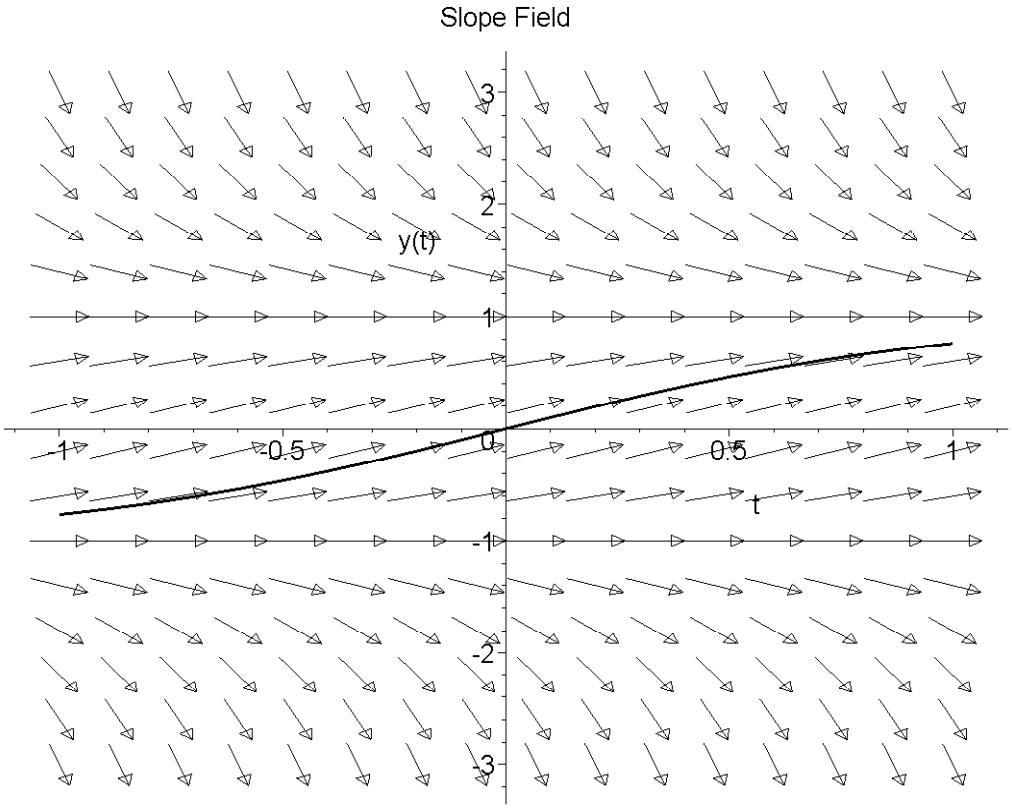
Here is the same example, with some additional arguments included. For example, the "dirgrid" option lets you decide how many arrows to include.

```
> DEplot(eq,y(t),t=-1..1,y=-3..3,title=`Slope
  Field`,arrows=MEDIUM,color=black,dirgrid=[16,16]);
```



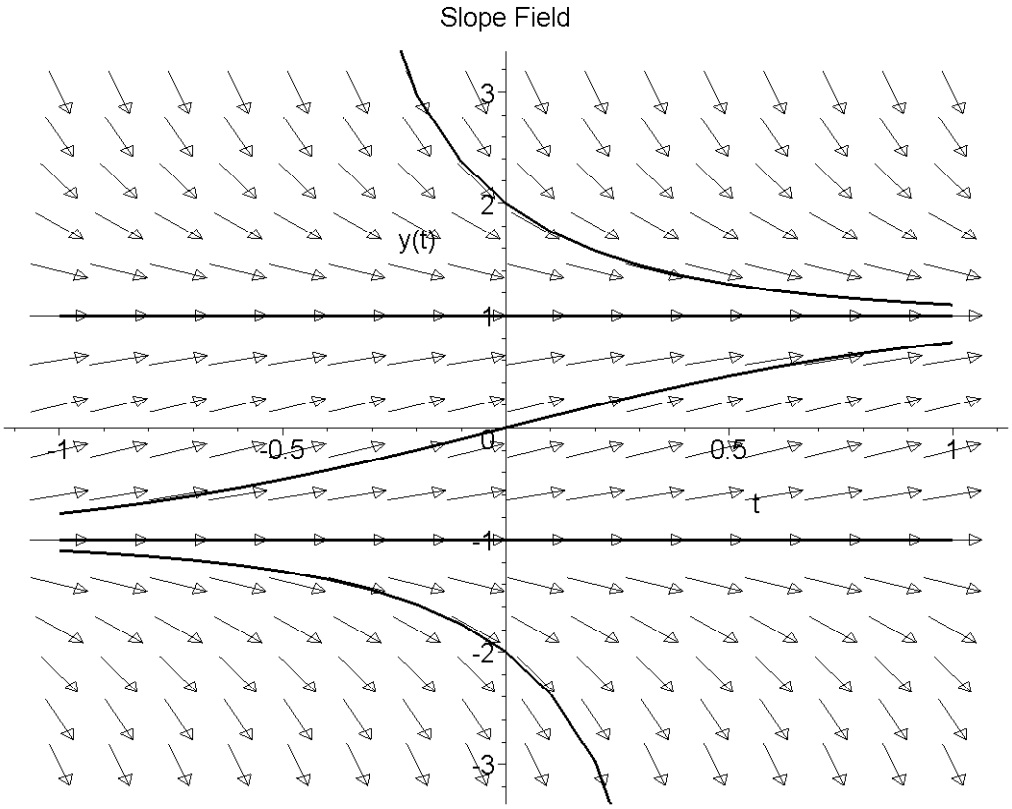
Let's add a solution curve to this plot. To do this, we must specify an initial condition. Let's try y(0)=0. To specify initial conditions in DEplot, you put them in a list, contained in square brackets, just after the t range. The added option "linecolor=black" tell the command to draw the solution curve in black.

```
> DEplot(eq,y(t),t=-1..1,[[y(0)=0]],y=-3..3,title=`Slope
  Field`,arrows=MEDIUM,color=black,dirgrid=[16,16],linecolor=black);
```



Note that I used double square brackets around the initial condition. This is because you can enter several initial conditions; each must be in square brackets:

```
> DEplot(eq,y(t),t=-1..1,[[y(0)=-2],[y(0)=-1],[y(0)=0],[y(0)=1],[y(0
  )=2]],y=-3..3,title=`Slope
  Field`,arrows=MEDIUM,color=black,dirgrid=[16,16],linecolor=black);
```



Important Note! The solution curves plotted by the DEplot command have been computed *numerically*, using an algorithm similar to Euler's Method. They are generally good approximations, but they are not exact, and they may occasionally be very bad approximations.

The Dsolve Command

The command "dsolve" can solve some differential equations *analytically*. Let's start out with a simple example that we are quite familiar with:

y' = -y.

First, we'll define the equation, and save it in a variable called "eq2":

```
> eq2 := diff(y(t),t) = -y(t);
```

$$eq2 := \frac{\partial}{\partial t} y(t) = -y(t)$$

Now we'll use "dsolve" to find the general solution. The first argument of "dsolve" is the equation, and the second is the function to solve for:

```
> dsolve(eq2,y(t));
```

$$y(t) = _C1\, e^{(-t)}$$

This should look familiar! Notice that Maple has called the arbitrary constant "_C1". This is a common Maple convention; when it creates a constant, it typically put an underscore in the beginning of the name.

We can also use "dsolve" to solve an initial value problem. For example, suppose

y(0)=2.

We specify this in the command by combining the differential equation with the initial condition in "curly brackets", thus creating the initial value problem:

```
> dsolve({eq2,y(0)=2},y(t));
```

$$y(t) = 2\,e^{(-t)}$$

Look at that example carefully. The first argument is {eq2,y(0)=2}. In Maple, curly brackets are commonly used to create lists of things. In this case, we make a list of the equation and the initial condition, and pass this to "dsolve" as its first argument. The second argument, y(t), stays the same.

How do we plot this solution? Unfortunately, the solution given above is not in a format that is easily plotted by the "plot" command. The plot command plots expressions, but the above solution is actually an equation (it has an equals sign sitting the middle). The expression that we want, namely 2*exp(-t), is the right-hand-side of the equation. Fortunately, there is a command in Maple that lets us take the right-hand-side from an equation. The function is called "rhs".

So here is one way to plot the solution above. First, save the solution in a variable, say "sol2" (because it solves eq2):

```
> sol2 := dsolve({eq2,y(0)=2},y(t));
```

$$sol2 := y(t) = 2\,e^{(-t)}$$

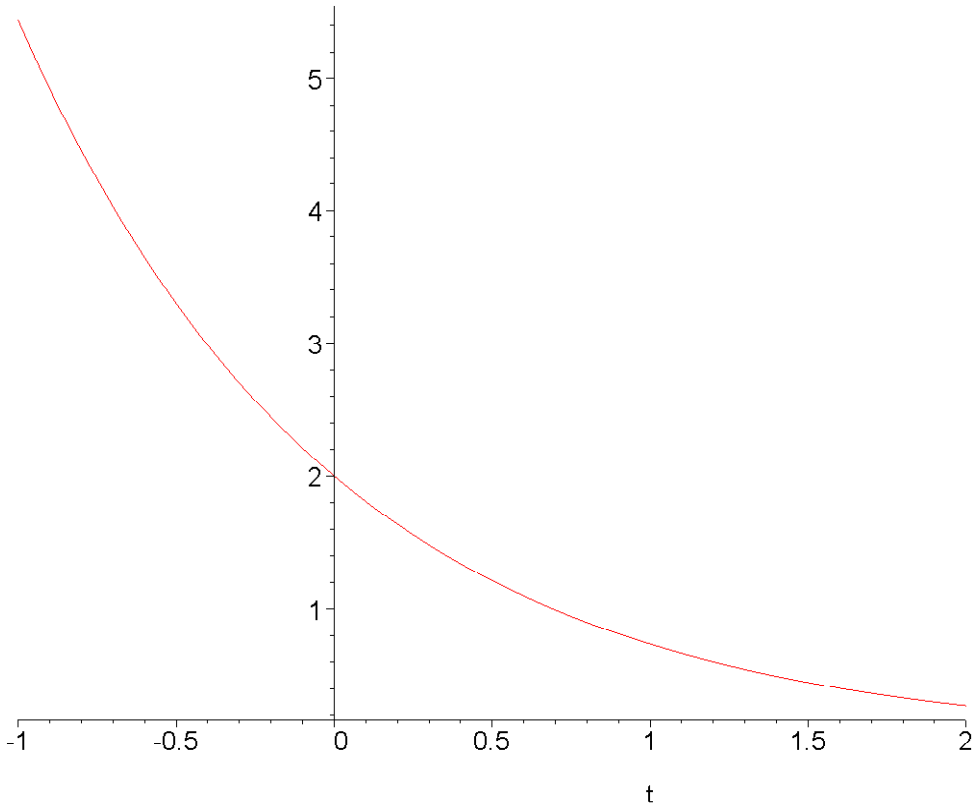
Now take out the right-hand-side, and save it in a new variable called rhsSol2:

```
> rhsSol2 := rhs(sol2);
```

$$rhsSol2 := 2\,e^{(-t)}$$

Now plot it:

```
> plot(rhsSol2,t=-1..2);
```

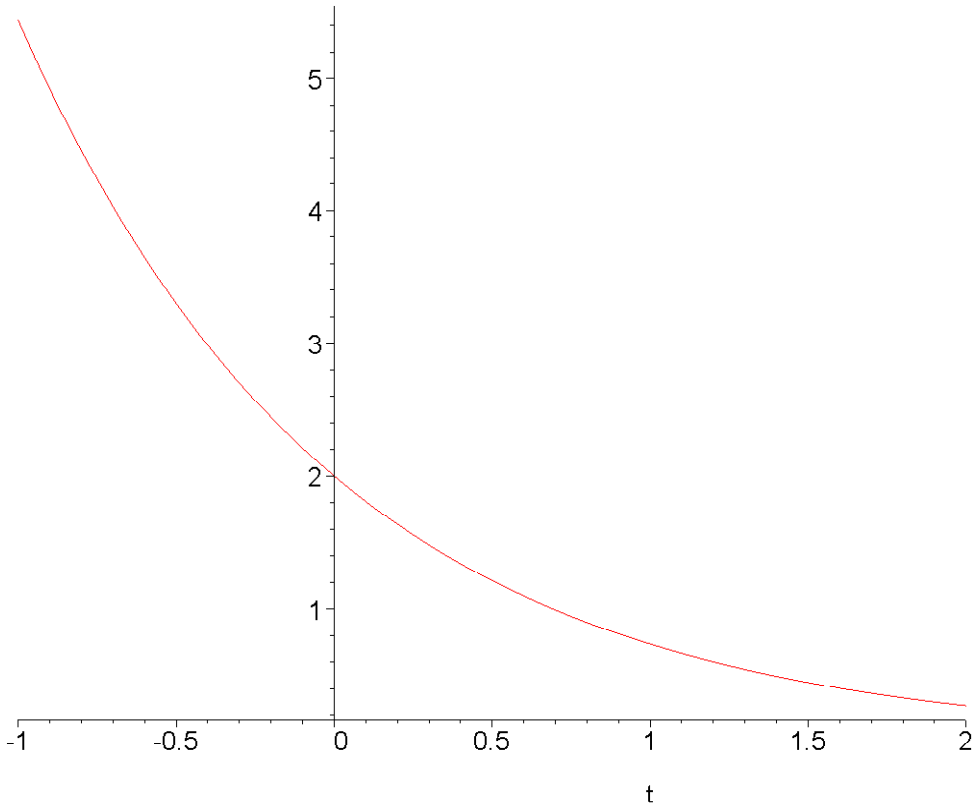


You can combine several of those steps:

```
> sol2 := rhs(dsolve({eq2,y(0)=2},y(t)));
```

$$sol2 := 2\,\mathbf{e}^{(-t)}$$

```
> plot(sol2,t=-1..2);
```



Let's try another example. Recall that "eq" is still defined:

```
> eq;
```

$$\frac{\partial}{\partial t}\mathrm{y}(t) = 1 - \mathrm{y}(t)^2$$

Let's give this to "dsolve":

```
> sol1 := rhs(dsolve(eq,y(t)));
```

$$sol1 := \tanh(t + _C1)$$

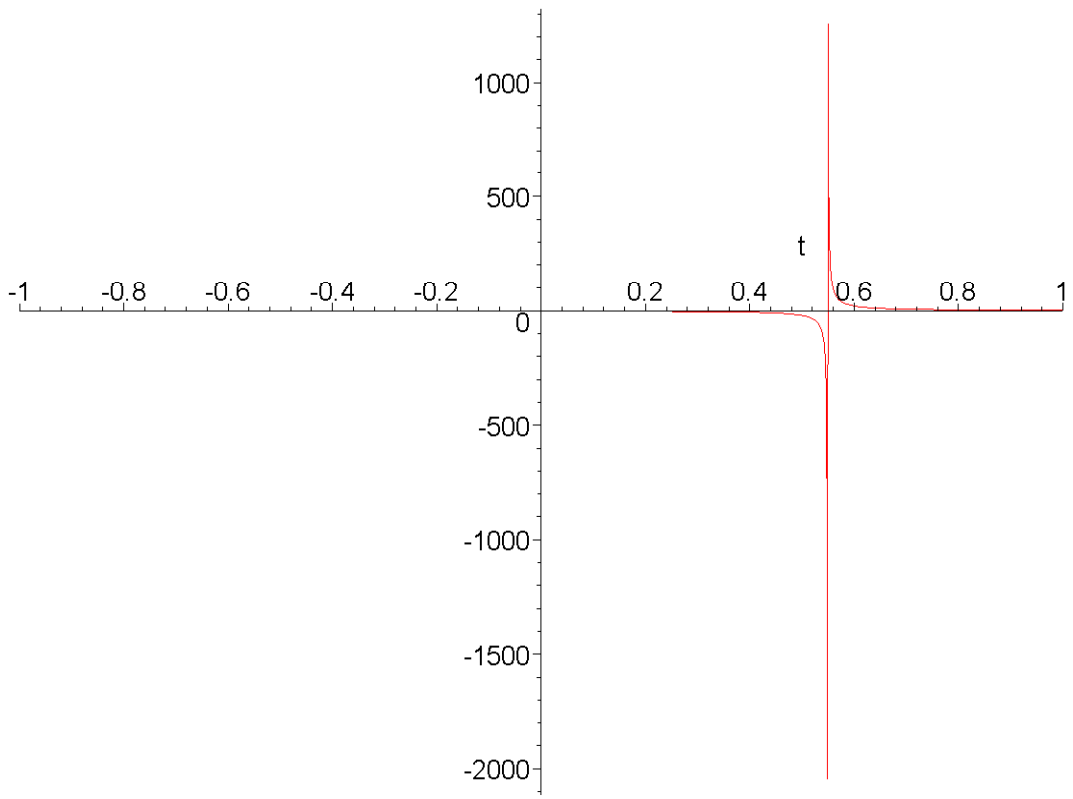
So the solution is the hyperbolic tangent function. Does this really give us all the solutions that we expect?

Let's try some of the same initial conditions that we used earlier:

```
> sol1a := rhs(dsolve({eq,y(0)=-2},y(t)));
```

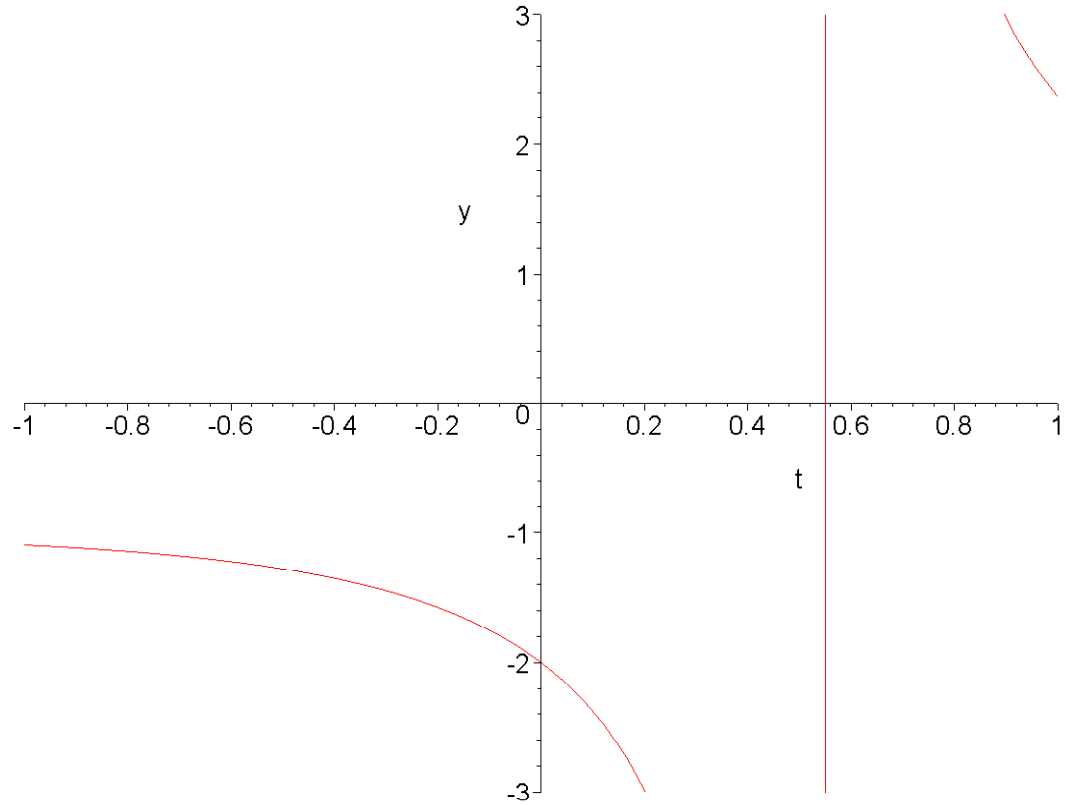
$$sol1a := \coth\left(t - \frac{1}{2}\ln(3)\right)$$

```
> plot(sol1a,t=-1..1);
```



What is going on here? This doesn't look like anything we saw with the "DEplot" command. Let's restrict the y range to be the same as the slope field plots shown earlier:

```
> plot(sol1a,t=-1..1,y=-3..3);
```



Aha! We see the solution where y(0)=-2. What is happening is that this solution "blows up" in finite

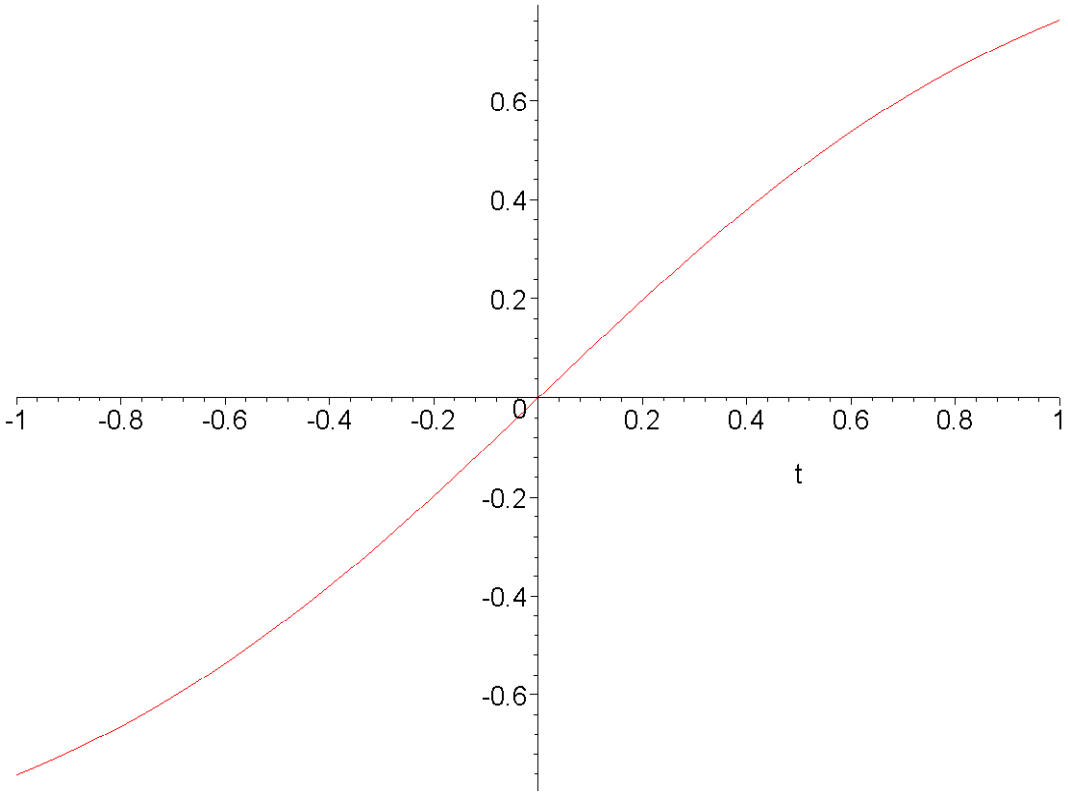
time. Maple doesn't know that we don't care about what happens after this point, so it also plots the function for values of t after the vertical asymptote.

Let's try the initial condition y(0)=0. This one shouldn't have any vertical asymptotes.

```
> sol1b := rhs(dsolve({eq,y(0)=0},y(t)));
```

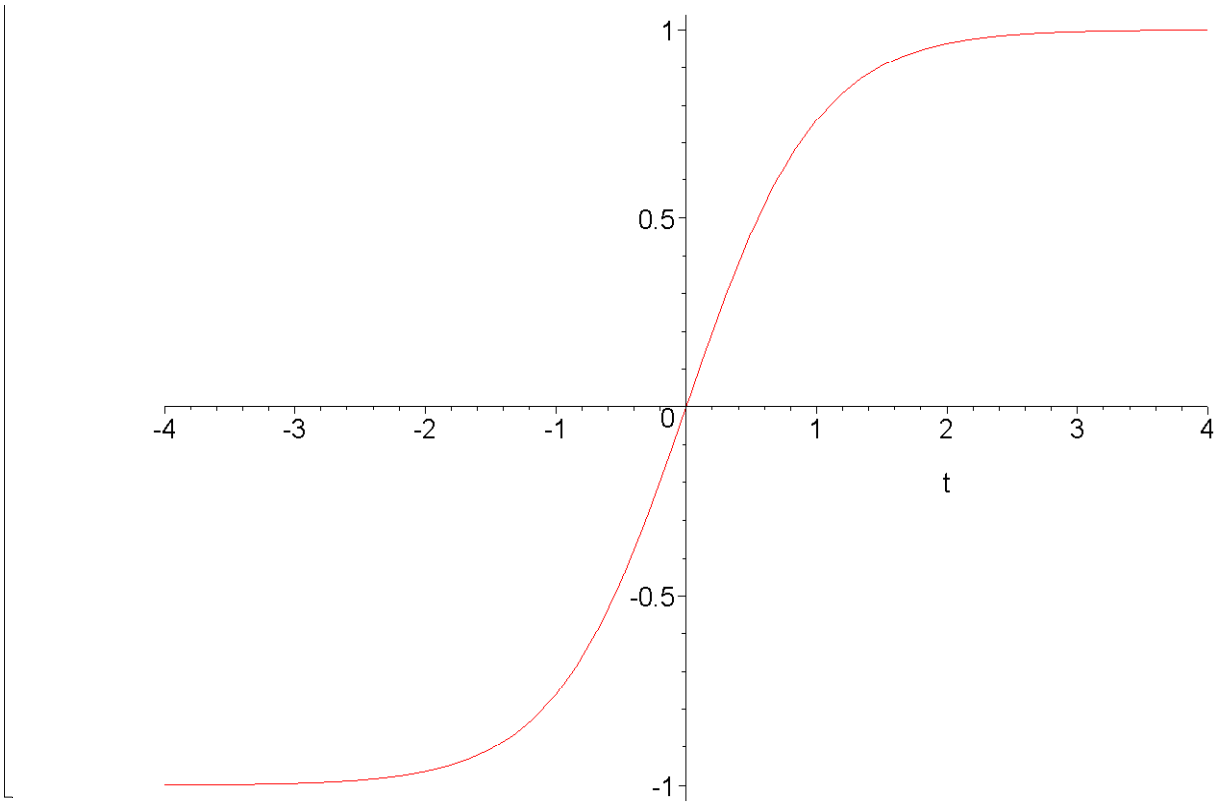
$$sol1b := \tanh(t)$$

```
> plot(sol1b,t=-1..1);
```



This one is well-behaved. If we use a longer range of time, we can clearly see what happens as t becomes very large or as -t becomes very large:

```
> plot(sol1b,t=-4..4);
```



Finally, let's put an initial condition into "dsolve" that should give us an equilibrium solution:

```
> sol1c := rhs(dsolve({eq,y(0)=1},y(t)));
```

$$sol1c := 1$$

That worked as expected.

Now, recall that the general solution given by dsolve was tanh(t+_C1). Unfortunately, this is only the correct solution if -1 < y(0) < 1. This is a example where we must be aware of Maple's quirks if we are to use it successfully.

Can Maple solve any first order differential equation? Unfortunately, no. Here is a simple example:

```
> eq3 := diff(y(t),t) = sin(y(t))+cos(t);
```

$$eq3 := \frac{\partial}{\partial t} y(t) = \sin(y(t)) + \cos(t)$$

```
> dsolve(eq3,y(t));
```

The function returns nothing; Maple can not solve this equation.