The Mass-Spring System

Math 308

This Maple session uses the mass-spring system to demonstrate the phase plane, direction fields, solution curves (``trajectories''), the extended phase space, and more.

```
> with(DEtools):
```

We first define the two equations that make up the system:

```
> eq1 := diff(y(t),t) = v(t);
```

$$eq1 := \frac{\partial}{\partial t} y(t) = v(t)$$

```
> eq2 := diff(v(t),t) = -(b/m)*v(t)-(k/m)*y(t);
```

$$eq2 := \frac{\partial}{\partial t} v(t) = -\frac{b\, v(t)}{m} - \frac{k\, y(t)}{m}$$

Now define a variable that holds the system:

```
> sys := [eq1,eq2];
```

$$sys := \left[\frac{\partial}{\partial t} y(t) = v(t), \frac{\partial}{\partial t} v(t) = -\frac{b\, v(t)}{m} - \frac{k\, y(t)}{m}\right]$$

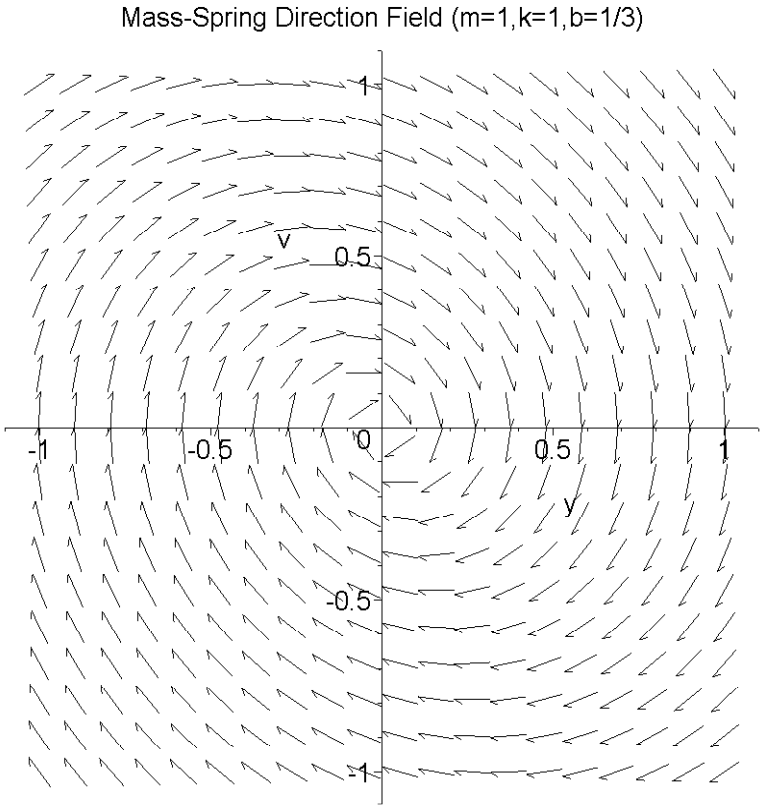
We substitute in some numerical values for the parameters, so we can plot some solutions.
(The "subs" command makes the substitutions given by the first several arguments in the expression given as the last argument.)

```
> sys1 := subs(b=1/3,m=1,k=1,sys);
```

$$sys1 := \left[\frac{\partial}{\partial t} y(t) = v(t), \frac{\partial}{\partial t} v(t) = -\frac{1}{3} v(t) - y(t)\right]$$

Let's first look at the direction field. This is the same as the vector field, but all the vectors are plotted with the same length. We use the DEplot command.

```
> DEplot(sys1,[y(t),v(t)],t=0..1,y=-1..1,v=-1..1,scaling=constrained
  ,color=black,title=`Mass-Spring Direction Field (m=1,k=1,b=1/3)`);
```

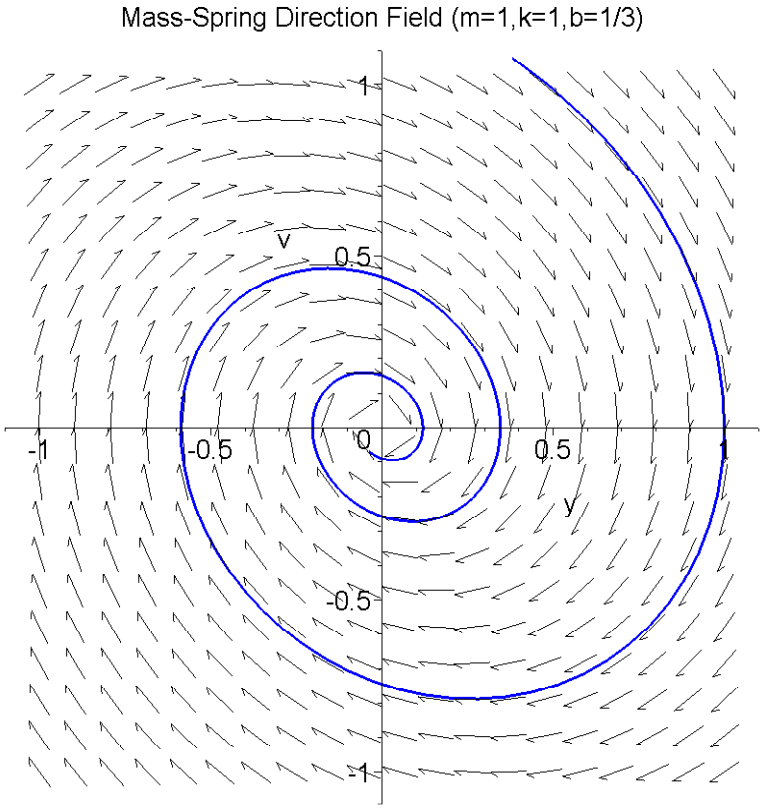


Let's add a solution curve, also called a *trajectory.* We will find the curve with initial condition y(0)=1, v(0)=0.

```
> ics := [[0,1,0]];
```

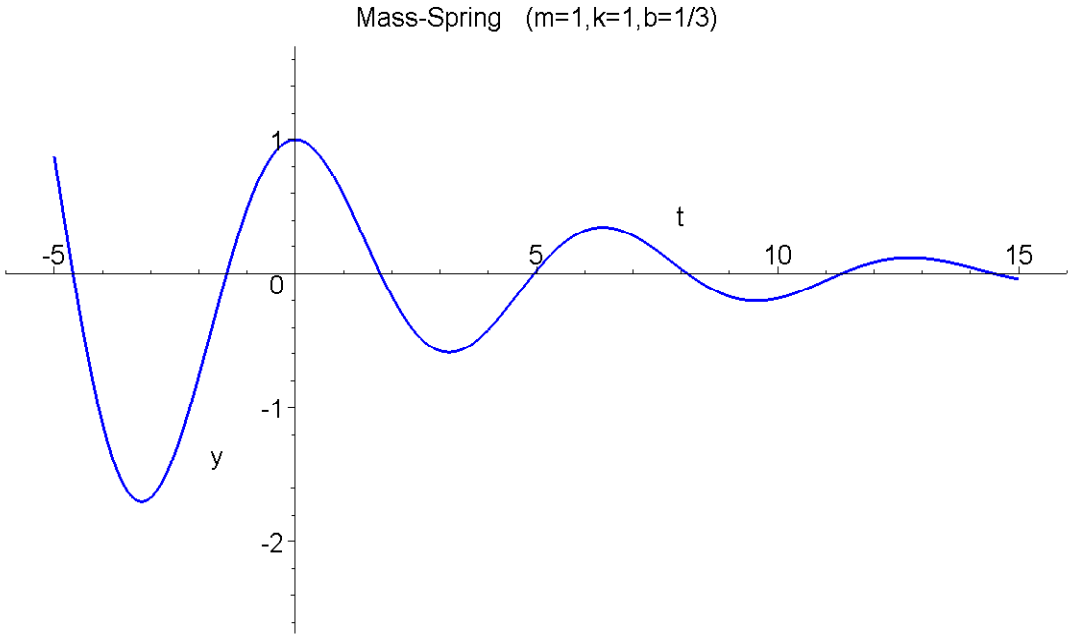
$$ics := [[0, 1, 0]]$$

```
> DEplot(sys1,[y(t),v(t)],t=-5..15,ics,y=-1..1,v=-1..1,scaling=const
  rained,color=black,title=`Mass-Spring Direction Field
  (m=1,k=1,b=1/3)`,linecolor=blue,stepsize=.1);
```



Let's plot y(t) vs. t for the same solution. The option "scene=[var1,var2]" let's us choose how the solution is plotted. If we say "scene=[t,y]", we get a plot of y(t) vs. t.

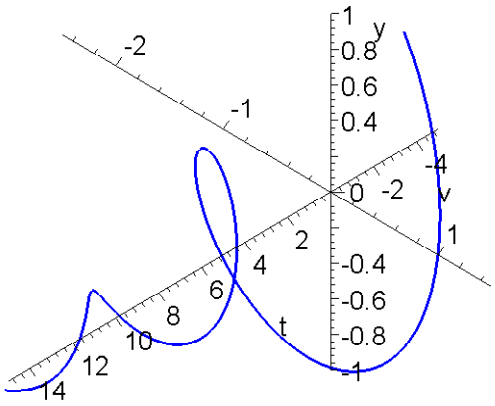
```
> DEplot(sys1,[y(t),v(t)],t=-5..15,ics,y=-2.5..1.5,v=-1..1,color=black,title=`Mass-Spring (m=1,k=1,b=1/3)`,linecolor=blue,stepsize=.1,scene=[t,y]);
```



Now let's see this solution in the extended phase space.

```
> DEplot3d(sys1,[y(t),v(t)],t=-5..15,ics,y=-2.5..1.5,v=-1..1,color=b
  lack,title=`Mass-Spring Extend Phase Space
  (m=1,k=1,b=1/3)`,linecolor=blue,stepsize=.1,scene=[t,y,v],axes=nor
  mal);
```

Mass-Spring Extend Phase Space (m=1,k=1,b=1/3)




```
>
```