Math 308 - Differential Equations **Fall 2002**

Homework Assignment 1

Due *Friday, September 13.*

Note: You will need a calculator for some of these problems. In calculations requiring several steps, keep at least four significant digits in the intermediate steps.

Text problems:

- Section 1.1, Exercise 4. (See the population model discussed in the text for one method to determine the constant k.)
- Section 1.1, Exercises 8, 9, and 12. In Exercise 9, first *find a formula for the decay rate in terms of an arbitrary half-life.* In other words, if h is the half-life, express λ as a function of h. Then you can use the formula to answer 9(a) and 9(b). Note that the answer to Exercise 9 is in the back of the book; in particular, the answer to 9(d) is "yes", but you must (briefly) explain why.
- Section 1.1, Exercise 14. (Do Exercise 13 first, and check your answer with the answer in the back of the book.)
- Section 1.2, Exercises 16, 18, 26, 34.

Additional problems:

1. Air is pumped into a spherical balloon at a constant rate of 10 cubic centimeters per second.
   (a) Find the differential equation for the volume v of the balloon.
   (b) Find the differential equation for the radius r of the balloon.
   (c) Solve the differential equations from (a) and (b). You can solve them individually, or you can use the solution of one to find the solution of the other.

   (The volume of a balloon with radius r is $4\pi r^3/3$.)

2. A common model for drug absorption in the body is that, after receiving a dose (by an injection, say), the drug will be "cleared" from the blood at a rate that is proportional to the amount present. In other words, if $a(t)$ is the amount of the drug in the bloodstream at time t (in hours), then
$$\frac{da}{dt} = -ka,$$
   where $k > 0$ is the proportionality constant.
   (a) Suppose that at $t = 0$, a person is given 5 units of the drug, and 24 hours later the amount of the drug remaining in the bloodstream is found to be 1.2 units. Use this data to determine k.
   (b) After the first 24 hours, the patient is again given a dose of 5 units. What will be the amount of the drug in the bloodstream at the end of the next 24 hours?
   (c) Suppose the patient continues to receive 5 units every 24 hours. What will the be the amount of the drug in the bloodstream just before each dose is given? Does this amount become arbitrarily large, or will it approach some limiting value?