Math 308 - Differential Equations **Fall 2002**

Homework Assignment 3

Due *Friday, September 27.*

1. Consider the initial value problem
$$\frac{dy}{dt} = \frac{y}{t}$$
   (a) Determine the region in the (t, y) plane where the Existence and Uniqueness Theorems apply.
   (b) Pick 16 or so points in the (t, y) plane, and plot a slope field. Include some negative values of t and y. Based on the slope field, can you guess the solutions?
   (c) Solve the differential equation (it is separable). Sketch the solutions if $y(1) = 0$, $y(1) = 1$, or $y(1) = 2$.
   (d) What is $y(0)$ for each of the solutions that you found in (c)? Does this contradict the Uniqueness Theorem?

2. Find *all* the solutions to the initial value problem
$$\frac{dy}{dt} = 5y^{\frac{4}{5}}, \qquad y(0) = 0.$$

3. For each of the following differential equations: sketch the phase line; find the equilibrium points and classify them as sinks, sources, or nodes; in one graph, sketch the equilibrium solutions along with several representative solution curves versus t. (Note: You do not have to solve the differential equations analytically.)
   (a) $\dfrac{dy}{dt} = y^2 - 6y - 16$
   (b) $\dfrac{dy}{dt} = y^2 + 2y + 10$
   (c) $\dfrac{dy}{dt} = (y + 2)(y - 1)^2$
   (d) $\dfrac{dy}{dt} = -2y + \sin y$

4. In Problem 3 of Homework 2, you solved the problem of an object falling that is also acted on by friction. The differential equation was
$$\frac{dv}{dt} = -g + \frac{F_d(v)}{m},$$
   where g is the gravitational acceleration ($g > 0$), m is the mass of the object, and $F_d(v)$ is the friction force for velocity v. The three forms of friction considered were $F_d(v) = 0$, $F_d(v) = -C_d v$ and $F_d(v) = -C_d v|v|$.

Now consider one more common model of friction: $F_d(v) = -C_d v^3$, where, as before, $C_d > 0$ is a constant. The differential equation becomes

$$\frac{dv}{dt} = -g - \frac{C_d}{m} v^3.$$

(a) Sketch the phase line for this equation. Find and classify the equilibria.

(b) Does this model show that there is a "terminal velocity"? If so, what is the formula for the terminal velocity?

Text Problems:

- Section 1.5/ 11, 14, 17
- Section 1.6/ 41, 43

Some notes on the problems from the text:
1.5/11: "Continuously differentiable" means that f and its first derivative are continuous.
1.6/41: Check out the hint in the back of the book.
1.6/43: The answers are in the back of the book; explain how you get these answers.