Homework Assignment 7

Due *Friday, November 1.*

For Questions 1-4,

(a) Find the general solution to the system $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$.

(b) Sketch the phase portrait. Include the "straight line" solutions, and sketch several additional qualitatively correct trajectories in the phase plane. (I recommend that you check your answer with Maple or with the software on the CD provided with the text. An example of using Maple to create a phase portrait is attached.)

(c) Classify the equilibrium point of the system as either a saddle, a source or a sink. (The matrices have been chosen so that these are the only possibilities.)

(d) Solve the initial value problem $\vec{Y}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Draw this solution in your phase portrait in part (b). (Use a different color or use a dashed line to indicate this solution.) Also sketch $x(t)$ and $y(t)$ (where $\vec{Y} = \begin{bmatrix} x \\ y \end{bmatrix}$) for this solution, including positive and negative values for t.

1. $A = \begin{bmatrix} 1 & -1 \\ -6 & -4 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$
3. $A = \begin{bmatrix} 1/2 & -1 \\ -5/2 & -4 \end{bmatrix}$
4. $A = \begin{bmatrix} -4 & -1 \\ -2 & -3 \end{bmatrix}$

5. Suppose that for some matrix A, $A\vec{v}_1 = \lambda_1\vec{v}_1$, and $A\vec{v}_2 = \lambda_2\vec{v}_2$, and $\lambda_1 \neq \lambda_2$. Show that $\vec{v}_1$ and $\vec{v}_2$ are linearly independent. (In other words, eigenvectors associated with *different* eigenvalues are always linearly independent.)

   Hint: Consider the equation

   $$k_1\vec{v}_1 + k_2\vec{v}_2 = \vec{0}. \tag{1}$$

   To show that $\vec{v}_1$ and $\vec{v}_2$ are linearly independent, we must show that the only solution to this equation is $k_1 = k_2 = 0$. As a start, consider multiplying both sides of (1) by A to get a new equation. Then consider multiplying (1) by, say, λ_1 to get another new equation. Now subtract one of the new equations from the other new equation, and see where that leads.

More on the other side!

Text Problems

- Section 3.1/ 35 (A brief solution is in the book. Give a more detailed solution in your own words.)
- Section 3.2/ 15, 16, 17, 18

Exercises - Do Not Hand In – Check the answers in the back of the book.

- Section 3.2/ 1–9 odds
- Section 3.3/ 1–7 odds