Homework Assignment 8

Due *Friday, November 8.*

For Questions 1-3,

(a) Find the general solution to the system $\frac{d\vec{Y}}{dt} = A\vec{Y}$.

(b) Sketch the phase portrait. Use nullclines to help make reasonably accurate plots of the trajectories.

(c) Classify the equilibrium point of the system as either a spiral sink, a spiral source, or a center. (The matrices have been chosen so that these are the only possibilities.)

(d) Solve the initial value problem $\vec{Y}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Draw this solution in your phase portrait in part (b). (Use a diferent color or use a dashed line to indicate this solution.) Also sketch $x(t)$ and $y(t)$ (where $\vec{Y} = \begin{bmatrix} x \\ y \end{bmatrix}$) for this solution, including positive and negative values for t.

1. $A = \begin{bmatrix} 2/3 & 4 \\ -4 & 2/3 \end{bmatrix}$

2. $A = \begin{bmatrix} 0 & 2 \\ -2 & -2 \end{bmatrix}$

3. $A = \begin{bmatrix} -2 & 3 \\ -3 & 2 \end{bmatrix}$

4. Let $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $b \neq 0$. Show that A must have complex eigenvalues.

5. Recall the second order differential equation for the mass-spring system:

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0,$$

where $m > 0$, $b \geq 0$, and $k > 0$.

   (a) Let $v = \frac{dy}{dt}$, and convert this equation into a 2×2 first order system for $\vec{Y}(t) = \begin{bmatrix} y(t) \\ v(t) \end{bmatrix}$.

   (b) What conditions on m, b, and k will ensure that $\vec{Y}(t) \to \vec{0}$ as $t \to \infty$?

   (c) Under what conditions on m, b, and k will the solutions exhibit a decaying oscillation?

For Questions 6-8,

(a) Find the general solution to the system $\frac{d\vec{Y}}{dt} = A\vec{Y}$.

(b) Sketch the phase portrait.

(d) Solve the initial value problem $\vec{Y}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Draw this solution in your phase portrait in part (b). (Use a different color or use a dashed line to indicate this solution.)

6. $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

8. $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

Recommended Exercises - Do Not Hand In – Check the answers in the back of the book.

- Section 3.4/ 1–13 odds
- Section 3.5/ 1–7 odds, 17, 19