Math 308 - Differential Equations **Fall 2002**

Homework Assignment 10

Due *Friday, December 6*

1. For each of the following second order differential equations, (i) use the method of undetermined coefficients to find the general solution, and then (ii) solve the initial value problem.

   (a) $y'' + 5y' + 4y = \sin(2t), \qquad y(0) = 1,\ y'(0) = -2$

   (b) $3y'' + 8y' + 5y = e^{-3t}\cos(t), \qquad y(0) = 0,\ y'(0) = 0$

   (c) $y'' + 9y = 6\cos(3t), \qquad y(0) = 1,\ y'(0) = 3$

2. For each of the following second order differential equations, determine if the method of undetermined coefficients can be used to find y_p. If so, write down the appropriate form for $y_p(t)$, but *do not solve for the coefficients.*

   (a) $y'' + 5y' + 4y = t^2\sin(3t)$

   (b) $y'' + 8y' + y = \ln(1 + t^2)$

   (c) $y'' + 2y' + y = te^t$

   (d) $y'' + 2y' + y = te^{-t}$

   (e) $y'' + 3y' + 2y = \dfrac{1}{1 + t^2}$

   (f) $y'' + 9y = (t^2 + 3)\sin(2t)$

   (g) $y'' + 4y' + 4y = 8$

3. (a) Suppose we have the equation

   $$y'' + py' + qy = g_1(t) + g_2(t). \tag{1}$$

   Show that if y_{p_1} is a particular solution to

   $$y'' + py' + qy = g_1(t).$$

   and y_{p_2} is a particular solution to

   $$y'' + py' + qy = g_2(t).$$

   then $y_{p_1} + y_{p_2}$ is a particular solution to (1).

   (b) Use the result of (a) to solve the initial value problem

   $$y'' + 3y' + 2y = 2\sin(t) + 10\sin(2t), \qquad y(0) = -3,\ y'(0) = 0.$$

4. Show that the expression $A\cos(\omega t) + B\sin(\omega t)$ may be written as $R\cos(\omega t - \phi)$, where $R = \sqrt{A^2 + B^2}$, and $\tan\phi = B/A$ (assuming $A \neq 0$). (The quantity R is the *amplitude* and ϕ is the *phase* of the sinusoidal function.)

   Hint: Use the trigonometric identity

   $$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

   and choose ϕ so that $A = R\cos(\phi)$ and $B = R\sin(\phi)$.

5. Use the trigonometric identities $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ with $a = (\omega_0 + \omega)t/2$ and $b = (\omega_0 - \omega)t/2$ to show that

$$\cos(\omega t) - \cos(\omega_0 t) = 2 \sin\left(\frac{(\omega_0 + \omega)t}{2}\right) \sin\left(\frac{(\omega_0 - \omega)t}{2}\right).$$

   (We used this formula in class in the discussion of *beats* in the undamped forced harmonic oscillator.)

6. A mass-spring system with $k = 16$ and $b = 1$ is acted on by an external force of $4 \cos 2t$. (Assume that all the units are consistent.)
   (a) Suppose the mass is $m = 1$. Find the steady state response of this system.
   (b) Determine the value of the mass m for which the amplitude of the steady state response is a maximum.

Text Problems

- Section 4.3/ 14, 18 (see note 1), 20–23 (see note 2)

Notes on the text problems

1. Change the wording of 18(c) from "rough sketch of a typical solution" to "rough sketch of the solution where $y(0) = 0$ and $y'(0) = 0$".
2. The gaps in the plot in Exercise 23 should not be there.