Homework Assignment 11

Due *Friday, December 13*

1. Consider the nonlinear system

$$\frac{dx}{dt} = x(1 - x - y)$$
$$\frac{dy}{dt} = y(3 - 2x - y)$$

   (a) Find all the equilibrium points.

   (b) Find the linearization at each equilibrium, classify the equilibrium point, and sketch the phase portrait for the linearized system.

   (c) Sketch the phase portrait for the nonlinear system. Use the results of (b) to determine the phase portrait near the equilibrium points. Use nullclines to help determine what happens elsewhere. Sketch and label the separatrices associated with any saddle points. (You may use Maple or the text's CD to check your answer, but you do not have to hand in a computer generated phase portrait.)

   (d) Consider the solution $(x(t), y(t))$ for which $x(0) = 1/2$ and $y(0) = 1/2$. Use your phase portrait in (c) to determine each of the following: $\lim_{t\to\infty} x(t)$, $\lim_{t\to-\infty} x(t)$, $\lim_{t\to\infty} y(t)$, $\lim_{t\to-\infty} y(t)$.

2. Repeat parts (a)-(d) of the previous question for the system

$$\frac{dx}{dt} = (x^3 - 1)(1 - y)$$
$$\frac{dy}{dt} = y(2x - 1)$$

3. Consider the linear system

$$\frac{dx}{dt} = ax,$$
$$\frac{dy}{dt} = by,$$

   where a and b are constants.

   (a) Show that the function $g(x, y) = x^{-b}y^a$ is a conserved quantity.

   (b) What condition on a and b will make this a Hamiltonian system?

Continued on the back...

Text Problems:

- Section 5.1/ 30
- Section 5.3/ 4, 5 (see note 1), 10, 12, 14

Notes for the text problems:

1. The answer in the back of the book for 5.3/5 is "No, longer." Give a more detailed explanation than this.

Exercises - do not hand in - check the answers in the back of the book

- Section 5.1/ 5, 7, 15, 17, 27
- Section 5.2/ 1, 3, 5, 13
- Section 5.3/ 1, 3