Math 308 - Differential Equation
Final Exam Topics

1. The basics
   - What is a differential equation (DE)?
   - What is a solution to a differential equation?
   - What is an initial value problem (IVP)?
   - Classifying differential equations: ordinary or partial, order, linear or nonlinear, autonomous or nonautonomous.
2. First order differential equations
   - Slope field
   - Analytical techniques
     - Separable equations: identify and solve.
     - Linear equations: identify and solve (integrating factor).
   - Qualitative techniques for autonomous first order equations $y' = f(y)$
     - Phase line (including interpretation of the graph of $f(y)$ vs. y)
     - Critical points (i.e. equilibrium solutions)
     - Stability of equilibria: asymptotically stable, unstable, semistable (Problem 7, p. 84)
   - Theory: existence, uniqueness
   - Modeling and applications
     - Convert a verbal description of a process into a differential equation.
     - Applications: exponential population growth, drug clearance, mixing, Newton's law of motion and the velocity of an object, Newton's law of cooling, logistic equation.
   - Euler's Method for numerical approximation (also applies to systems)
3. Linear second order differential equations
   - Homogeneous or nonhomogeneous, constant coefficients or variable (i.e. "time"-dependent) coefficients
   - Theory: principle of superposition, fundamental set of solutions, linear independence, Wronskian
   - Find the general solution to $ay'' + by' + cy = 0$ in all cases; solve initial value problems.
   - Nonhomogeneous: the method of undetermined coefficients
   - Mechanical vibrations
     - Mass-spring system: mass, damping or friction coefficient, spring constant.
     - Free vibrations (i.e. no external forcing), damped or undamped; natural frequency, "quasi-frequency"
     - Forced vibrations (external forcing, beats, resonance, transient response, steady state response, amplitude, phase shift)
4. Systems of first order equations (topics applicable to linear or nonlinear systems)
   - What is a system of first order differential equations?
   - What is a solution to a system of first order differential equations?
   - Convert a second order differential equation into a first order system.

5. Phase plane for autonomous first order systems (linear or nonlinear)
    - Vector field and direction field
    - Understand how to go back and forth between a solution plotted in the phase plane and plots of the components of a solution as functions of time.
    - Critical points (i.e. equilibrium solutions)
    - Nullclines
6. Systems of first order linear differential equations
    - Linear algebra background: solving linear algebraic equations, linear independence of vectors, eigenvalues and eigenvectors
    - General solution to $\vec{\mathbf{x}}' = A\vec{\mathbf{x}}$ for any 2×2 real matrix A.
    - Phase portraits for linear systems
    - Classification of critical points
        - Stability: asymptotically stable, stable, unstable
        - Type: saddle, (nodal) sink, (nodal) source, spiral sink, spiral source, center, proper node, improper node, "zero eigenvalue"
7. Nonlinear systems of first order differential equations
    - Find critical points.
    - Linearization at critical points (Jacobian matrix)
    - Classification of critical points based on the linearization. (What does the linearization tell us about the nonlinear system near the critical point?)
    - Separatrices (trajectories that approach a saddle as $t \to \infty$ or $t \to -\infty$)
    - Find nontrivial solutions that correspond to special cases (if possible). (An example is the behavior of trajectories on the x and y axes of the competing species model.)
    - Check for a function $H(x, y)$ whose level curves $H(x, y) = c$ are trajectories. (Examples include the predator/prey system and the pendulum with no friction.)
    - Use all of the above to create a qualitatively correct phase portrait for a nonlinear system.
    - Examples: pendulum, competing species, predator/prey.

Text chapters:

- 1.1 – 1.3
- 2.1 – 2.5, 2.7, 2.8 (but not in as much detail as the text)
- 3.1 – 3.4, 3.5 (but skip "reduction of order"), 3.6, 3.8, 3.9
- 7.1 – 7.6, 7.8. Also, we covered zero eigenvalues in more detail than the text.
- 9.1 – 9.3 (but we did not specifically talk about "almost linear"; we focused on linearization using the Jacobian matrix), 9.4, 9.5.